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CASIMIR FORCE, CASIMIR OPERATORS AND THE RIEMANN HYPOTHESIS

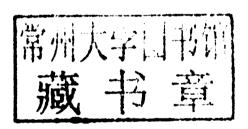
MATHEMATICS FOR INNOVATION IN INDUSTRY AND SCIENCE



Casimir Force, Casimir Operators and the Riemann Hypothesis

Mathematics for Innovation in Industry and Science

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Preface

This volume contains the proceedings of the conference 'Casimir Force, Casimir Operators and the Riemann Hypothesis – Mathematics for Innovation in Industry and Science', held in November 2009 at Fukuoka (Japan). The motive for the conference was the following. The year 2009 marked the 100th birthday of Casimir and the 150th birthday of the Riemann hypothesis. Actually the paper in which Riemann proposed the hypothesis was published in November 1859. It was also the year when he was appointed as full professor at Göttingen. Casimir, known for the Casimir force in physics and Casimir operators in mathematics, received his PhD in November 1931 at Leiden University and the named operators appeared in his thesis. There is also a nice connection between Casimir and Riemann. The Casimir force was first proven to exist theoretically essentially using the analytic continuation of the Riemann zeta function. Recently this force was measured experimentally. The conference was aimed to highlight some aspects of Casimir's and Riemann's heritage, by focusing on the following topics:

- · Casimir operators in harmonic analysis and representation theory,
- · Number theory, in particular zeta functions and cryptography,
- · Casimir force in physics and its relation with nano-science,
- · Mathematical biology,
- · Importance of mathematics for innovation in industry.

The latter topic was inspired both by the call for innovation in industry worldwide and by the fact that Casimir, who was the director of Philips research for a long time in his career, had an outspoken opinion on the importance of fundamental science for industry. Let us elaborate a little on this theme. In former days science, and in particular mathematics, was closely connected with applications. Attempts to cut this connection, to do mathematics for its own sake, were strongly criticized. Most known is the statement (in French, the language of science in those days) of the Dutch philosopher and mathematician Willem Jacob 's Gravesande (1688–1742) saying that he 'méprisait ces calculateurs de profession qui passent leur vie à la recherche de vérités de pure spéculation, et dont la découverte n'est d'aucune utilité soit pour les autres sciences, soit pour les besoins de la vie'. Riemann (1826–1866) was a pure mathematician, did not respond to 's Gravesande's picture, though he did, however, spend some time with proving the correctness of the first book of the Bible, Genesis, in a mathematical way. On the other hand, surprisingly enough, Riemann's geometry turned out to be of great

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importance for Einstein's relativity theory, and then for the correct functioning of navigation systems. Casimir had a position in between both. He laid the foundation for what became known as the science-technology spiral. Technology uses science with a time delay of say ten years; science in turn is driven by new developments in technology; and both progress together. It is our opinion that industry cannot survive without science and science not without industry. The emancipation of mathematics into an individual discipline is fine, but the discipline should not forget to contribute to wealth creation and the quality of life. We are pleased to see that in some countries, like Great Britain, this is a main point in the review of research in the Mathematical Sciences (see the 2010 EPSRC Review of the Mathematical Sciences). The ties between mathematics and industrial research should be maintained, sometimes repaired. One clever way to achieve this is to adapt our graduate education programs by letting students participate for a few months in some industrial company, as, for example, Kyushu University does. This benefits both students and industry: students get acquainted with the jobs that are available for mathematicians and companies make acquaintance with young talent that can contribute to innovation. This even meets the complaints sometimes heard from the side of the students: one does not know exactly what mathematics is good for. It might also be a solution for countries where a considerable decline of influx of first year students in mathematics is seen. The above considerations may explain why the conference was sponsored by the Global COE Program 'Education and Research Hub for Math-for-Industry', Kyushu University and supported by MEXT, the Japanese Ministry of Education, Culture, Sports, Science and Technology. It is a pleasure to thank these sponsors for their support.

We thank Kazufumi Kimoto, one of the invited speakers, for his technical support in the editing process. We also want to thank the publishing house De Gruyter for the excellent cooperation in publishing the proceedings of the conference. Finally we want to express our appreciation for the great help of Seiko Sasaguri during the conference and its organization.

Fukuoka, June 2010

Masato Wakayama Gerrit van Dijk

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Raising the profile of mathematics

Eng Chye Tan

Abstract. I would like to share the typical government's and the university's perspective on the desired functions of a mathematics department. Being a top-rate academic department is a given, but in a knowledge economy, there has been more demands. This is especially crucial for a country like Singapore, since we are small and do not have the luxury of scale. I would like to give a macro perspective what we hope Singapore Mathematics Departments could do, over and above the traditional functions of teaching and academic research. This should apply to all departments in a developed country, if they hope to continue to sustain the high level of (research) support and to attract strong students.

