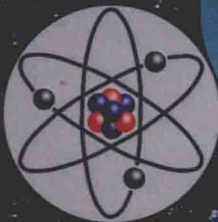
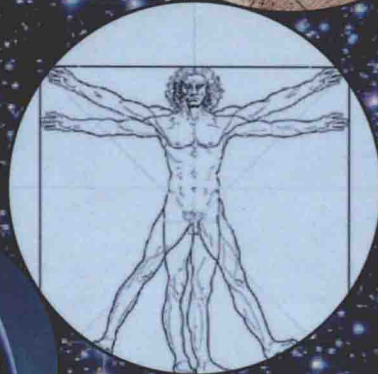


# Fractional Calculus View of Complexity

Tomorrow's Science

Bruce J. West



CRC Press  
Taylor & Francis Group

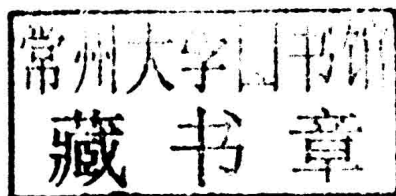
A SCIENCE PUBLISHERS BOOK

# Fractional Calculus View of Complexity

## Tomorrow's Science

**Bruce J. West**

Information Sciences Directorate  
Army Research Office  
Research Triangle Park, NC



**CRC Press**

Taylor & Francis Group

Boca Raton London New York

CRC Press is an imprint of the  
Taylor & Francis Group, an **informa** business

A SCIENCE PUBLISHERS BOOK

CRC Press  
Taylor & Francis Group  
6000 Broken Sound Parkway NW, Suite 300  
Boca Raton, FL 33487-2742

© 2016 by Taylor & Francis Group, LLC  
CRC Press is an imprint of Taylor & Francis Group, an Informa business

No claim to original U.S. Government works

Printed on acid-free paper  
Version Date: 20150728

International Standard Book Number-13: 978-1-4987-3800-2 (Hardback)

This book contains information obtained from authentic and highly regarded sources. Reasonable efforts have been made to publish reliable data and information, but the author and publisher cannot assume responsibility for the validity of all materials or the consequences of their use. The authors and publishers have attempted to trace the copyright holders of all material reproduced in this publication and apologize to copyright holders if permission to publish in this form has not been obtained. If any copyright material has not been acknowledged please write and let us know so we may rectify in any future reprint.

Except as permitted under U.S. Copyright Law, no part of this book may be reprinted, reproduced, transmitted, or utilized in any form by any electronic, mechanical, or other means, now known or hereafter invented, including photocopying, microfilming, and recording, or in any information storage or retrieval system, without written permission from the publishers.

For permission to photocopy or use material electronically from this work, please access [www.copyright.com](http://www.copyright.com) (<http://www.copyright.com/>) or contact the Copyright Clearance Center, Inc. (CCC), 222 Rosewood Drive, Danvers, MA 01923, 978-750-8400. CCC is a not-for-profit organization that provides licenses and registration for a variety of users. For organizations that have been granted a photocopy license by the CCC, a separate system of payment has been arranged.

**Trademark Notice:** Product or corporate names may be trademarks or registered trademarks, and are used only for identification and explanation without intent to infringe.

**Visit the Taylor & Francis Web site at**  
**<http://www.taylorandfrancis.com>**

**and the CRC Press Web site at**  
**<http://www.crcpress.com>**

# **Fractional Calculus View of Complexity**

**Tomorrow's Science**



# Foreword

---

Change, evolutionary or revolutionary, helpful or harmful, is how we track events through time to our universe. In this volume, Bruce West confronts change within science and shows us ways to look at the future of science that are different than the pathways of its past. Science in itself may be an evolutionary process, but the ways humans perform science, utilize it, and understand it can be and, according to West, are currently revolutionary. West shows the revolutionary fervor of modern science by describing some of the dramatic changes in scientific needs, utilizations and methodologies. Even though Bruce West does not exactly say this, I will—we are in the midst of a paradigm shift in science. The revolution in science has started. Elements of this shift include moving from little science (single investigators) to big science (multi-disciplinary teams of scientists), the increased role of complexity, and connections in understanding the informational and networked nature of science, and the human utilization of science within the context of the rest of society.

