BEGINNING ALGEBRA

Patricia J. Cass

Elizabeth R. O'Connor

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Columbus State Community College

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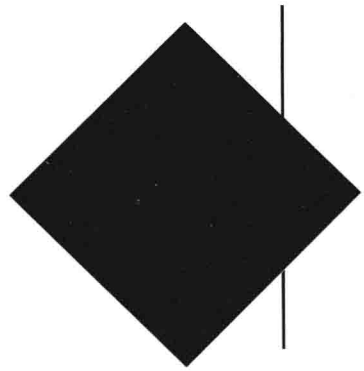
BEGINNING ALGEBRA

We dedicate this book to the many people who have supported us during the writing of this book—to our students who told us that they needed a book that could help them to learn algebra, to our colleagues who believed that our approaches were helping them to reach their students, and to our families who were willing to share their part of our time and attention with this project. Without all of you, none of this would be possible.



Preface

The content of beginning algebra courses varies little from college to college. There are many textbooks on the market that cover the material adequately, but most have one shortcoming–they do not offer enough worked examples for each topic presented.

As an instructor teaching this course, you know the difficulty of presenting the many concepts and problem-solving techniques to students with wide ranges of abilities, math experiences, and maturity. You never have enough class time to answer all the questions or work all the problems that the students want to see. Therefore, it is important that the students have a resource available—other than you—in order to get the help and answers they need. Students need a textbook they can read and understand, which will fill in the gaps and explain further what was not clear to them in class. *Beginning Algebra* meets the needs of both students and instructor by offering many more worked examples than traditional textbooks.

The material is presented in informal, non-threatening language that offers students reassurance and encouragement, without compromising the mathematical integrity. Especially aimed at mature, yet developing math students, the approach will not offend better-prepared students. These students can skip some of the explanations when the material is familiar, but will have the resource available for topics that are difficult for them.

In keeping with the NCTM Standard of helping students view math as problem solving, techniques are developed throughout the book that emphasize the analysis and organization of information. This text includes extensive coverage of word problems that have real-world applications in order to hold students' interest.

A unique feature, located at the end of each chapter, are the Thought-Provoking Problems. These involve written responses, analysis activities, calculator experimentation, and skill-stretching problem-solving. An instructor wanting to expand these areas might use the Thought-Provoking Problems as a starting point for class discussion questions, small group activities, extra credit questions, or other creative activities.

The algebra topics are presented in a traditional order, with a heavy concentration on formulas, evaluating, equation solving, and word problems. Since critical thinking and decision-making processes are being developed through this course, extensive practice with word problems is essential. Word problems are presented throughout the entire text, and include a wide variety of types so that teachers can pick and choose the ones most appropriate for their students.

The presentation of polynomial operations, which includes a chapter on factoring, gives the students a good foundation, using both trial-and-error and product-sum methods. Algebraic fractions follow immediately so that students can apply factoring skills to simplify and perform operations with rational expressions. The graphing chapter provides explanation and practice not only in graphing equations and inequalities, but also in writing equations from given information. Graphing is also used as a foundation for solving systems of equations.

Since the NCTM Standards recommend elimination of simplification processes that could be done by calculator, parts of Chapters 2, 7 and 13 can be considered optional. Chapter 14 details quadratic equations, develops the quadratic formula, solves quadratic equations, and solves word problems requiring a quadratic equation.

Chapter 15, "Practical Geometry," is included as a resource for students who need the basics of geometry for future courses. The presentation and exercises do not involve any algebraic skills so this chapter can be used at any point during the course.

This text is an excellent resource for students. It includes extensive examples with complete solutions. Chapter tests at the end of each chapter can be used as practice. The cumulative review exercises are grouped according to type so a student can see the connection among, for example, addition of whole numbers, fractions, decimals, signed numbers, and expressions. The continuous emphasis on word problems and critical thinking carries into the Thought-Provoking Problems. The appendices contain sections on math anxiety, math study skills, and an extensive fraction and decimal review for those students who need it.

Supplements

This text is accompanied by a comprehensive set of supplements available to all adopters:

