# Scattering of Electromagnetic Waves

Numerical Simulations

Leung Tsang
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For ordering and customer service, call 1-800-CALL-WILEY.

### Library of Congress Cataloging in Publication Data

```
Tsang, Leung.
```

Scattering of electromagnetic waves: Numerical simulations / L. Tsang . . . [et al.]

p. cm. — (Wiley series in remote sensing)

Includes index.

ISBN 0-471-38800-9 (cloth: alk. paper)

1. Electromagnetic waves—Scattering—Mathematical models. I. Tsang, Leung. II. Series.

QC665.S3 S23 2000 621.36'78'015118—dc21

00-040864

Printed in the United States of America.

10 9 8 7 6 5 4 3 2 1

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To my family, Hannah, Clarisse, and Kaleb for their love.

— L. Tsang

To our families.

— J. A. Kong, K. H. Ding, C. O. Ao

### Scattering of Electromagnetic Waves

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### **PREFACE**

Electromagnetic wave scattering is an active, interdisciplinary area of research with myriad practical applications in fields ranging from atomic physics to medical imaging to geoscience and remote sensing. In particular, the subject of wave scattering by random discrete scatterers and rough surfaces presents great theoretical challenges due to the large degrees of freedom in these systems and the need to include multiple scattering effects accurately. In the past three decades, considerable theoretical progress has been made in elucidating and understanding the scattering processes involved in such problems. Diagrammatic techniques and effective medium theories remain essential for analytical studies; however, rapid advances in computer technology have opened new doors for researchers with the full power of Monte Carlo simulations in the numerical analysis of random media scattering. Numerical simulations allow us to solve the Maxwell equations exactly without the limitations of analytical approximations, whose regimes of validity are often difficult to assess. Thus it is our aim to present in these three volumes a balanced picture of both theoretical and numerical methods that are commonly used for tackling electromagnetic wave scattering problems. While our book places an emphasis on remote sensing applications, the materials covered here should be useful for students and researchers from a variety of backgrounds as in, for example, composite materials, photonic devices, optical thin films, lasers, optical tomography, and X-ray lithography. Introductory chapters and sections are also added so that the materials can be readily understood by graduate students. We hope that our book would help stimulate new ideas and innovative approaches to electromagnetic wave scattering in the years to come.

The increasingly important role of numerical simulations in solving electromagnetic wave scattering problems has motivated us to host a companion web site that contains computer codes on topics relevant to the book. These computer codes are written in the MATLAB programming language and are available for download from our web site at www.emwave.com. They are provided to serve two main purposes. The first is to supply our readers a hands-on laboratory for performing numerical experiments, through which the concepts in the book can be more dynamically relayed. The second is to give new researchers a set of basic tools with which they could quickly build on projects of their own. The fluid nature of the web site would also allow us to regularly update the contents and keep pace with new research developments.

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The present volume covers numerical simulation techniques and results for electromagnetic wave scattering in random media and rough surfaces. Due to the large degree of freedom associated with these systems, especially for 3-D scattering problems, fast computational methods are essential for maximizing returns from limited computational resources. Indeed, the subject of numerical electromagnetics has seen explosive growth in recent years. For lack of space, we choose to focus here on methods and techniques which are more directly related to our own research.

We begin in Chapter 1 with Monte Carlo simulations of a simple onedimensional random medium — a layered medium characterized by permittivity fluctuations. Simulation results are used to explain passive remote sensing measurements of the Antarctic firn. For two- and three-dimensional scattering, it is advantageous to formulate the problem in terms of surface integral equations where the unknowns are confined to a lower dimensionality. Numerical solutions of surface integral equations are often obtained through the method of moments (MoM). We also discuss a useful technique known as the discrete dipole approximation (DDA) for solving volume integral equation. The DDA can be used to model inhomogeneous, irregularily shaped object by discretizing it as a collection of point dipoles. In MoM and DDA, numerical solutions are obtained by approximating the integral equations with a set of linear equations. Thus matrix computation is an essential aspect of numerical electromagnetics. When the size of the system becomes very large, direct matrix inversion becomes inefficient, and iterative methods such as the conjugate gradient methods are often used instead. Iterative methods usually require repeated computations of matrix-vector multiplication, and for problems with translational invariance, it is possible to utilize fast Fourier transform (FFT) to speed up this operation. The use of FFT in conjunction with iterative solvers is the cornerstone of fast computational methods introduced later in this book. Therefore we discuss these topics at some length in Chapter 2.