Major world trends

There are two main trends shaping the world – globalization and the rise of knowledge economies. These two trends must shape our thinking on how we groom our graduates. Professor Charles Vest, President-emeritus of MIT, in a speech to Singapore's National Science Foundation, emphasized the growing importance of a class of challenges and problems which are complex, and would need teams of multi-disciplinary talents, not just scientists and engineers, to resolve. Further, all around the world, we see increasingly multi-national teams working on some of these complex issues. We surmised that it is thus beneficial for our students to be exposed to working in diversified teams with different cultures and expertise.

NUS Engineering curriculum revamp

NUS Engineering, in response to these trends, embarked on a design-centric curriculum. The idea is to incorporate plenty of opportunities for students to apply engineering ideas and design to practical problems. Three themes were selected:

- future transportation systems,
- · engineering in medicine, and
- · smart and sustainable cities.

We had experience with engineering design for several years, but we have not made it an integral part of our curriculum. As an example, the Formula Society Automotive Engineering (FSAE) project has already gone into its 9th year, and the various versions of Formula One cars made clearly illustrate the engineering sophistication of

The article is based on the opening address given by the author at the conference "Casimir Force, Casimir Operators and the Riemann Hypothesis – Mathematics for Innovation in Industry and Science".

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our mechanical engineering students, and also the healthy culture of a senior group of students passing their expertise to the next group. This year, in a competition in Michigan, our FSAE car was ranked 19th, our best result so far since our participation in 2001, and the best from Asia. We hope to get better each year.

Another project on wing-in-ground-effect crafts is being conceived together with the help of a private company *WigetWorks*. Wing-in-ground-effect crafts (WIGs) are advanced hybrid air cushion crafts that offer the highest combination of speed, fuel efficiency and ride smoothness. Interest is driven by a local company WigetWorks which owns the IP needed to design and build an 8-seater WIG craft already classified by Lloyd's Register. The owner of this company was a mathematician who was a faculty member in NUS for some time, and is one of the wealthiest man in Singapore. WigetWorks is working with the Faculty of Engineering to develop the next generation of WIGs including one with a higher pay-load capacity.

As this theme will involve the design and development of a complete engineering system, it will offer many opportunities for graduate and undergraduate students to learn and put into practice technical concepts that span multiple disciplines (for instance, from Architecture, Design, Business) from engineering to design. WIG crafts have the potential to transform the tourism, defence and marine-transport industries in a big-way.

The important role of mathematics

And do mathematics play a part in all these – well, the answer is clearly "yes". The modeling of the aerodynamics cannot do without mathematics. But do mathematics students play a part in this, the answer is unfortunately "no".

Why have I linked this to mathematics? Well, it has something to do with the "culture" in mathematics. I have been a Deputy Chair of the Mathematics Department, the Dean of the Faculty of Science, of which mathematics is a department. And currently, I am the Provost. We are here in a conference organized by the Centre of Excellence Program in Math-for-Industry. This Program is meant promote the importance of mathematics in industry, and to enhance collaborations between mathematics and other disciplines, i.e., to add a different dimension to mathematics, transforming it in the process.

Public interest in mathematics

Singapore, like Japan, has a very much exam-oriented system. There are several critical examinations for our children, and at age 12, the Primary School Leaving Examination (PSLE) is the key set of exams which determines the high school a child would go to. It is not usual to hear parents complaining on the difficulty of this examination, especially for mathematics. The following is one such problem which appeared this year:

"Jim bought some chocolates and gave half of it to Ken. Ken bought some sweets and gave half of it to Jim. Jim ate 12 sweets and Ken ate 18 chocolates. The ratio of Jim's sweets to chocolates became 1:7 and the ratio of Ken's sweets to chocolates became 1:4. How many sweets did Ken buy?"

Students have about 5 minutes to attempt this problem, and they cannot use algebra which was not taught in primary schools. It is fair to say that in the Singapore's context, public understanding of mathematics is rudimentary, and their interest is probably because their children need to pass these exams. I suspect it is the same elsewhere.

Perception of mathematics in university

My sense is that most senior management of universities are well disposed towards mathematics – every top-rate university would want a top-notched economics department, physics and mathematics departments in the physical sciences, several top professional schools, etc. However, the other colleagues (outside of mathematics) are less appreciative of the usefulness of mathematics research, and would treat the teaching of mathematics as the more crucial function of a mathematics department. Thus, sometimes, even the engineers wonder whether if there is a need to teach so much mathematics.

