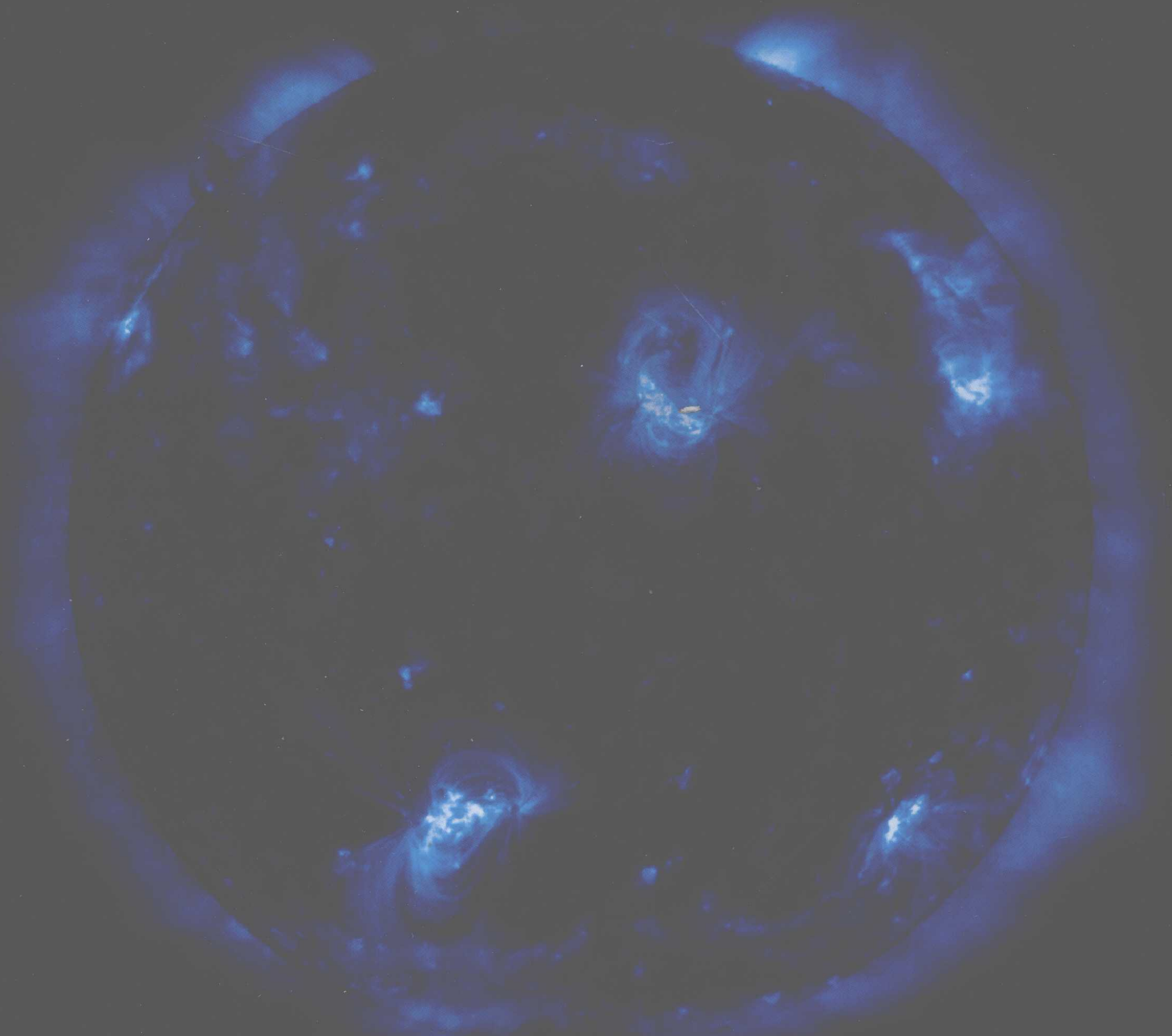
PRELIMINARY EDITION

Volume 2

COLLEGE MATHEMATICS

APPLIED TO THE REAL WORLD



Stefan Waner Steven R. Costenoble

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College Mathematics

Applied to the Real World

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Brooks/Cole
Thomson Learning™

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Preface

College Mathematics Applied to the Real World, Preliminary Second Edition, is a substantial revision of the First Edition of Finite Mathematics and Calculus Applied to the Real World. The book is intended for a two-semester finite mathematics and calculus course for the same audience. Like the First Edition, the Second Edition is designed to address the considerable challenge of generating enthusiasm and developing mathematical sophistication in an audience that is often ill-prepared for and disaffected by the traditional applied mathematics courses offered on many college campuses. Unlike the First Edition—and indeed unlike any other text on the market to date—the book is supported by, and linked with, an extensive and highly developed Internet web site (the "RealWorld" web site) containing interactive tutorials, on-line technology, optional supplementary material, and much more. As with the First Edition, the book is usable by itself, independent of the web site, but those instructors who wish to use graphing calculators or computers will find plenty of support both within the book and on the web site.