The remainder of the book is divided into two main parts. Chapters 3–6 deal with simulations of rough surface scattering, while volume scattering simulations involving random discrete scatterers are studied in Chapters 7–13 (except Chapter 11 — which contains aspects of both rough surface and volume scattering). The topic of electromagnetic wave interactions with rough surfaces has important applications in microwave remote sensing of ocean surface, geophysical terrain, and agricultural fields as well as in the design and manufacturing of optical systems and X-ray lithography. In Chapter 3, we discuss scattering and emission by periodic rough surfaces. Two

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solution methods are used to solve this problem. The first is the T-matrix method, which makes use of Floquet mode expansions and the extended boundary condition. The T-matrix formulation is exact, but the resulting equations become ill-conditioned when the surface is very rough. The second method uses a surface integral equation approach with MoM. Although computationally more intensive than the T-matrix method, the surface integral equation approach is applicable to surfaces with deep corrugation. We also describe Ewald's method for speeding up calculations of the Green's function in periodic medium. This has applications in active research areas such as frequency selective surfaces and photonic bandgap materials.

In Chapter 4, we discuss one-dimensional random rough surface scattering. The core ideas behind rough surface scattering simulations are introduced here. We describe in details the discretization procedure for the surface integral equations in the Dirichlet, Neumann, and two-media problems. Numerical methods for generating Gaussian and fractal rough surface profiles are described. The issue of truncating the rough surface and limiting the computational domain is also an important one. We discuss two popular approaches. The first approach uses a tapered incident wave that illuminates only a part of the entire rough surface, while the second approach uses a periodic boundary condition. As described in Volume I, random rough surfaces are often characterized by their power spectra. This is convenient for theoretical work, but how well does it model reality? We include discussion of wave scattering from real-life rough surface profiles. In addition to simulating bistatic scattering from rough surfaces, we also take an in-depth look at emissivity calculations based on rough surface simulations, which impose much more stringent energy conservation requirement.

Chapters 5 and 6 are devoted respectively to fast computational methods in 1-D and 2-D rough surface scattering simulations. The development of fast computational methods is particularly important in scattering by 2-D rough surfaces (3-D scattering problem) where the number of unknowns can quickly escalate as we increase the surface size. Since real-life surfaces are 2-D, we emphasize in this book fast computational methods that can be applied to scattering by both 1-D and 2-D rough surfaces. We introduce the sparse matrix iterative approach with canonical grid (SMCG). In this method, the impedance matrix is split into a strong part that consists of near-neighbor interactions and a weak part that consists of all the rest. An iterative scheme such as the conjugate gradient method is adopted to solve the matrix equation. The strong matrix is sparse and can be easily handled. However, the weak interactions require the multiplication of the dense weak

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matrix with successive iterates and could therefore present a major computational bottleneck. To speed up such calculations, the concept of canonical grid (CG) is introduced. The essential nature of CG is that it is translationally invariant. In rough surface scattering problems, the CG is usually taken to be the mean flat surface. By translating the unknowns to the CG, the weak interactions can be performed simultaneously for all unknowns using FFT. This reduces memory requirements from  $\mathcal{O}(N^2)$  to  $\mathcal{O}(N)$  and operation counts from  $\mathcal{O}(N^2)$  to  $\mathcal{O}(N \log N)$ . We also introduce the physics-based two-grid (PBTG) method for dealing with lossy dielectric surfaces. In this method, a dense grid suitable for the lower half-space and a coarse grid suitable for the upper half-space are chosen. By taking advantage of the attenuative nature of the Green's function in the lower half-space and the slowly varying nature of the Green's function in the upper half-space with respect to the dense grid, one can achieve the accuracy of a single dense grid with the computational efficiency of a single coarse grid. Other fast methods discussed and illustrated in Chapter 5 include the steepest descent fast multipoles method (SDFMM) and the method of ordered multiple interactions (MOMI).

In contrast to rough surface scattering, volume scattering involving dense distributions of discrete scatterers is often a full-fledged 3-D scattering problem. The additional degree of freedom makes direct simulations of scattering coefficients rather difficult. Radiative transfer theory is commonly used for such problems, but the conventional approach fails to take into account of coherent multiple interactions between the scatterers. A better approach is to perform the scattering simulations on a test volume that contains a large number of scatterers but forms only a small part of the whole system. Coherent interactions are captured through the simulated extinction coefficients and phase functions, which can then be used in the dense medium radiative transfer equation (rigorously derived in Volume III) to solve the large-scale problem. These concepts are discussed in Chapter 7, where idealized randomly distributed point scatterers are used to illustrate the methods. The multiple scattering problem is formulated using the Foldy-Lax self-consistent equations.

