THOMAS / FINNEY 9TH EDITION

PART I

9TH EDITION

Calculus and Analytic Geometry

PART I

George B. Thomas, Jr.
Massachusetts Institute of Technology

Ross L. Finney

With the collaboration of
Maurice D. Weir
Naval Postgraduate School

Addison-Wesley Publishing Company
Reading, Massachusetts • Menlo Park, California • New York
Don Mills, Ontario • Wokingham, England • Amsterdam
Bonn • Sydney • Singapore • Tokyo • Madrid
San Juan • Milan • Paris

Acquisitions Editor	Laurie Rosatone	*Production Editorial Services*	Barbara Pendergast
Development Editor	Marianne Lepp	*Art Editors*	Susan London-Payne, Connie Hulse
Managing Editor	Karen Guardino	*Copy Editor*	Barbara Flanagan
Senior Production Supervisor	Jennifer Bagdigian	*Proofreader*	Joyce Grandy
Senior Marketing Manager	Andrew Fisher	*Text Design*	Martha Podren, Podren Design;
Marketing Coordinator	Benjamin Rivera		Geri Davis, Quadrata, Inc.
Prepress Buying Manager	Sarah McCracken	*Cover Design*	Marshall Henrichs
Art Buyer	Joseph Vetere	*Cover Photo*	John Lund/Tony Stone Worldwide
Senior Manufacturing Manager	Roy Logan	*Composition*	TSI Graphics, Inc.
Manufacturing Coordinator	Evelyn Beaton	*Technical Illustration*	Tech Graphics

Photo Credits: **142, 238, 408, 633, 722,** From *PSSC Physics 2/e,* 1965; D.C. Heath & Co. with Education Development Center, Inc., Newton, MA. Reprinted with permission **186,** AP/Wide World Photos **266,** Scott A. Burns, Urbana, IL **287,** Joshua E. Barnes, University of Hawaii **354,** Marshall Henrichs **398,** © Richard F. Voss/IBM Research **442,** © Susan Van Etten

Library of Congress Cataloging-in-Publication Data

Thomas, George Brinton, 1914–

Calculus and analytic geometry / George B. Thomas, Ross L. Finney.
—9th ed.
p. cm.
Includes index.
ISBN 0-201-53175-5
1. Calculus. 2. Geometry, Analytic. I. Finney, Ross L.
II. Title.
QA303.T42 1996
515'. 15—dc20 94-30543
CIP

1 2 3 4 5 6 7 8 9 10 VH 98979695

CAS Explorations and Projects (Listed by chapter and section)

Preliminaries

P.4 How the graph of $y = f(ax)$ is affected by changing a

P.5 How the graph of $f(x) = A \sin((2\pi/B)(x - C)) + D$ responds to changes in A, B, and D

Chapter 1 Limits and Continuity

1.1 Comparing graphical estimates of limits with CAS symbolic limit calculations

1.3 Exploring the formal definition of limit by finding deltas for specific epsilons graphically

1.6 Observing the convergence of secant lines to tangent lines

Chapter 2 Derivatives

2.1 Given $f(x)$, find $f'(x)$ as a limit. Compare the graphs of f and f' and plot selected tangents

2.6 Differentiate implicitly and plot implicit curves together with tangent lines

Chapter 3 Applications of Derivatives

3.1 Finding absolute extrema by analyzing f and f' numerically and graphically

3.7 Estimating the error in a linearization by plotting $f(x)$, $L(x)$, and $|f(x) - L(x)|$

Chapter 4 Integration

4.4 Find the average value of $f(x)$ and the point or points where it is assumed

4.5 Exploring Riemann sums and their limits

4.7 a) Investigating the relationship of $F(x) = \int_a^x f(t)\,dt$ to $f(x)$ and $f'(x)$

b) Analyzing $F(x) = \int_a^{u(x)} f(t)dt$

Chapter 5 Application of Integrals

5.1 Finding intersections of curves

5.5 Arc length estimates

Chapter 6 Transcendental Functions

6.1 Graphing inverse functions and their derivatives

6.12 Exploring differential equations graphically and numerically with slope fields and Euler approximations

Chapter 7 Techniques of Integration

7.5 Using a CAS to integrate. An example of a CAS-resistant integral

7.6 Exploring the convergence of improper integrals

Chapter 8 Infinite Series

8.1 Exploring the convergence of sequences. Compound interest with deposits and withdrawals. The logistic difference equation and chaotic behavior