Bruce West has an exceptional way of weaving science history, theory, application, philosophy, and management into one cohesive story. One of the main characters in West's story is the mathematics of fractional calculus. West makes many compelling cases to convince readers of the power and upcoming role of fractional calculus methodology. Much like Isaac Asimov who saw his role as a futurist "to reconnoiter the territory up ahead so that humanity, in its travel through time, may have a better notion of what to aim for and what to avoid" [1], West provides both an aim point and a path for future science with fractional calculus and complexity playing major roles. In the last chapter of the book, the future nature and philosophy of science are clearly shown as only a futurist like West can describe. Most exciting for me is West's analysis of the underlying philosophy and principles of tomorrow's science. His four hypothesized changes in scientific principles in the last chapter are profound and impactful and my favorite element of the book.

I recently heard a colleague explain the interdisciplinary role of mathematics in science: “Mathematics is part of all science, but it is also only a part of any science”. I agree with West and my colleague—modern science is inherently interdisciplinary. It relies on combining concepts, models, methods, knowledge, and perspectives from all angles, dimensions, and far out regions of scientific inquiry. The collective science enterprise, as it begins to encompass large cooperative organizations and teams to form the concept of big science, is where future science will make its mark. In the past, these kinds of large-scale interdisciplinarity were limited to outsized projects like the Manhattan Project, or building the Internet, or reserved for government-led studies conducted by the National Academy of Science, or the National Research Council [4], but today big science is more and more the norm of industrial research centers and university laboratories. While science education continues to use a more narrow disciplinary focus to produce organized sets of departments, courses, and topics that are accessible, structural, sequential, and assessable for education, real science is very different. In the research world, science is completely unaware of these artificial disciplinary boundaries. Through use of vivid examples and an insightful description of complexity, West describes this holistic and interconnected view of the modern science enterprise. As West clearly demonstrates in his historical anecdotes and future projections that science contains many complex, layered, multi-scaled, dynamic, multi-dimensional, and contextual components, it is the collective endeavors that will make further progress possible. Edward Teller (1980) wrote back in the industrial age that even though science should pursue simplicity in its models and theories in order to provide clarity and understanding for society, there were many elements in science (like the understanding of life itself) that are inherently and purposefully complex [5]. The difference between Teller’s industrial age view of science and West’s more modern and futurist view is in the expanse of the complexity and connectivity of science. Teller saw complexity filling a limited role and connectivity only needed for few large-scale efforts, whereas West sees complexity and connectivity as central elements of science.

As the book unfolds, it becomes more and more obvious that there is much more to performing science than structured processes or strictly empirical and quantitative foundations and hierarchies. According to Bruce West, research in science must shift its principles to allow for incorporation of modern perspectives that lead to inventive processes where established methodologies and knowledge are combined with novel procedures and entirely new contexts. While this assembly can lead to more chaotic, qualitative and emergent methods, West explains how society and science will benefit.

Modern science is handling the paradigm shift by building connections among disparate concepts, fields, and contexts to construct explanations of the world in ways that are not possible through strictly disciplinary means.

Scientists synthesize these connections to deepen understanding of underlying scientific principles. An example is the entry of information and computer science into the fold of science over the past half-century. These new elements will play significant roles in the future, and as Denning and Martell explain, “the laws of information reveal new possibilities and constraints that are not apparent from the laws of physics.” [2] West lays out a future path showing how science will construct a system of shared knowledge, appropriate levels of abstraction, and an enlightened human context that enables science to engage society. As to whether science is unified into one science with many modes of thought or there are many different, yet connected science disciplines may be mostly semantics, but what is clear and foundational are the strengths and necessities of the connections and integrations.

It is no accident that this move towards integration of science coincides with globalization and the proliferation of information networks. Integration has become sciences’ answer to the frustrations of science in the fragmented industrial-age where each discipline was hamstrung as a self-contained entity. The previous disciplinary, linear, empirical, assembly-line science has been challenged and overwhelmed by the dynamic societal changes at work in the information age. The much more collective form of modern science embraces the complexity and gathers information and ideas from all of science so that powerful scientific processes are created, stored, accessed and shared. Some of the largest, most successful mass collaborations in science are made through entirely open, virtual and social networks [3]. In place of disciplinary limitations, modern science has built a collective power that is more creative, more original, and more effective than any and all single disciplinary perspectives of the past. Bruce West’s excellent book embraces the complexity, connectedness, and fractional calculus mathematics to assemble future aim points and pathways for science to follow and scientists to contemplate.

