HZ BOOKS

McGraw Hill Education

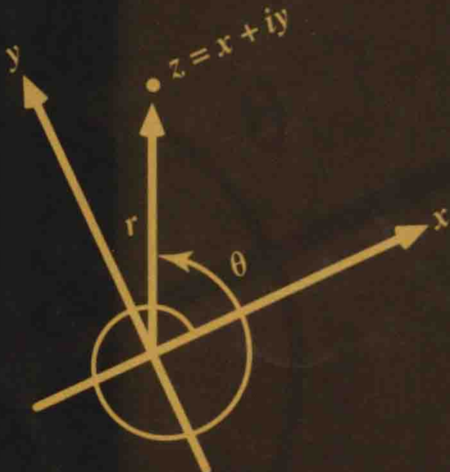
华章数学原版精品系列

复变函数及应用

（英文版·第9版）

Ninth Edition

Complex Variables *and Applications*



（美）James Ward Brown · Ruel V. Churchill 著
密歇根大学

华章数学原版精品系列

复变函数及应用

（英文版 · 第9版）

Complex Variables and Applications

(Ninth Edition)

（美）James Ward Brown Ruel V. Churchill 著
密歇根大学

机械工业出版社
China Machine Press

图书在版编目（CIP）数据

复变函数及应用（英文版·第9版）/（美）布朗（Brown，J. W.），（美）丘吉尔（Churchill, R. V.）著. —北京：机械工业出版社，2014.7
（华章数学原版精品系列）
书名原文：Complex Variables and Applications, Ninth Edition

ISBN 978-7-111-47087-8

I. 复… II. ① 布… ② 丘… III. 复变函数－英文 IV. O174.5

中国版本图书馆 CIP 数据核字（2014）第 131683 号

本书版权登记号：图字：01-2014-3567

复变函数及应用

[美] James Ward Brown 等著

出版发行：机械工业出版社（北京市西城区百万庄大街 22 号 邮政编码：100037）

责任编辑：明永玲　　责任校对：殷 虹

印　刷：北京瑞德印刷有限公司　　版　次：2014 年 7 月第 1 版第 1 次印刷

开　本：170mm × 242mm 1/16　　印　张：29.5

书　号：ISBN 978-7-111-47087-8　　定　价：85.00 元

凡购本书，如有缺页、倒页、脱页，由本社发行部调换

客服热线：（010）88378991 88361066　　投稿热线：（010）88379604

购书热线：（010）68326294 88379649 68995259　　读者信箱：hzjsj@hzbook.com

PREFACE

This book is a thorough revision of its earlier eighth edition, which was published in 2009. That edition has served, just as the earlier ones did, as a textbook for a one-term introductory course in the theory and application of functions of a complex variable. This new edition preserves the basic content and style of the earlier ones, the first two of which were written by the late Ruel V. Churchill alone.

The book has always had two main objectives.

(*a*) The first is to develop those parts of the theory that are prominent in applications of the subject.

(*b*) The second objective is to furnish an introduction to applications of residues and conformal mapping. The applications of residues include their use in evaluating real improper integrals, finding inverse Laplace transforms, and locating zeros of functions. Considerable attention is paid to the use of conformal mapping in solving boundary value problems that arise in studies of heat conduction and fluid flow. Hence the book may be considered as a companion volume to the authors' text *Fourier Series and Boundary Value Problems,* where another classical method for solving boundary value problems in partial differential equations is developed.

The first nine chapters of this book have for many years formed the basis of a three-hour course given each term at The University of Michigan. The final three chapters have fewer changes and are mostly intended for self-study and reference. The classes using the book have consisted mainly of seniors concentrating in mathematics, engineering, or one of the physical sciences. Before taking the course, the students have completed at least a three-term calculus sequence and a first course in ordinary differential equations. If mapping by elementary functions is desired earlier in the course, one can skip to Chap. 8 immediately after Chap. 3 on elementary functions and then return to Chap. 4 on integrals.