8.5 Exploring $\sum_{n=1}^{\infty}(1/(n^3\sin^2 n))$, a series whose convergence or divergence has not yet been determined

8.10 Comparing functions' linear, quadratic, and cubic approximations

Chapter 9 Conic Sections, Parametrized Curves, and Polar Coordinates

9.5 Exploring the geometry of curves that are defined implicitly or explicitly by parametric equations. Numerical estimates of the lengths of nonelementary paths

9.8 How the graph of $r = ke/(1 + e\cos\theta)$ is affected by changes in e and k. How changes in a and e affect the ellipse $r = a(1 - e^2)/(1 + e\cos\theta)$.

To the Instructor

This Is a Major Revision

Throughout the 40 years that it has been in print, Thomas/Finney has been used to support a variety of teaching methods from traditional to experimental. In response to the many exciting currents in teaching calculus in the 1990s, the new edition is the most extensive revision of Thomas/Finney ever. We have built on the traditional strengths of the book—excellent exercises, sound mathematics, variety in applications—to produce a flexible text that contains all the elements needed to teach the many different kinds of courses that exist today.

A book does not make a course: The instructor and the students do. With this in mind we have added features to Thomas/Finney 9th edition to make it the most flexible calculus teaching resource yet.

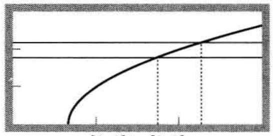
- The exercises have been reorganized to facilitate assigning a subset of the material in a section.
- The grapher explorations, all accessible with any graphing calculator, many suitable for in-class and group work, have been expanded.
- New Computer Algebra System (CAS) explorations and projects that require a CAS have been included. Some of these can be done quickly while others require several hours. All are suitable for either individual or group work. You will find a list of CAS exercise topics following the Table of Contents.
- Technology Connection notes appear throughout the text suggesting experiments students might do with a grapher to supplement their understanding of a given topic. These notes are meant to encourage students to think of their grapher as a casually available tool, like a pencil.
- We revised the entire first semester and large parts of the second semester to provide what we believe is a cleaner, more visual, and more accessible book.

With all these changes, we have not compromised our belief that the fundamental goal of a calculus book is to prepare students to enter the scientific community.

Students Will Find Even More Support for Creative Problem Solving

Throughout this book, we have included examples and discussions that encourage students to think visually and numerically. Almost every exercise set has easy to

67



[0, 3] by [0, 3]

Keeping x between 1.75 and 2.28 will keep y between 1.8 and 2.2.

Technology *Target Values* You can experiment with target values on a graphing utility. Graph the function together with a target interval defined by horizontal lines above and below the proposed limit. Adjust the range or use zoom until the function's behavior inside the target interval is clear. Then observe what happens when you try to find an interval of x-values that will keep the function values within the target interval. (See also Exercises 7–14 and CAS Exercises 61–64.)

For example, try this for $f(x) = \sqrt{3x - 2}$ and the target interval (1.8, 2.2) on the y-axis. That is, graph $y_1 = f(x)$ and the lines $y_2 = 1.8$, $y_3 = 2.2$. Then try the target intervals (1.98, 2.02) and (1.9998, 2.0002).