Our Approach to Pedagogy

Readability We would like students to read this book. We would like students to *enjoy* reading this book. Thus we have written the books in a conversational and student-oriented style, and have made frequent use of a question-and-answer dialogue format in order to encourage the development of the student's mathematical curiosity and intuition. We hope that this text will give the student insight into how a mathematician develops and thinks about mathematical ideas and their applications.

Real World Orientation We are particularly proud of the diversity, breadth and abundance of examples and exercises we have been able to include in this edition. A large number of these are based on real, referenced data from business, economics, the life sciences and the social sciences.

Adapting real data for pedagogical use can be tricky; available data can sometimes be numerically complex, intimidating for students, or incomplete. In this regard, we have learned many lessons from our experience in the First Edition, and we have modified and streamlined many of the real world applications, rendering them as tractable as any "made-up" application. At the same time, we have been careful to strike a pedagogically sound balance between applications based on real data and more traditional "generic" applications. Thus the density and selection of real data-based applications has been tailored to the pedagogical goals and appropriate difficulty level for each section.

Five Elements of Mathematical Pedagogy The "Rule of Three" is a common pedagogical theme in current reform-oriented texts. Accordingly, many of the central concepts, such as functions, solutions of systems of equations, limits, derivatives, and integrals, are discussed numerically, graphically and algebraically, and we go to some lengths to clearly delineate these distinctions. (See, for instance, the section headings in Chapters 1 and 2 in the Table of Contents.) As a Fourth element, we continue to incorporate verbal communication of mathematical concepts through our emphasis on verbalizing mathematical concepts, our discussions of rephrasing sentences into forms that easily go over to mathematical statements, and the communication and reasoning exercises at the end of each section.

The Fifth element, interactive discourse, is new to the Second Edition and, we believe, unique to the market. The interactive discourse element is implemented within the printed text through expanded use of question-and-answer dialogs, but most dramatically through the RealWorld web site. At the web site, students can interact with the material in several ways: through true-false quizzes on every topic; through interactive review exercises, and questions and answers in the tutorials (including multiple choice items with detailed feedback and Javascript-based interactive elements); and through on-line utilities that automate a variety of tasks, from graphing to regression and matrix algebra.

Exercise Sets We regard the strength of our exercise sets as one of the best features of the First Edition. Our substantial collection of exercises provides a wealth of material that can be used to challenge students at almost every level of preparation, and includes everything from straightforward drill exercises to interesting and rather challenging applications. We have also included, in virtually every section of every chapter, applications based on real data, communication and reasoning exercises useful for writing assignments, graphing calculator exercises, and amusing exercises.

Many of the scenarios used in application examples and exercises are revisited several times throughout the book. Thus, for instance, students will find themselves using a variety of techniques, from graphing through the use of derivatives to elasticity of demand, to maximize revenue in the same application. Reusing scenarios and important functions this way provides unifying threads and shows students the complex texture of real-life problems.

Some Distinguishing Features

- **Question-and-Answer Dialogue** One of the pedagogical tools we employ is the frequent use of informal question-and-answer dialogues which often anticipate the kind of questions that may occur to the student and also guide the student through the development of new concepts.
- **Extended Applications and Projects** Each chapter begins with the statement of an interesting problem that is returned to at the end of that chapter in a section titled "You're the Expert." This extended application uses and illustrates the central ideas of the chapter. The themes of these applications are varied, and they are designed to be as non-intimidating as possible. Thus, for example, we avoid pulling complicated formulas out of thin air, but focus instead on the development of mathematical models appropriate to the topics. Among the more interesting of these applications are an example using marginal analysis to design a strategy for regulating sulfur emissions, and the use of multiple linear regression to model household income in the US. These applications are ideal for assignment as projects, and to this end we have included groups of exercises at the end of each.
- **Before We Go On** Most examples are followed by supplementary discussions under the heading "Before we go on." These discussions may include a check on the answer, a discussion of the feasibility and significance of a solution, or an in-depth look at what the solution means.
- **Communication and Reasoning Exercises for Writing and Discussion** These are exercises designed to broaden the student's grasp of the mathematical concepts, and include

exercises in which the student is asked to provide his or her own examples to illustrate a point, design an application with a given solution, "fill in the blank" type exercises, and exercises that invite discussion and debate. These are often exercises with no single correct answer.