1. *Instructor's Manual*: The Instructor's Manual includes notes and approaches for each chapter, one readiness and three mastery tests for each chapter, and answers to all even-numbered problems in the text.
2. *Transparency Masters*: The Transparency Masters have been extensively class-tested. They correspond with the topics covered in each chapter and provide full explanations of key concepts, and problems for the students to work during the instructor's presentation.
3. *EXPTest*: The EXPTest (available for IBM-PCs and compatibles on both 5 1/4″ and 3 1/2″ disks) is a computerized test bank containing hundreds of questions keyed specifically to the text from which instructors can choose, delete, and edit multiple choice, true-false, and open-ended questions. Additional questions may also be added for a more customized test.
4. *EXAMbuilder*: The EXAMbuilder is a computerized test bank similar to EXPTest, available for MacIntosh users.
5. *Developmental Mathematics Video Series*: These videos are available for qualified adopters. Through the departmental or college audiovisual library, students can check out these videos and use them to review material when they need additional help.
6. *Tutorial Software*: This text-specific, interactive tutorial software in Windows and MacIntosh formats allows students to practice the skills taught in the textbook. For each topic in the text, the tutorial provides multiple-choice, true-false, and short-answer exercises. The student is presented with a step-by-step solution to incorrectly answered problems. The program keeps track of right and wrong responses, and can report to the instructor on the students' progress.

Acknowledgments

We would like to thank our many reviewers, who have been so encouraging and generous in giving their time and expertise to suggest changes that improved these materials. Some of the educators who have played a particularly important role in helping to develop this text include:

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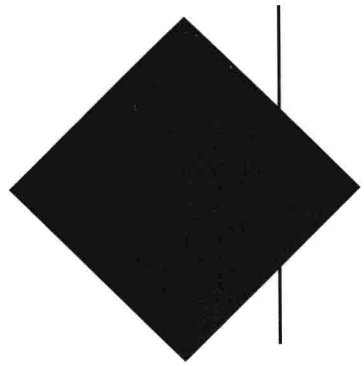
We would also like to thank our editors, Al Bruckner, Susan Gay, Kirby Lozyniak, and the staff at PWS. They have all been helpful and supportive during this interesting experience. Their expertise has added to the quality of our efforts and we greatly appreciate their time and talent.

We hope that you enjoy working with this text. Our faculty and students have. Many students have said that it is the first time that they really understood a math book. We hope that your experiences with this text are at least as good as ours have been.

Patricia J. Cass
Elizabeth R. O'Connor



To the Student

In order to succeed in this or any math course, you must start from the same point as every other student in your class and use the same vocabulary and the same rules. Since each of your past math experiences has been different, we need to begin with a few basics to insure that everyone is starting with a solid foundation. It is likely that many of the topics in these first few chapters will be familiar to you. Nevertheless, please read each section as if the material was new to you. Pay attention to details. Underline the important statements and outline each topic. If you study all of the sections, you will begin to fill in the gaps which exist in your own background. You will see how topics are related and you can progress with confidence. The topics which are truly review will bring quick success. The weaker areas will be repaired and reinforced and will soon become sources of confidence as well.

In spite of your past experiences with math texts, we urge you to READ this one. It is written with you, the adult learner, in mind. It doesn't matter if you are an adult of 17 or an adult of 45. You can pick things up quickly if they are explained clearly. You don't want to waste your time. You want to learn the facts and how to apply them so you can move on to a successful working knowledge of math. To help you accomplish this, we have included many example problems with detailed explanations. Chapter objectives are stated clearly at the beginning of each chapter and are summarized in the chapter review. Many practice problems in a variety of types and real-life applications are included, with the odd-numbered answers at the back of the book. Ask your instructor if the videotape and computer software supplements are available on your campus.

Whether or not you have had difficulty with mathematics courses in the past, we urge you to read and use the Appendix materials. They are not in the back of the book because we thought that they were unimportant, but rather because they are support sections for anyone learning math. You will find help in the Appendices to develop good study skills and some suggestions for overcoming math anxiety.

We wish you success in your mathematical endeavors, and hope that this text helps you to accomplish your goals.

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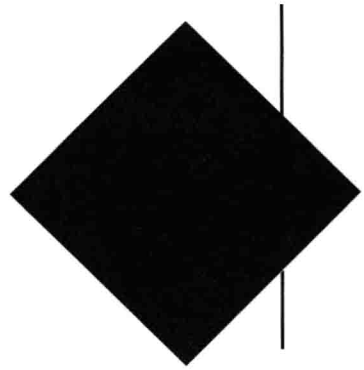

Contents

1.2 The "Why" of Algebra

You may think that you have never had any experience with algebra other than in a classroom. That, in fact, is probably not the case. Think about how you figure your grades in a class. If your grade for the term is based simply on the average of your scores on three tests, how do you calculate your grade? To find your average, you add the three scores together and divide the total by three. For instance, if your grades were 87, 95, and 94, you would figure the average this way:

$$87 + 95 + 94 = 276 \qquad 276 \div 3 = 92$$

Therefore, your average would be 92.