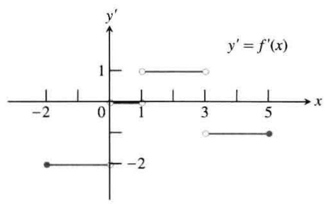
32. *Recovering a function from its derivative*

a) Use the following information to graph the function f over the closed interval $[-2, 5]$.

i) The graph of f is made of closed line segments joined end to end.

ii) The graph starts at the point $(-2, 3)$.

iii) The derivative of f is the step function in Fig. 2.13.



2.13 The derivative graph for Exercise 32.

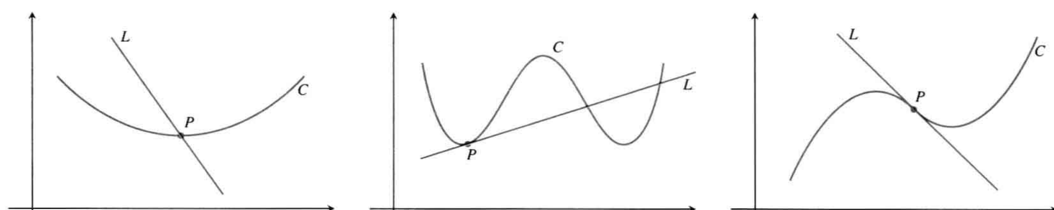
b) Repeat part (a) assuming that the graph starts at $(-2, 0)$ instead of $(-2, 3)$.

118

mid-level exercises that require students to generate and interpret graphs as a tool for understanding mathematical or real-world relationships. Many sections also contain a few more challenging problems to extend the range of the mathematically curious.

This edition has more than 1600 figures to appeal to the students' geometric intuition. Drawing lessons aid students with difficult 3-dimensional sketches, enhancing their ability to think in 3-space. In this edition we have increased the use of visualization internal to the discussion. The burden of exposition is shared by art in the body of the text when we feel that pictures and text together will convey ideas better than words alone.

Throughout the text, students are asked to experiment, investigate, and explain. Writing exercises are placed throughout the text. In addition, each chapter end contains a list of questions that ask students to review and summarize what they have learned. Many of these exercises make good writing assignments.



L meets C only at P but is not tangent to C.

L is tangent to C at P but meets C at several points.

L is tangent to C at P but lies on two sides of C, crossing C at P.

98

1.49 Exploding myths about tangent lines.

Students Will Master Techniques

Problem-Solving Strategies We believe that the students learn best when procedural techniques are laid out as clearly as possible. To this end we have revisited the summaries of the steps used to solve problems, adding some where necessary, deleting some where a thought process rather than a technique was at issue, and making each one clear and useful. As always, we are especially careful that examples in the text follow the steps outlined by the discussion.

Exercises Every exercise set has been reviewed and revised. Exercises are now *grouped by topic,* with special sections for grapher explorations. Many sections also have a set of Computer Algebra System (CAS) Explorations and Projects, a new feature for this edition. Within each group, the exercises are graded and paired. Within this framework, the exercises generally follow the order of presentation of the text.

Hidden Behavior

Sometimes graphing f' or f'' will suggest where to zoom in on a computer generated graph of f to reveal behavior hidden in the grapher's original picture.

Checklist for Graphing a Function $y = f(x)$

1. Look for symmetry.
   Is the function even? odd?
2. Is the function a shift of a known function?
3. Analyze dominant terms.
   Divide rational functions into polynomial + remainder.
4. Check for asymptotes and removable discontinuities.
   Is there a zero denominator at any point?
   What happens as $x \to \pm\infty$?
5. Compute f' and solve $f' = 0$. Identify critical points and determine intervals of rise and fall.
6. Compute f'' to determine concavity and inflection points.
7. Sketch the graph's general shape.
8. Evaluate f at special values (endpoints, critical points, intercepts).
9. Graph f, using dominant terms, general shape, and special points for guidance.

230

Exercises that require a calculator or computer are identified by icons: calculator exercise, graphing utility (such as graphing calculator) exercise, and Computer Algebra System exercise.