- **Conceptual and Computational Devices** The text features a wide variety of novel devices to assist the student in overcoming hurdles. These include a calculation thought experiment for the analysis and differentiation of complicated functions, the use of the remarkably quick and efficient "column integration" method for integration by parts, and a "template" method for evaluating definite integrals that avoids the common errors in signs.
- **Quick Examples** Most definition boxes include one or more straightforward examples that a student can use to solidify each new concept as soon as it is encountered.
- **Footnotes** We use footnotes throughout the text to provide interesting background, extended discussion, and various asides.
- **Thorough Integration of Spreadsheet and Graphing Technology** Guidance on the use of spreadsheets (we use Microsoft® Excel as our primary example) and graphing calculators (we use the TI-83 as our example) is thoroughly integrated throughout the discussion, examples, and exercise sets. In many examples we include a discussion of the use of spreadsheets or graphing technology to aid in the solution. Groups of exercises for which the use of technology is suggested or required appear throughout the exercise sets. Moreover, technology plays an important conceptual and pedagogical role in our presentation of some topics. For example, our discussion of curve sketching (Analyzing the Graph of a Function) has been written with graphing technology (spreadsheet or calculator) in mind, and we have used an approach that is well-suited to the sensible and increasingly popular practice of first using graphing technology to draw the graphs and then using calculus to explain the results. As a result, our approach to curve sketching is more concise, less rigid, and also less long-winded than the standard treatments. Some of the real power of technology is be seen in the chapter on applications of the integral, where we guide the student in the use of technology to analyze mathematical models based on real data, make projections, and calculate and graph moving averages. Discussion and exercises on the proper entry of functions into spreadsheets and graphing calculators and computers are included in the Algebra Review section of the appendix.

The RealWorld Web Site

Our site at http://www.hofstra.edu/~matscw/RealWorld has been evolving for several years with growing recognition in cyberspace. The interactive element that the RealWorld web site adds to our pedagogical approach is one that is naturally attuned to a new generation of college students raised in an environment in which computers suffuse both work and play. Students no longer need to be told to surf the Internet to search for assistance with school projects; the Internet has become a natural extension of their everyday environment.

- **On-Line Chapters** Our web site includes three complete on-line chapters on Logic, Trigonometric Functions, and Calculus Applied to Probability. These chapters incorporate the same pedagogy as the printed material, and can be readily printed out and distributed.
- **Extra Topics** We include complete interactive text and exercise sets for a selection of topics not ordinarily included in printed texts, but nevertheless often requested by

instructors. The text itself refers to these topics at appropriate points, and the instructor can decide whether to include this material.

- **Detailed Chapter Summaries** Our on-line comprehensive summaries review all the basic definitions and problem solving techniques discussed in each chapter. The summaries include additional examples for review and can be readily printed out. They also include interactive elements that reinforce the main ideas of each chapter.
- **Interactive True False Chapter Quizzes** We provide on-line interactive true-false quizzes based on the material in each chapter to assist the student in reviewing all the pertinent concepts and avoiding common pitfalls.
- **Chapter Review Exercises** The site also includes a collection of review questions taken from past tests and exams that do not appear in the printed text. This portion of the site is gradually growing as we add additional topics and new exercises.
- **On-Line Interactive Tutorials** The site includes highly interactive tutorials on several major topics. The student is guided through exercises in a manner that parallels the text section-by-section.
- **Downloadable Excel Tutorials** At the web site, the student will find detailed Excel tutorials for almost every section of the book. These interactive tutorials expand on the examples given in the text.
- **On-Line Utilities** Our collection of easy-to-use on-line utilities, written in Java™ and Javascript, allow the student to solve many of the technology-based application exercises directly on the web page. The utilities now available include an on-line Java grapher, a function evaluator, a matrix algebra utility, a linear programming utility, a simple regression page (linear, exponential, quadratic and cubic), a complete pivoting utility, a complete game theory utility (which can also play against the user), a Markov process utility, dice casting applets, a Monty Hall Java simulation, and much more. None of these utilities require anything more than a standard, Java-capable web browser such as the current versions of Netscape and Microsoft® Explorer.
- **Downloadable Software** In additional to the on-line utilities (which run on the web page) our site includes a suite of free and intuitive stand-alone Macintosh® programs for the Gauss-Jordan algorithm, the simplex method, matrix algebra, and function graphing.