How mathematics is perceived within the university is thus one critical component. I think there is much scope for mathematicians to influence our other colleagues on the special role which mathematics can play in the academic endeavour of many other disciplines. Instead of waiting for them to come to us, we could also reach out to them!

I do not have to convince all of you here, that there has been a tremendous explosion of mathematics outside of the physical sciences in biology, economics, business, and computer science, as well as dramatic successes in the physical sciences. But how often have our colleagues try to bring some of the more exciting applications of mathematics to our classrooms. To our students and colleagues, nothing beats knowing some current exciting applications of mathematics. Let us face it? we would not win the students' hearts with the harmonic oscillator. That is an ancient application, and students would probably have seen it anyway before they came to the university. Many of the new applications are exciting and serious but use only undergraduate mathematics.

I have alluded to the perception of mathematics amongst other colleagues within the University. We usually talk about closer working relationships between departments, and joint appointments are useful to facilitate this. Not surprisingly, if we were to look at joint appointments, mathematics would not do so well. Why are joint appointments difficult with mathematics, but much easier with statistics or operations research, etc.?

Perhaps, mathematics departments have much higher quality control, and they jealously protect this quality component – they would ask if a faculty could produce the high quality papers in top journals in mathematics, rather than if the faculty could add value to the department by adding a different academic dimension. As an illustration, I tried to enthuse my department to a joint appointment with a very well-funded Centre 4 E. C. Tan

for Quantum Technology. Some mathematics departments do have faculty in this area, but most are found in physics. It did not go through for a variety of reasons.

A nice application which I have just read

One of the interesting piece of research which I read about is Barry Cipra's article on fluid dynamics explaining the mystery in insects' motion. This can be found in "What is Happening in the Mathematical Sciences", a very useful publication by the American Mathematical Society (AMS). He explained the work of John Bush of MIT and his students Brian Chan and David Hu. While people understand how the water strider is able to stand on water because of the surface tension of water, less is known about how the strider walks on water.

The water strider's situation is interesting and far more complex mathematically that one may think. How does one even walk on a surface that is practically frictionless. The trick is that the strider creates vortices propelling them backwards, and by conservation of momentum, this strider could move forward.

Brian even developed a robotic water strider – he called it "Robostrider" – using a piece of elastic string from his sock. While John's work is fundamental, research groups elsewhere are trying to design micro-air machines and micro-fluidic devices using the same principles.

Link between content and applications

The example above is just one of the many which we can easily find. The link between research and education, as well as content and applications, is strong, but are rarely conveyed in our classrooms. *Mathematical Moments* by the AMS provide excellent examples which could be downloaded very easily. But I do not see such posters in our departments, and many departments for that matter.

We pretty much teach mathematics the same way we were taught, albeit in a neater and sophisticated presentation package using Tex and Powerpoint. I have illustrated the example of wing-in-ground-effect crafts in the Engineering curriculum to give a sense on how things do more in other disciplines. I think there is a fundamental need for mindset change in terms of how we teach mathematics, and if we do not, the drastic decline of Science, Technology, Engineering and Mathematics (STEM) enrolment will hit us real bad.

Falling student enrolment in STEM

Falling mathematics enrolment is a matter of fact, and this is consistent with the falling STEM numbers. For a knowledge society, maintaining a strong STEM pipeline is essential. But things are not looking good. In Australia, there is a 15% drop in mathematics majors from 2001 to 2008. In USA, mathematical sciences, and this includes statistics, enrolment fell from 4% in 1967 to less than 1% in 2007 of the graduating

cohort. Interestingly in Japan, the mathematical sciences enrolment increased from a low base of 0.6% in the 60s to slightly less than 0.9% in 2007. A similar story in Singapore, mathematical sciences enrolment is a little more than 2%, but the quality has been deteriorating. Typical mathematics departments in Asia, particularly Japan and Singapore, are strongly oriented towards pure mathematics, and pay less attention to applied mathematics. Thus, the likelihood of students, who have studied mathematics under such an environment, being interested in technologies is low. This situation does limit the contribution of mathematics and our graduates to the country's endeavors toward maintaining prosperity through advancement in science and technology.

Global centre of excellence program "Education and Research Hub for Math-for-Industry" at Kyushu University

Kyushu University's efforts in this area are impressive. The university inaugurated the Mathematical Research Center for Industrial Technology (MRIT) in April 2007 as a university-wide shared education and research facility to further the objectives of their 21st Century Centre of Excellence Program.