Within the exercise sets, we have practice exercises, exercises that encourage critical thinking, more challenging exercises (in subsections marked "Applications and Theory"), and exercises that require writing in English about concepts. Writing exercises are placed both throughout the exercise sets, and in an end-of-chapter feature called "Questions to Guide Your Review."

Chapter End At the end of each chapter are three features with questions that summarize the chapter in different ways.

Questions to Guide Your Review ask students to think about concepts and verbalize their understanding without trying to calculate numeric answers. These are, as always, suitable for writing exercises.

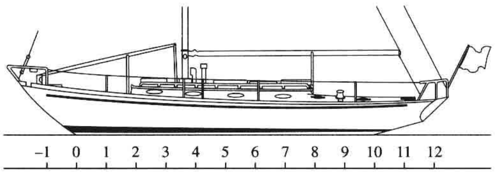
Practice Exercises provide a review of the techniques, ideas, and key applications.

Additional Exercises—Theory, Examples, Applications supply challenging applications and theoretic problems that deepen the understanding of mathematical ideas.

Applications, Technology, History—Features That Bring Calculus to Life

Applications and Examples It has been a hallmark of this book through the years that we illustrate applications of calculus with real data based on already familiar situations or situations students are likely to encounter soon. Throughout the text, we

17. *A sailboat's displacement.* To find the volume of water displaced by a sailboat, the common practice is to partition the waterline into 10 subintervals of equal length, measure the cross section area $A(x)$ of the submerged portion of the hull at each partition point, and then use Simpson's rule to estimate the integral of $A(x)$ from one end of the waterline to the other. The table here lists the area measurements at "Stations" 0 through 10, as the partition points are called, for the cruising sloop *Pipedream*, shown here. The common subinterval length (distance between consecutive stations) is $h = 2.54$ ft (about 2′ 6 1/2″, chosen for the convenience of the builder).



a) Estimate *Pipedream*'s displacement volume to the nearest cubic foot.

Station	Submerged area (ft^2)
0	0
1	1.07
2	3.84
3	7.82
4	12.20
5	15.18
6	16.14
7	14.00
8	9.21
9	3.24
10	0

b) The figures in the table are for seawater, which weighs 64 lb/ft^3. How many pounds of water does *Pipedream* displace? (Displacement is given in pounds for small craft, and long tons [1 long ton = 2240 lb] for larger vessels.)

443

cite sources for the data and/or articles from which the applications are drawn, helping students understand that calculus is a current, dynamic field. Most of these applications are directed toward science and engineering, but there are many from biology and the social sciences as well.

Technology: Graphing Calculator and Computer Algebra Systems Explorations *Virtually every section* of the text contains calculator exercises that explore numerical patterns and/or graphing calculator exercises that ask students to generate and interpret graphs as a tool to understanding mathematical and real-world relationships. Many of the calculator and graphing calculator exercises are suitable for classroom demonstration or for group work by students in or out of class.

Computer Algebra System (CAS) exercises have been added to every chapter. Numbering more than 130, these exercises have been tested on both Mathematica and Maple. A full list of CAS exercise topics follows the Table of Contents.

As in previous editions, $\sec^{-1}x$ has been defined so that its range, $[0, \pi/2) \cup (\pi/2, \pi]$, and derivative $1/(|x|\sqrt{x^2-1})$, agree with the results returned by Computer Algebra Systems and scientific calculators.

Notes appear throughout the text encouraging students to explore with graphers.

History Any student is enriched by seeing the human side of mathematics. As in earlier editions, we feature history boxes that describe the origins of ideas, conflicts concerning ownership of ideas, and interesting sidelights into popular topics such as fractals and chaos.

The Many Faces of This Book

Mathematics Is a Formal and Beautiful Language A good part of the beauty of the calculus lies in the fact that it is a stunning creation of the human mind. As in previous editions we have been careful to say only what is true and mathematically sound. In this edition we reviewed every definition, theorem, corollary, and proof for clarity and mathematical correctness.