We mention here a sample of the changes in this edition, some of which were suggested by students and people teaching from the book. A number of topics have been moved from where they were. For example, although harmonic functions are still introduced in Chap. 2, harmonic conjugates have been moved to Chap. 9, where they are actually needed. Another example is the moving of the derivation of an important inequality needed in proving the fundamental theorem of algebra (Chap. 4) to Chap. 1, where related inequalities are introduced. This has the advantage of enabling the reader to concentrate on such inequalities when they are grouped together and also of making the proof of the fundamental theorem of algebra reasonably brief and efficient without taking the reader on a distracting side-trip. The introduction to the concept of mapping in Chap. 2 is shortened somewhat in this edition, and only the mapping $w = z^2$ is emphasized in that chapter. This was suggested by some users of the last edition, who felt that a detailed consideration of $w = z^2$ is sufficient in Chap. 2 in order to illustrate concepts needed there. Finally, since most of the series, both Taylor and Laurent, that are found and discussed in Chap. 5 rely on the reader's familiarity with just six Maclaurin series, those series are now grouped together so that the reader is not forced to hunt around for them whenever they are needed in finding other series expansions. Also, Chap. 5 now contains a separate section, following Taylor's theorem, devoted entirely to series representations involving negative powers of $z - z_0$. Experience has shown that this is especially valuable in making the transformation from Taylor to Laurent series a natural one.

This edition contains many new examples, sometimes taken from the exercises in the last edition. Quite often the examples follow in a separate section immediately following a section that develops the theory to be illustrated.

The clarity of the presentation has been enhanced in other ways. Boldface letters have been used to make definitions more easily identified. The book has fifteen new figures, as well as a number of existing ones that have been improved. Finally, when the proofs of theorems are unusually long, those proofs are clearly divided into parts. This happens, for instance, in the proof (Sec. 49) of the three-part theorem regarding the existence and use of antiderivatives. The same is true of the proof (Sec. 51) of the Cauchy-Goursat theorem. Finally, there is a Student's Solutions Manual (ISBN: 978-0-07-352899-1; MHID: 0-07-352899-4) that is available. It contains solutions of selected exercises in Chapters 1 through 7, covering the material through residues.

In order to accommodate as wide a range of readers as possible, there are footnotes referring to other texts that give proofs and discussions of the more delicate results from calculus and advanced calculus that are occasionally needed. A bibliography of other books on complex variables, many of which are more advanced, is provided in Appendix 1. A table of conformal transformations that are useful in applications appears in Appendix 2.

As already indicated, some of the changes in this edition have been suggested by users of the earlier edition. Moreover, in the preparation of this new edition, continual interest and support has been provided by a variety of other people, especially the staff at McGraw-Hill and my wife Jacqueline Read Brown.

James Ward Brown

CONTENTS

CHAPTER

1

COMPLEX NUMBERS

In this chapter, we survey the algebraic and geometric structure of the complex number system. We assume various corresponding properties of real numbers to be known.

1. SUMS AND PRODUCTS

Complex numbers can be defined as ordered pairs (x, y) of real numbers that are to be interpreted as points in the ***complex plane***, with rectangular coordinates x and y, just as real numbers x are thought of as points on the real line. When real numbers x are displayed as points $(x, 0)$ on the ***real axis***, we write $x = (x, 0)$; and it is clear that the set of complex numbers includes the real numbers as a subset. Complex numbers of the form $(0, y)$ correspond to points on the y axis and are called ***pure imaginary numbers*** when $y \neq 0$. The y axis is then referred to as the ***imaginary axis***.

It is customary to denote a complex number (x, y) by z, so that (see Fig. 1)

(1) $$z = (x, y).$$

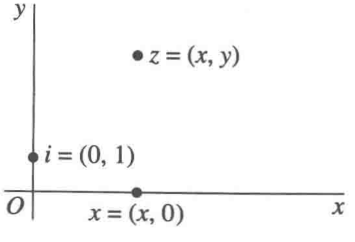


FIGURE 1

The real numbers x and y are, moreover, known as the ***real and imaginary parts*** of z, respectively, and we write

$$x = \operatorname{Re} z, \qquad y = \operatorname{Im} z. \tag{2}$$

Two complex numbers z_1 and z_2 are ***equal*** whenever they have the same real parts and the same imaginary parts. Thus the statement $z_1 = z_2$ means that z_1 and z_2 correspond to the same point in the complex, or z, plane.

The ***sum*** $z_1 + z_2$ and ***product*** $z_1 z_2$ of two complex numbers

$$z_1 = (x_1, y_1) \quad \text{and} \quad z_2 = (x_1, y_1)$$

are defined as follows:

$$(x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2), \tag{3}$$

$$(x_1, y_1)(x_2, y_2) = (x_1 x_2 - y_1 y_2, y_1 x_2 + x_1 y_2). \tag{4}$$

Note that the operations defined by means of equations (3) and (4) become the usual operations of addition and multiplication when restricted to the real numbers:

$$(x_1, 0) + (x_2, 0) = (x_1 + x_2, 0),$$
$$(x_1, 0)(x_2, 0) = (x_1 x_2, 0).$$

The complex number system is, therefore, a natural extension of the real number system.