The purpose of MRIT is to conduct collaborative research as well as to facilitate human interactions between mathematics and other disciplines. Although such education and research institutes have become quite common in the West, MRIT is the first of its kind in Japan. Observing its progress, I am envious that Kyushu has embarked on this brave experiment, and has seen some early signs of success. I am sure many could learn from your experience.

I do hope that mathematics departments and our colleagues could pay more attention to what I have spoken about. Thank You!

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Casimir and lessons for innovation

Martin F. H. Schuurmans

Abstract. The European vision on Innovation and Technology as proposed by EIT is presented and connected to work and life of Hendrik Brugt Gerhard Casimir (1909–2000), who has been recognized as an excellent leader of industrial research at Royal Philips Electronics and for fundamental contributions to the foundations of Quantum Mechanics and Solid State Physics. The paper will also emphasize the importance of Casimir-like simplification in science, with the zero-point fluctuations of the electromagnetic field a case in point. This builds on earlier scientific work of Casimir and the author and on private and scientific contacts between them.

Introduction and contents

When Hendrik Brugt Casimir died in 2000 at the age of 90, his obituary appeared in Physics Today [13] and also in the New York Times [1]. Indeed Henk Casimir not only received 6 honorary doctoral degrees but also the Pake Price of the American Physical Society 1999 as an excellent leader of industrial research at Royal Philips Electronics and for fundamental contributions to the foundations of quantum mechanics and Solid State Physics. He is best known for predicting a true effect of quantum mechanics that even the seeming emptiness of a vacuum between two objects can generate electromagnetic forces that pull two objects toward each other. The prediction [4] published by Casimir and his then student Dik Polder in 1948, some 60 years ago, was verified in 1996 by Steve Lamoreaux [8]. The interpretation is in quantum mechanics as follows. Due to the Heisenberg uncertainty principle of quantum mechanics, vacuum is filled by zero-point fluctuation electromagnetic waves. Within the constrained space between two parallel metal plate objects fewer zero-point fluctuation waves can arise than outside of the parallel plates where there is much more 'volume'. In between the plates only short wavelength waves can exist. Outside all waves exist. Together the net effect is an attraction between the plates with an inverse dependence on the fourth power of the distance between the plates. Since Lamoreaux's initial measurement this dependence has been verified to 1% accuracy and has also been applied in other fields of physics like wetting of surfaces by liquids and in theories of the origin of the universe and of time-travel.

As already evidenced by the Pake Price, Henk Casimir had also major contributions in the leadership of industrial research. For example he laid the foundation for what became known as the science-technology spiral. Technology uses science with a time delay of say 10 years; science in turn is driven by new developments in technology; and both progress together. For example radio tubes made it possible for new aspects

of atomic and nuclear physics to be researched. The resulting science-technology spiral is largely responsible for the technological progress of the previous century. A comprehensive description of Casimir's views can be found in his excellent book Haphazard Reality-Half a Century of Science [3].

The author has enjoyed private and scientific contacts with Henk Casimir, considers himself a student of Dik Polder, has had an active scientific career in solid state physics and has been a leader of Research and Development activities at Royal Philips Electronics and at Philips Health Care. Today he leads the European Institute for Innovation and Technology, EIT, an initiative of the European Commission and its president Barroso to boost innovation in Europe.

This paper is divided into three sections:

- 1. innovation as it has changed over time and the role of science in innovation,
- 2. simplification in science using Casimir's zero-point fluctuations,
- 3. the EIT approach to innovation and the changing role of science in innovation.

Section 2 can be skipped by those that are first and foremost interested in the management of innovation or have no science background.

1 Innovation as it has changed over time

Definition of innovation

Let me start with my simple definition of innovation: Innovation is bringing 'something' new to the market. Already in 1934 Schumpeter [12] defined the 'something' new as a new product (good, service, process or organization), a new production process, new supply sources, a new market and new market structures. The market refers to public and societal use. Bringing is done by the various business parties and this covers a vast range of industries like manufacturing industries, service industries and even public domain industries. New refers to really new, not existing before, but also refers to improved.

My definition is very much in line with the one given by the OECD Oslo Manual [9], namely 'Innovation is the Implementation of a new or significantly improved product (good or service), or process, a new marketing method, or a new organizational method in business practice, work place organization or external relations'. For the purpose of this paper I prefer to use my simple definition of innovation.