Even Better Suited to Be the Reference Text in a Reform Course Whether calculus is taught by a traditional lecture or entirely in labs with individual and group learning which focuses on numeric and graphical experimentation, ideas and techniques need to be articulated clearly. This book provides the exercises for computer and grapher experiments and group learning and, in a traditional format, the summation of the lesson—the formal statement of the mathematics and the clear presentation of the technique.

Students Will Learn from This Book for Many Years to Come We provide far more material than any one instructor would want to teach. We do this intentionally. Students can continue to learn calculus from this book long after the class has ended. It provides an accessible review of the calculus a student has already studied. It is a resource for the working engineer or scientist.

Content Features of the Ninth Edition

Preliminary Material

- Lines, functions, and graphs are reviewed briefly.
- Trigonometric functions (formerly treated in an Appendix) are included.

Limits

- The limit is introduced through rates of change (Section 1.1) but defined before the derivative.
- The initial discussion of the limit is intuitive, using numeric and graphic examples of rates of change.
- The basic rules for working with limits are presented in Section 1.2.
- The limit is presented formally in Section 1.3, using input/output control systems to motivate the ε - δ definition. Covering the formal definition of the limit is optional.
- Chapter 1 concludes with the definition of the tangent line and instantaneous rate of change at a point, bringing to a close the investigation begun in Section 1.1.

Derivatives

- Chapter 2 opens with the concept and definition of the derivative as a function.
- The treatment of implicit differentiation has been revised (Section 2.6).
- The treatment of related rates has been moved earlier in the text (Section 2.7).

Applications of the Derivative

- Extrema (Section 3.1) and the Mean Value Theorem (Section 3.2) are now treated in separate sections. The first section presents the motivating problems of maximization and antidifferentiation. The second section provides motivating questions about antidifferentiation, and the Mean Value Theorem provides the answers. Testing critical points is the subject of Section 3.3.
- The graphing sections (Sections 3.4, 3.5) have been revised to emphasize the qualitative reading of graphs.
- Section 3.5 on asymptotes and dominant terms has been rewritten to present a unified approach to graphing rational functions.
- Presentation of L'Hôpital's Rule is postponed to Chapter 6 where it is applied to comparisons with exponential and logarithmic functions.
- Quadratic approximation, formerly in Chapter 3, has been included with Taylor polynomials in Chapter 8.

Integration

- As in previous editions, the indefinite integral is covered before the definite integral (Section 4.1).
- Differential equations and initial value problems are presented immediately after the indefinite integral (Section 4.2).
- Substitution is introduced for indefinite integrals in Section 4.3 and discussed again for definite integrals in Section 4.8.
- The definite integral is motivated by estimating with finite sums (Section 4.4).
- Some techniques of integration have been moved into Chapter 6 (Transcendental Functions), making Chapter 7 (Techniques of Integration) a shorter, more focused chapter.
- Integration using a Computer Algebra System (CAS) is covered in Section 7.5 along with integral tables.

Sequences and Series

- The introduction to sequences has been spread over two sections (Sections 8.1, 8.2), providing more time for this idea.

- Chapter 8 has been reorganized to allow one section per lecture. (See the Table of Contents.)
- Power series are applied to solve differential equations and initial value problems (Section 8.11).

Conic Sections

- The geometry of conic sections is treated in Section 9.1.
- Eccentricity is covered separately in Section 9.2, where it is used to classify the conics.

Differential Equations

- Differential equations and initial value problems are previewed in Section 3.2 by introducing students to the idea of determining functions from derivatives and initial values. Section 3.4 continues the preparation for differential equations by showing how to sketch the graph of a function given the formula for its first derivative and a point through which the graph must pass.
- Differential equations, initial value problems, and their applications then become the central topics of the following sections:

4.2	Differential Equations, Initial Value Problems, and Mathematical Modeling
6.5	Growth and Decay (the initial value problem $y' = ky$, $y(0) = y_0$ and its applications)
6.11	First Order Differential Equations
6.12	Euler's Numerical Method; Slope Fields

Solutions of differential equations and initial value problems appear as appropriate in exercise sets throughout the remainder of the text.