6 April 2015

**Chris Arney**  
Professor of Mathematics  
Chair of Network Science  
United States Military Academy  
West Point, New York

## References

- [1] Asimov, I., *Change! Seventy-one Glimpses of the Future*, Houghton Mifflin (1981).
- [2] Denning, P.J., C.H. Martell, *Great Principles of Computing*, MIT Press (2015).
- [3] Nielsen, M., *Reinventing Discovery: The New Era of Networked Science*, Princeton University Press (2012).



- [4] Oleson, A., S. Brown. *The Pursuit of Knowledge in the Early American Republic*. Johns Hopkins Press, (1976).
- [5] Teller, E.. *The Pursuit of Simplicity*, Pepperdine University Press (1980).

# Preface

---

This book is based, in large part, on a lecture I gave at the *Network Frontier Workshop* at Northwestern University in December of 2013. The lecture was a tutorial intended to explain the quantitative reasoning entailed by the fractional calculus applied to complex physical, social and biological phenomena. The reason to study the fractional calculus is its inextricable link to complexity and that it provides a way to think systematically about complex phenomena in general and complex networks in particular. The ordinary calculus provided ways to organize our thinking about such physical phenomena as acoustic and electromagnetic waves, diffusion and even quantum mechanics. However, it does not do well with the complexity of earthquakes, avalanches, turbulence, cognition and stock market crashes. For these and other complex phenomena the notions of scaling and fractals were introduced and through them new ways of thinking were developed.

As is sometimes the case, the talk was so well received that I decided to write it up and submit it for publication. The *Colloquium* section of the *Reviews of Modern Physics* was interested in the manuscript and after a round of responding to reviewer comments and suggestions it was accepted for publication [22]. What became clear in responding to the reviewers' concerns was the existence of a wide interest in the material. After completing the paper it was obvious that an even more extended version of the presentation might have some value for the broader scientific community.

In fleshing out the paper to make the book reasonably self-contained an unintended perspective of science emerged. The view point that kept asserting itself is a resurgence of the *Natural Philosophy* of the seventeenth and eighteenth centuries. A perspective in which the pursuit of scientific knowledge and understanding has wisdom as its ultimate goal, and not merely the ordered accumulation of empirical facts necessary for a rational model of the world. The scientist that most personifies this ideal, in my view, is Leonardo da Vinci. As a scientist he was able to incorporate his artistic skills in his pursuit of understanding anatomy, botany and physics. As an engineer

his application of the arts and sciences into the planning of entertainments at the dinner parties of his benefactors and his equally gifted development of fortifications and armaments for their city states were unparalleled. Above it all was his singular ability to record his observation on how the world worked.

Herein I present the case for science regaining its position as *Natural Philosophy* and for reasserting its integrated nature, thereby pulling away from the disciplinary constructs devised to make the understanding of scientific growth and the complexity of phenomena intellectually manageable. The approach is unlike a text devoted to a pedagogical presentation of a specialized topic or a monograph focused on an author's area of research, the method of the present book is an effort to accomplish both these things while providing a rationale for why the reader may be interested in learning more about the fractional calculus. This book is for the researcher who has heard about many of these scientifically exotic activities but could not see how they fit into their own scientific interests or how they could be made compatible with the way they understand science. It is also for the novice who has not yet decided where their talents could be most productively applied.

I sent a draft of the manuscript for this book to a number of colleagues for comments. I am very grateful to those that responded, who include: Professors Chris Arney, Mario Bologna, Richard Magin and Michael Shlesinger. They provided multiple suggestions that vastly improved the book, but as always, any residual errors of fact or flaws in presentation are entirely my own.

# Acknowledgement

---

I wish to acknowledge the love and encouragement of my wife Sharon and to thank her for her good humor regarding what must seem to be a never ending stream of scientific thinking.