Let us elaborate on the phrase 'new'. New can be the result of *exploitation* i.e. the refinement and extension of know-how. But new can also be the result of *experimentation* with new alternatives. The latter typically requires fundamental curiosity driven research and the application of such science. Note the know-how and the research can be but need not be of a technological nature. Until the nineties, when speaking about innovation, people were typically speaking about technology driven innovation by business enterprises. Today the scope of innovation is much broader. In any case

exploitation and experimentation right from the start of the use of the phrase innovation have been important. The two together we will call R&D, Research and Development.

Innovation is often coupled with entrepreneurship and partnerships. Already Anton and Gerard Philips who established the Royal Philips Electronics company in the early 20th century, were clearly entrepreneurs with a founding belief that by daring to make choices that improve the lives of people both inside and outside the company, they would be successful not by coincidence but by design, read by innovation.

Science and innovation

It is interesting to pause here for a moment and state what innovation is and is not! Innovation is not invention, not technology, not research, not science and not engineering. Innovation means the creation of an impact on the society by giving the society a (useful) public good called 'something new'. It is a public good because once the 'something new' has been produced it can also benefit others (non-excludability) and the value of its use is not depleted by the fact that others use it (non-rivalry).

So what is then the role of science in innovation? Upfront I need to say that few activities have had a deeper and broader impact on society than science. Without science there would be no birth control, no disease control, no ubiquitous communication, no internet, no globalization and so on. The impact of science on the economy has been equally dramatic. In a sense one can say that the 20th century is the golden age of the applications of science that have led to public goods that have changed and shaped our present day society. For better and for worse one might say. The outcome has not always been in the best interest of a sustainable society. Indeed we are facing now huge challenges around climate mitigation and adaptation, sustainable energy and demographic and food issues related to overpopulation. My position is that it will be again science that will help us to gain control over these issues and move forward with a sustainable world society. But not only science since innovation has been changing over time and so has the role of innovation. Let me describe that development.

Linear innovation

Let us go back in time to the 1950s when it was assumed that in innovation there was a linear relation between science, engineering, manufacturing and marketing/sales.



The model of innovation was that for a manufacturing industry (like Philips at the time), and technology driven innovation. It was the time of the big R, experimentation, and the big industrial research laboratories like Bell Labs, IBM Yorktown and the Philips Nat. Lab. (Physics Laboratory). Believe prevailed that new inventions would further revolutionize life in the society. This surely happened. In 1948 the transis-

tor was invented in Bell Labs and this has led to the formidable development of new miniaturized applications like today's cell phone and laptop. A few years later the solid state laser was invented and this completely revolutionized both the music and the computer storage industry. In the beginning science was not only seen as necessary but also as a sufficient condition for technology innovation; R, experimentation was the only thing that really mattered. However, over time leading manufacturing companies started to include the D, exploitation, both development and industrial engineering, as an important element to the innovation chain. In the 60s and 70s, this gave rise to the notion that innovation is driven by R&D. This notion to some extent is still pervasive today. Many government innovation policies measure the size of public and private R&D (Research and Development) as a percentage of GDP (Gross Domestic Product) of the country or nation. A good example is the Lisbon agenda of the European Community striving for a 2% + 1% objective of the ratio of R&D and GDP. While there is some truth here, the model today is far too simple as we will see in the following.

The Casimir spiral

In the 80s most R&D managers had realized that innovation is not linear and certainly not driven only by experimentation, R, read science but also very much by exploitation, D. One started to realize that in a complete picture not only innovation draws on science but also that the demands of science often force the creation of innovation. This was in fact already suggested by Henk Casimir, then the head of Philips Research and the Philips Nat. Lab., at the time of the seventies when the linear model still seemed largely valid. Technology uses science with a time delay of say 10 years, he argued. And science in turn is driven by new developments in technology; and both progress together. He was of course right but is seldom cited for this early wisdom.

The technology spiral persists until this very day. The invention of the transistor some 10 years after the first transistor science has led to computers that in turn have brought about new nano-science and bio-science. This new science promises to revolutionize our society and play a role in answering with innovation to the different forms of crisis (energy, carbon, food, health) facing us right now. Today we realize that the pitch of the spiral is different in different fields: fast pitch in bio and slow pitch in electronics, a more mature field.

New models of innovation

Meanwhile new types of innovations have swept across society. One is the advent of the mobile telephony where very quickly companies like Nokia have captured the possibilities of existing and emerging technologies (R&D) and invented phone after phone suited to an ever more thirsty society for seamless communication everywhere and with everyone. The 'something new' consisted not only of improved product upon product but also of new markets and new supply chain models for the R&D. Indeed