- Section 4.7 solves initial value problems using the Fundamental Theorem of Calculus, and Section 8.11 solves differential equations and initial value problems with power series.

Supplements for the Instructor

OmniTest^3 in DOS-Based Format This easy-to-use software is developed exclusively for Addison-Wesley by ips Publishing, a leader in computerized testing and assessment. Among its features are the following.

- **DOS interface is easy to learn and operate.** The windows look-alike interface makes it easy to choose and control the items as well as the format for each test.
- **You can easily create make-up exams, customized homework assignments, and multiple test forms to prevent plagiarism.** OmniTest^3 is algorithm driven—meaning the program can automatically insert new numbers into the same equation—creating hundreds of variations of that equation. The numbers are constrained to keep answers reasonable. This allows you to create a virtually endless supply of parallel versions of the same test. This new version of OmniTest also allows you to "lock in" the values shown in the model problem, if you wish.
- **Test items are keyed by section** to the text. Within the section, you can select questions that test individual objectives from that section.
- **You can enter your own questions** by way of OmniTest^3's sophisticated editor—complete with mathematical notation.

Instructor's Solutions Manual by Maurice D. Weir (Naval Postgraduate School). This two-volume supplement contains the worked-out solutions for *all* the exercises in the text.

Answer Book contains short answers to most exercises in the text.

Supplements for the Instructor and Student

Student Study Guide by Maurice D. Weir (Naval Postgraduate School). Organized to correspond with the text, this workbook in a semiprogrammed format increases student proficiency with study tips and additional practice.

Student Solutions Manual by Maurice D. Weir (Naval Postgraduate School). This manual is designed for the student and contains carefully worked-out solutions to all of the odd-numbered exercises in the text.

Differential Equations Primer A short, supplementary manual containing approximately a chapter's-worth of material. Available should the instructor choose to cover this material within the calculus sequence.

Technology-Related Supplements

Analyzer* This program is a tool for exploring functions in calculus and many other disciplines. It can graph a function of a single variable and overlay graphs of other functions. It can differentiate, integrate, or iterate a function. It can find roots, maxima and minima, and inflection points, as well as vertical asymptotes. In addition, Analyzer* can compose functions, graph polar and parametric equations, make families of curves, and make animated sequences with changing parameters. It exploits the unique flexibility of the Macintosh wherever possible, allowing input to be either numeric (from the keyboard) or graphic (with a mouse). Analyzer* runs on Macintosh II, Plus, or better.

The Calculus Explorer Consisting of 27 programs ranging from functions to vector fields, this software enables the instructor and student to use the computer as an "electronic chalkboard." The Explorer is highly interactive and allows for manipulation of variables and equations to provide graphical visualization of mathematical relationships that are not intuitively obvious. The Explorer provides user-friendly operation through an easy-to-use menu-driven system, extensive on-line documentation, superior graphics capability, and fast operation. An accompanying manual includes sections covering each program, with appropriate examples and exercises. Available for IBM PC/compatibles.

InSight A calculus demonstration software program that enhances understanding of calculus concepts graphically. The program consists of ten simulations. Each presents an application and takes the user through the solution visually. The format is interactive. Available for IBM PC/compatibles.

Laboratories for Calculus I Using Mathematica By Margaret Höft, The University of Michigan-Dearborn. An inexpensive collection of *Mathematica* lab experiments consisting of material usually covered in the first term of the calculus sequence.

Math Explorations Series Each manual provides problems and explorations in calculus. Intended for self-paced and laboratory settings, these books are an excellent complement to the text.

Exploring Calculus with a Graphing Calculator, Second Edition, by Charlene E. Beckmann and Ted Sundstrom of Grand Valley State University.