If your instructor wanted to find the averages for all the students in her class, she would have to figure each total separately and then divide by three. She might decide to use her computer to help her, instructing it to do the task over and over again with each student's grades. One way to do this would be to write a general statement that the computer could "understand" and would respond to by adding each set of three numbers and dividing the total by three to get the average. She would write a *formula*, using letters to keep it general, and the formula would give exactly those instructions. She might use this one:

$$A = \frac{a + b + c}{3}$$

where A stands for average and a, b, and c are the three scores. Any letter could be used to stand for the average, as long as what that letter represents is clearly defined, but A seems appropriate to stand for "average."

The use of letters and mathematical symbols to describe a process is **algebra**. Algebra and its formulas and symbols state in general terms what arithmetic numbers and symbols state in specific terms.

This isn't as complicated as it sounds. Think about the last vacation you planned. You knew approximately how fast you could travel in an hour (miles per hour) and about how many hours you could drive each day. Knowing that information, you could get an approximate idea of how many miles you could travel per day.

If you know you can drive about 50 miles in an hour, you know you can drive 100 miles in 2 hours and 150 miles in 3 hours, and so on. You can multiply the rate at which you travel by the number of hours you drive and find out how many miles you can travel. In general, the *distance* you can travel is equal to the *rate* at which you travel *times* your traveling *time*. In an algebraic formula,

$$\text{Distance} = \text{rate} \times \text{time}, \quad \text{or } D = rt$$

Again, you do not have to use D, r, and t, but they are the most logical choices to help you remember what the parts of the formula represent.

You could write this information in table form as well. In this example, the average rate at which you drive never changes, so the only things that vary are the time and the distance. The following table displays the relationship between the time and the distance traveled.

t (hours)	1	2	3	4	5	6	7
D (miles)	50	100	150	200	250	300	350

In Chapter 11 you will learn how to display this type of information on a two-dimensional graph called a Cartesian coordinate system. You could indicate the hours (t) on the horizontal scale and the miles or distance (D) on the vertical scale. Figure 1 shows how this graph might be drawn.

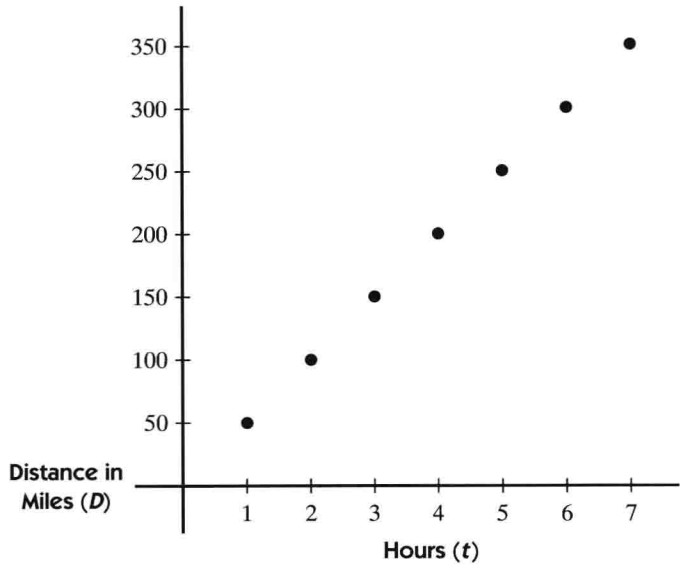

FIGURE 1

In any situation in which formulas are used to describe in a general manner something that is always true, algebraic techniques are used. Think about a sale at a store. The sale price is equal to the regular price minus the discount. You could also say $S = P - D$, where S stands for the sale price, P stands for the regular price, and D stands for the amount of the discount.

Computers were mentioned earlier. Any time a computer is used to keep records, figure prices, or do any other repetitive tasks, the person programming the computer must communicate with it in mathematical, algebraic formulas.

Think of other situations wherein you do a task over and over. Could you use a general statement and write a formula that would describe what you are doing?

These examples do not, of course, tell the whole story about algebra, but they do form a basis for understanding the difference between arithmetic and algebra. Arithmetic deals with very specific situations, and algebra generalizes those situations. Remember that the use of a letter symbol (called a **variable**) instead of a number symbol (called a **numeral**) generalizes what you are doing and says that the relationship can be true in more than one instance.

Algebra	Arithmetic
General statement of a relationship between quantities that is always true.	Specific problem involving calculations to arrive at an answer.
Sale price = Regular price − Discount $S = P - D$	Find the sale price of a \$120 coat if a \$30 discount is offered.
Distance = rate × time $D = rt$	How far can you travel in 6 hours if you average 55 miles per hour?

In order to translate from words to symbols, you should become familiar with certain terms commonly used to indicate arithmetic operations. You need to recognize these terms and to know which operation each implies.

Sum ⟶ addition
Difference ⟶ subtraction
Product ⟶ multiplication
Quotient ⟶ division

Remember these terms so that you can translate correctly from words to symbols when writing formulas.