Acknowledgments

This project would not have been possible without the contributions and suggestions of numerous colleagues, students and friends. We are particularly grateful to our many colleagues who class tested the various preliminary versions of the First Edition, and to those who are class testing this preliminary version of the Second Edition. We are also quite grateful to our editors at BrooksCole for their encouragement and guidance throughout the project. Specifically, we would like to thank Margot Hanis, for her enthusiasm and for believing this would work, and Curt Hinrichs for seeing the project through to completion.

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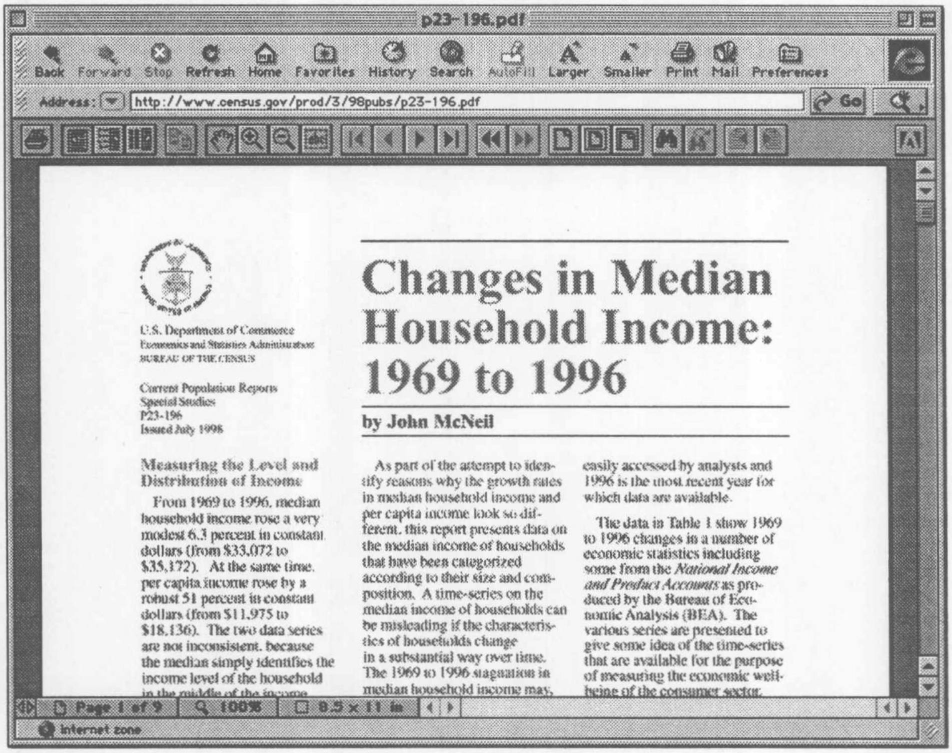
Real World Web Site

URL: **http://www.hofstra.edu/~matscw/RealWorld**

Note on the chapter numbering: The chapters are numbered as they will appear in the second editions of *Calculus Applied to the Real World* and *Finite Mathematics Applied to the Real World*.

Part I
Calculus

✣ Chapter 5—The Integral



As Assistant Personnel Manager for a large corporation, you have been asked to estimate the average annual wage earned by a worker in your company, from the time the worker is hired to the time the worker retires. You have data about wage increases. How will you estimate this average?

Introduction

Roughly speaking, calculus is divided into two parts: **differential calculus** (the calculus of derivatives) and **integral calculus**, which is the subject of this chapter and the next. Integral calculus is concerned with problems that are in some sense the reverse of the problems seen in differential calculus. For example, where differential calculus shows you how to compute the rate of change of a quantity, integral calculus shows you how to find the quantity if you know its rate of change. This is made precise in the **Fundamental Theorem of Calculus**. Integral calculus and the Fundamental Theorem of Calculus allow us to solve many problems in economics, physics, and geometry, including one of the oldest problems in mathematics, that of computing areas of complicated regions.

5.1 The Indefinite Integral

Having studied differentiation in the preceding chapters, we now discuss how to *reverse* the process. For instance, we might ask ourselves the following question.

Question If the derivative of $F(x)$ is $4x^3$, what was $F(x)$?
Answer After a moment's thought, we recognize $4x^3$ as the derivative of x^4. So, we might have $F(x) = x^4$. However, on thinking further, we realize that $F(x) = x^4 + 7$ would work just as well. In fact, $F(x) = x^4 + C$ would work for any number C. Thus, there are *infinitely many* possible answers to this question.

In fact, we will see shortly that the formula $F(x) = x^4 + C$ covers *all* possible answers to the question.

Let us give a name to what we have just seen.