Exploring Calculus with Mathematica, by James K. Finch and Millianne Lehmann of the University of San Francisco.

Exploring Calculus with Derive, by David C. Arney of the United States Military Academy at West Point.

Exploring Calculus with Maple, by Mark H. Holmes, Joseph G. Ecker, William E. Boyce, and William L. Seigmann of Rensselaer Polytechnic Institute.

Exploring Calculus with Analyzer*, by Richard E. Sours of Wilkes University.

Exploring Calculus with the IBM PC Version 2.0, by John B. Fraleigh and Lewis I. Pakula of the University of Rhode Island.

Acknowledgments

We would like to express our thanks for the many valuable contributions of the people who reviewed this book as it developed through its various stages:

Manuscript Reviewers

Erol Barbut, *University of Idaho*
Neil E. Berger, *University of Illinois at Chicago*
George Bradley, *Duquesne University*
Thomas R. Caplinger, *Memphis State University*
Curtis L. Card, *Black Hills State University*
James C. Chesla, *Grand Rapids Community College*
P.M. Dearing, *Clemson University*
Maureen H. Fenrick, *Mankato State University*
Stuart Goldenberg, *CA Polytechnic State University*
Johnny L. Henderson, *Auburn University*
James V. Herod, *Georgia Institute of Technology*
Paul Hess, *Grand Rapids Community College*
Alice J. Kelly, *Santa Clara University*
Jeuel G. LaTorre, *Clemson University*
Pamela Lowry, *Lawrence Technological University*
John E. Martin, III, *Santa Rosa Junior College*
James Martino, *Johns Hopkins University*
James R. McKinney, *California State Polytechnic University*
Jeff Morgan, *Texas A & M University*
F. J. Papp, *University of Michigan—Dearborn*
Peter Ross, *Santa Clara University*
Rouben Rostamian, *University of Maryland—Baltimore County*
William L. Siegmann, *Rensselaer Polytechnic Institute*
John R. Smart, *University of Wisconsin—Madison*
Dennis C. Smolarski, S. J., *Santa Clara University*
Bobby N. Winters, *Pittsburgh State University*

Technology Notes Reviewers

Lynn Kamstra Ipina, *University of Wyoming*
Robert Flagg, *University of Southern Maine*
Jeffrey Stephen Fox, *University of Colorado at Boulder*
James Martino, *Johns Hopkins University*
Carl W. Morris, *University of Missouri—Columbia*
Robert G. Stein, *California State University—San Bernardino*

Accuracy Checkers

Steven R. Finch, *Massachusetts Bay Community College*
Paul R. Lorczak, *MathSoft, Inc.*
John R. Martin, *Tarrant County Junior College*
Jeffrey D. Oldham, *Stanford University*

Exercises

In addition, we thank the following people who reviewed the exercise sets for content and balance and contributed many of the interesting new exercises:

Jennifer Earles Szydlik, *University of Wisconsin—Madison*
Aparna W. Higgins, *University of Dayton*
William Higgins, *Wittenberg University*
Leonard F. Klosinski, *Santa Clara University*
David Mann, *Naval Postgraduate School*
Kirby C. Smith, *Texas A & M University*

Kirby Smith was also a pre-revision reviewer and we wish to thank him for his many helpful suggestions.

We would like to express our appreciation to David Canright, Naval Postgraduate School, for his advice and his contributions to the CAS exercise sets, and Gladwin Bartel, at Otero Junior College, for his many helpful suggestions.

Answers

We would like to thank Cynthia Hutcherson for providing answers for exercises in some of the chapters in this edition. We also appreciate the work of an outstanding team of graduate students at Stanford University, who checked every answer in the text for accuracy: Miguel Abreu, David Cardon, Tanya Kalich, Jeffrey D. Oldham, and Julie Roskies. Jeffrey D. Oldham also tested all the CAS exercises, and we thank him for his many helpful suggestions.