# Nomenclature

---

AMI	:	atrial myocardial infarction
AR	:	allometry relation
ARMA	:	auto-regressive integrated moving average
ARIFMA	:	auto-regressive integrated fractional moving average
CBF	:	cerebral blood flow
CLT	:	central limit theorem
CTRW	:	continuous time random walk
DMM	:	decision making model
ERH	:	echo response hypothesis
IPL	:	inverse power law
FDR	:	fluctuation-dissipation relation
FDWE	:	fractional diffusion-wave equation
FPE	:	Fokker-Planck equation
FFPE	:	fractional Fokker-Planck equation
FGn	:	fractional Gaussian noise
FK	:	fractional kinetics
FLE	:	fractional Langevin equation
FLogE	:	fractional logistic equation
FME	:	fractional master equation
FP	:	Fokker-Planck
FPSE	:	fractional phase space equation
FPU	:	Fermi, Pasta, Ulam
FRW	:	fractional random walk
FSH	:	fractional search hypothesis
GLE	:	generalized Langevin equation
GWF	:	generalized Weierstrass function
HRV	:	heart rate variability
LE	:	Langevin equation
LFD	:	local fractional derivative
ML	:	Mittag-Leffler

MLF	:	Mittag-Leffler function
MLM	:	Mittag-Leffler matrix
MLMF	:	Mittag-Leffler matrix function
MRI	:	magnetic resonance imaging
MRL	:	modified Riemann-Liouville
MW	:	Montroll-Weiss
NMR	:	nuclear magnetic resonance
OP	:	operations research
PCM	:	principle of complexity management
PDF	:	probability density function
PSE	:	phase space equation
RG	:	renormalization group
RGK	:	renormalization group kinetics
RL	:	Riemann-Liouville
RRW	:	Rayleigh random walk
SCLT	:	stochastic central limit theorem
TBI	:	traumatic brain injury
TBM	:	total body mass

# Contents

---

Foreword .....	v
Preface .....	ix
Acknowledgement .....	xi
Nomenclature.....	xiii
<b>Chapter 1</b> The Challenge of Complexity.....	1
1.1 Little Science, Big Science .....	5
1.2 Complexity.....	7
1.2.1 Networks.....	8
1.2.2 Information .....	9
1.2.3 Unintended Consequences .....	10
1.2.4 Mathematical Framework.....	12
1.3 Chapter Overviews.....	13
1.4 After Thoughts .....	18
References.....	21
<b>Chapter 2</b> Yesterday's Science .....	23
2.1 Simple Linearity.....	25
2.1.1 Linear Superposition .....	26
2.2 Complicated Linearity.....	29
2.2.1 FPU Problem .....	29
2.2.2 Langevin Equation .....	31
2.2.3 Fokker-Planck Equation .....	35
2.3 Nonlinear Dynamics.....	39



	2.3.1 A Little about Chaos.....	40
	2.3.2 Carleman Embedding .....	42
	2.3.3 Infinite-order Linear Representation.....	46
	2.4 After Thoughts .....	48
	2.5 Appendix Chapter 2.....	50
	References.....	52
<b>Chapter 3</b>	<b>New Ways of Thinking.....</b>	<b>55</b>
	3.1 Why Now? .....	56
	3.2 Through the Looking Glass .....	57
	3.2.1 A Previous Paradigm Shift.....	59
	3.2.2 A Contemporary Shift .....	62
	3.3 Non-differentiability can be Physical .....	67
	3.3.1 Fractal Evolution.....	68
	3.3.2 Complex Fractal Dimension .....	72
	3.4 The Size Effect .....	75
	3.4.1 Sleep-wake Scaling.....	78
	3.4.2 Natural Science.....	78
	3.4.3 Information Transfer Hypothesis.....	81
	3.4.4 Social Allometry .....	83
	3.4.5 Strength of Materials.....	84
	3.5 After Thoughts .....	84
	References.....	86
<b>Chapter 4</b>	<b>Simple Fractional Operators .....</b>	<b>91</b>
	4.1 Random Walks.....	92
	4.1.1 Rayleigh Type .....	93
	4.1.2 Fractional Type .....	95
	4.1.3 Climate Change.....	97
	4.2 Fractional Derivatives .....	99
	4.2.1 Fractional Differentials and Limits.....	100
	4.2.2 Differentiating Fractal Functions.....	102
	4.3 Fractional Rate Equations .....	105
	4.3.1 Distribution of Rates.....	105
	4.3.2 Viscoelastic Material Experiments .....	107
	4.3.3 Fractional-Order Bloch Equations.....	109
	4.3.4 Three-scale Brownian Motion.....	111
	4.4 After Thoughts .....	115
	References.....	118
<b>Chapter 5</b>	<b>Tomorrow's Dynamics.....</b>	<b>121</b>
	5.1 What We Think We Know; Linear Systems .....	122
	5.2 Fractional Linear System .....	124