Antiderivative
An **antiderivative** of a function f is a function F such that $F' = f$.

Quick Examples

1. An antiderivative of $4x^3$ is x^4 — Because the derivative of x^4 is $4x^3$

2. Another antiderivative of $4x^3$ is $x^4 + 7$ — Because the derivative of x^4+7 is $4x^3$

3. An antiderivative of $2x$ is $x^2 + 12$. — Because the derivative of x^2+12 is $2x$

Thus,

If the derivative of A(x) is B(x), then an antiderivative of B(x) is A(x).

We call the set of *all* antiderivatives of a function the **indefinite integral** of the function.

Indefinite Integral

$$\int f(x)\, dx$$

is read "the **indefinite integral** of $f(x)$ with respect to x," and stands for the set of all antiderivatives of f. Thus, $\int f(x)\, dx$ is a *collection of functions*; it is not a single function, nor a number. The function f that is being **integrated** is called the **integrand**, and the variable x is called the **variable of integration**. (The expression dx is short for "with respect to x.")

Quick Examples

1. $\int 4x^3\, dx = x^4 + C$ — Every possible antiderivative of $4x^3$ has the form $x^4 + C$.

2. $\int 2x\, dx = x^2 + C$ — Every possible antiderivative of $2x^2$ has the form $x^2 + C$.

The **constant of integration**, C, reminds us that we can substitute any number for C and get a different antiderivative.

Example 1

Check that $\int x\, dx = \frac{x^2}{2} + C$.

Solution We check by taking the derivative of the right-hand side:

$$\frac{d}{dx}\left(\frac{x^2}{2} + C\right) = \frac{2x}{2} = x \checkmark$$

Question If $F(x)$ is one antiderivative of $f(x)$, why must all other antiderivatives have the form $F(x) + C$?
Answer Suppose that $F(x)$ and $G(x)$ are both antiderivatives of $f(x)$, so that $F'(x) = G'(x)$. Consider what this means by looking at Figure 1.

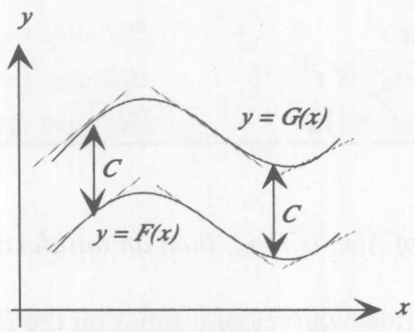


Figure 1

If $F'(x) = G'(x)$ for all x, then F and G have the *same slope* at each point. This means that their graphs must be *parallel*, and remain exactly the same vertical distance apart. But that is the same as saying that the functions differ by a constant[1], i.e., that $G(x) = F(x) + C$ for some constant C.

Now, we would like to make the process of finding indefinite integrals (antiderivatives) more mechanical. For example, it would be very nice to have a power rule for indefinite integrals, similar to the one we already have for derivatives. Two cases suggested by the examples above are:

[1] This argument can be turned into a more rigorous proof—that is, a proof that does not rely on geometric concepts such as "parallel graphs." We should also say that the result requires that F and G have the same derivative *on an interval* $[a, b]$.

$$\int x\,dx = \frac{x^2}{2} + C, \qquad \int x^3\,dx = \frac{x^4}{4} + C.$$

You should check the last equation by taking the derivative of its right-hand side. These cases suggest the following general statement.

Power Rule for the Indefinite Integral, Part I

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad \text{if } n \neq -1$$

In Words

To find the integral of x^n, add one to the exponent, then divide by the new exponent. This rule works provided n is not -1.

Quick Examples

1. $\int x^{55}\,dx = \dfrac{x^{56}}{56} + C$

2. $\int \dfrac{1}{x^{55}}\,dx = \int x^{-55}\,dx$ Exponent form

$= \dfrac{x^{-54}}{-54} + C$ Note: when we add 1 to -55, we get -54, *not* -56.

$= -\dfrac{1}{54x^{54}} + C$

3. $\int 1\,dx = x + C$ Since $1 = x^0$. This is an important special case.

Notes

The integral $\int 1\,dx$ is commonly written as $\int dx$.

Similarly, the integral $\int \dfrac{1}{x^{55}}\,dx$ may be written as $\int \dfrac{dx}{x^{55}}$.

We can easily check the power rule formula by taking the derivative of the right-hand side:

$$\frac{d}{dx}\left(\frac{x^{n+1}}{n+1} + C\right) = \frac{(n+1)x^n}{n+1} = x^n \ ✔$$

Question Why is there the restriction that $n \neq -1$?