Other Contributors

We are particularly grateful to Maurice D. Weir, Naval Postgraduate School, who shared his teaching ideas throughout the preparation of this book. He produced the final exercise sets and wrote most of the CAS exercises for this edition. We appreciate his constant encouragement and thoughtful advice.

We thank Richard A. Askey, University of Wisconsin—Madison, David McKay, Oregon State University, and Richard G. Montgomery, Southern Oregon State College, for sharing their teaching ideas for this edition.

We are also grateful to Erich Laurence Hauenstein, College of DuPage, for generously providing an improved treatment of chaos in Newton's method, and to Robert Carlson, University of Colorado, Colorado Springs, for improving the exposition in the section on relative rates of growth of functions.

To the Student

What Is Calculus?

Calculus is the mathematics of motion and change. Where there is motion or growth, where variable forces are at work producing acceleration, calculus is the right mathematics to apply. This was true in the beginnings of the subject, and it is true today.

Calculus was first invented to meet the mathematical needs of the scientists of the sixteenth and seventeenth centuries, needs that were mainly mechanical in nature. Differential calculus dealt with the problem of calculating rates of change. It enabled people to define slopes of curves, to calculate velocities and accelerations of moving bodies, to find firing angles that would give cannons their greatest range, and to predict the times when planets would be closest together or farthest apart. Integral calculus dealt with the problem of determining a function from information about its rate of change. It enabled people to calculate the future location of a body from its present position and a knowledge of the forces acting on it, to find the areas of irregular regions in the plane, to measure the lengths of curves, and to find the volumes and masses of arbitrary solids.

Today, calculus and its extensions in mathematical analysis are far reaching indeed, and the physicists, mathematicians, and astronomers who first invented the subject would surely be amazed and delighted, as we hope you will be, to see what a profusion of problems it solves and what a range of fields now use it in the mathematical models that bring understanding about the universe and the world around us. The goal of this edition is to present a modern view of calculus enhanced by the use of technology.

How to Learn Calculus

Learning calculus is not the same as learning arithmetic, algebra, and geometry. In those subjects, you learn primarily how to calculate with numbers, how to simplify algebraic expressions and calculate with variables, and how to reason about points, lines, and figures in the plane. Calculus involves those techniques and skills but develops others as well, with greater precision and at a deeper level. Calculus introduces so many new concepts and computational operations, in fact, that you will no longer be able to learn everything you need in class. You will have to learn a fair amount on your own or by working with other students. What should you do to learn?

1. Read the text. You will not be able to learn all the meanings and connections you need just by attempting the exercises. You will need to read relevant

passages in the book and work through examples step by step. Speed reading will not work here. You are reading and searching for detail in a step-by-step logical fashion. This kind of reading, required by any deep and technical content, takes attention, patience, and practice.

2. Do the homework, keeping the following principles in mind.
   a) Sketch diagrams whenever possible.
   b) Write your solutions in a connected step-by-step logical fashion, as if you were explaining to someone else.
   c) Think about why each exercise is there. Why was it assigned? How is it related to the other assigned exercises?
3. Use your calculator and computer whenever possible. Complete as many grapher and CAS (Computer Algebra System) exercises as you can, *even if they are not assigned.* Graphs provide insight and visual representations of important concepts and relationships. Numbers can reveal important patterns. A CAS gives you the freedom to explore realistic problems and examples that involve calculations that are too difficult or lengthy to do by hand.
4. Try on your own to write short descriptions of the key points each time you complete a section of the text. If you succeed, you probably understand the material. If you do not, you will know where there is a gap in your understanding.

Learning calculus is a process—it does not come all at once. Be patient, persevere, ask questions, discuss ideas and work with classmates, and seek help when you need it, right away. The rewards of learning calculus will be very satisfying, both intellectually and professionally.

G.B.T., Jr., *State College, PA*
R.L.F., *Monterey, CA*

Contents

3 Applications of Derivatives

4 Integration

5 Applications of Integrals

6 Transcendental Functions