In a dense medium, the correlation of scatterer positions could significantly affect the scattering results. The pair-distribution function quantifies the two-particle correlation property of the scatterers. In Chapter 8, we introduce the Percus-Yevick equation for the pair-distribution function and give closed-form solutions for hard and sticky spheres. For Monte Carlo simulations, statistical realizations of scatterer configurations are needed. Two

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methods are commonly employed to generate the particle positions: sequential addition and Metropolis shuffling, the latter method being more efficient when the particles are very closely packed. We show simulation results of the pair distribution functions for hard spheres and spheroids as well as sticky The simulated pair distribution functions are found to compare well with the Percus-Yevick pair distribution functions. Before dealing with 3-D dense media scattering, it is instructional to first study, in Chapter 9, the simpler problem of 2-D dense media scattering, where the volume scatterers are chosen to be infinitely long cylinders. We describe analytical pair distribution function and Monte Carlo simulations of particle positions in the 2-D case. The Foldy-Lax multiple scattering equations are then used to simulate extinction coefficients for densely packed hard and sticky cylinders. Finally, the SMCG method used in rough surface scattering is generalized to the volume scattering simulations. In Chapter 10, we perform 3-D dense media scattering calculations with dielectric spheres and spheroids. volume integral equation approach as well as the T-matrix approach based on the Foldy-Lax equations are described in details. Simulation results for the extinction coefficients and phase matrices are shown and compared with analytical approximations.

In Chapter 11, we describe the novel correlation phenomenon in random media scattering known as the memory effect, which manifests itself in wave scattering through the angular correlation function (ACF). ACF has been discussed in Chapter 6 of Volume I in the context of single scattering by point scatterers. Here, we provide a general derivation of the memory effect based on the statistical translational invariance of the random medium. The special property of ACF for random medium makes it a good candidate for the detection of a target embedded in random clutter. We explore such ideas by studying targets buried under rough surface and volume scatterers.

The subject of multiple scattering by finite cylinders has important applications in the remote sensing of vegetation as well as signal coupling among multiple vias in high frequency circuits. In Chapter 12, we consider scattering by vertical cylinders in the presence of reflective boundaries, which introduce additional complications. We discuss Monte Carlo simulations of these systems as well as simple analytical results that take into account of first and second order scattering. In Chapter 13, more realistic modeling of vegetation structures through stochastic Lindenmayer systems are presented. We compare scattering results from such systems obtained using the methods of DDA, the coherent addition approximation, and independent scattering.

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This book should provide a good mix of basic principles and current research topics. An introductory course in Monte Carlo simulations can cover most of Chapters 1, 2, 4, 5, 7, and 9.

### Acknowledgments

We would like to acknowledge the collaboration with our colleagues and graduate students. In particular, we wish to thank Professor Chi Chan of City University of Hong Kong, Professor Joel T. Johnson of Ohio State University, Dr. Robert T. Shin of MIT Lincoln Laboratory, and Dr. Dale Winebrenner of University of Washington. The graduate students who completed their Ph.D. theses from the University of Washington on random media scattering include Boheng Wen (1989), Kung-Hau Ding (1989), Shu-Hsiang Lou (1991), Charles E. Mandt (1992), Richard D. West (1994), Zhengxiao Chen (1994), Lisa M. Zurk (1995), Kyung Pak (1996), Guifu Zhang (1998), and Qin Li (2000). Much of their dissertation works are included in this book. Financial supports from the Air Force Office of Scientific Research, Army Research Office, National Aeronautics and Space Administration, National Science Foundation, Office of Naval Research, and Schlumberger-Doll Research Center for research materials included in this book are gratefully acknowledged. We also want to acknowledge the current UW graduate students who have helped to develop the numerical codes used throughout this book. These include Chi-Te Chen, Houfei Chen, Jianjun Guo, Chung-Chi Huang, and Lin Zhou. Special thanks are also due to Tomasz Grzegorczyk for proofreading on parts of the manuscript and Bae-Ian Wu for production assistance.

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February 2001

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