Any complex number $z = (x, y)$ can be written $z = (x, 0) + (0, y)$, and it is easy to see that $(0, 1)(y, 0) = (0, y)$. Hence

$$z = (x, 0) + (0, 1)(y, 0);$$

and if we think of a real number as either x or $(x, 0)$ and let i denote the pure imaginary number (0,1), as shown in Fig. 1, it is clear that*

$$z = x + iy. \tag{5}$$

Also, with the convention that $z^2 = zz$, $z^3 = z^2 z$, etc., we have

$$i^2 = (0, 1)(0, 1) = (-1, 0),$$

or

$$i^2 = -1. \tag{6}$$

Because $(x, y) = x + iy$, definitions (3) and (4) become

$$(x_1 + iy_1) + (x_2 + iy_2) = (x_1 + x_2) + i(y_1 + y_2), \tag{7}$$

$$(x_1 + iy_1)(x_2 + iy_2) = (x_1 x_2 - y_1 y_2) + i(y_1 x_2 + x_1 y_2). \tag{8}$$

*In electrical engineering, the letter j is used instead of i.

Observe that the right-hand sides of these equations can be obtained by formally manipulating the terms on the left as if they involved only real numbers and by replacing i^2 by -1 when it occurs. Also, observe how equation (8) tells us that *any complex number times zero is zero.* More precisely,

$$z \cdot 0 = (x + iy)(0 + i0) = 0 + i0 = 0$$

for any $z = x + iy$.

2. BASIC ALGEBRAIC PROPERTIES

Various properties of addition and multiplication of complex numbers are the same as for real numbers. We list here the more basic of these algebraic properties and verify some of them. Most of the others are verified in the exercises.

The commutative laws

$$z_1 + z_2 = z_2 + z_1, \quad z_1 z_2 = z_2 z_1 \tag{1}$$

and the associative laws

$$(z_1 + z_2) + z_3 = z_1 + (z_2 + z_3), \quad (z_1 z_2) z_3 = z_1 (z_2 z_3) \tag{2}$$

follow easily from the definitions in Sec. 1 of addition and multiplication of complex numbers and the fact that real numbers have corresponding properties. The same is true of the distributive law

$$z(z_1 + z_2) = zz_1 + zz_2. \tag{3}$$

EXAMPLE. If

$$z_1 = (x_1, y_1) \quad \text{and} \quad z_2 = (x_2, y_2),$$

then

$$z_1 + z_2 = (x_1 + x_2, y_1 + y_2) = (x_2 + x_1, y_2 + y_1) = z_2 + z_1.$$

According to the commutative law for multiplication, $iy = yi$. Hence one can write $z = x + yi$ instead of $z = x + iy$. Also, because of the associative laws, a sum $z_1 + z_2 + z_3$ or a product $z_1 z_2 z_3$ is well defined without parentheses, as is the case with real numbers.

The additive identity $0 = (0, 0)$ and the multiplicative identity $1 = (1, 0)$ for real numbers carry over to the entire complex number system. That is,

$$z + 0 = z \quad \text{and} \quad z \cdot 1 = z \tag{4}$$

for every complex number z. Furthermore, 0 and 1 are the only complex numbers with such properties (see Exercise 8).

There is associated with each complex number $z = (x, y)$ an additive inverse

$$-z = (-x, -y), \tag{5}$$

satisfying the equation $z + (-z) = 0$. Moreover, there is only one additive inverse for any given z, since the equation

$$(x, y) + (u, v) = (0, 0)$$

implies that

$$u = -x \quad \text{and} \quad v = -y.$$

For any *nonzero* complex number $z = (x, y)$, there is a number z^{-1} such that $zz^{-1} = 1$. This multiplicative inverse is less obvious than the additive one. To find it, we seek real numbers u and v, expressed in terms of x and y, such that

$$(x, y)(u, v) = (1, 0).$$

According to equation (4), Sec. 1, which defines the product of two complex numbers, u and v must satisfy the pair

$$xu - yv = 1, \quad yu + xv = 0$$

of linear simultaneous equations; and simple computation yields the unique solution

$$u = \frac{x}{x^2 + y^2}, \quad v = \frac{-y}{x^2 + y^2}.$$

So *the* multiplicative inverse of $z = (x, y)$ is

$$z^{-1} = \left(\frac{x}{x^2 + y^2}, \frac{-y}{x^2 + y^2}\right) \qquad (z \neq 0). \tag{6}$$

The inverse z^{-1} is not defined when $z = 0$. In fact, $z = 0$ means that $x^2 + y^2 = 0$; and this is not permitted in expression (6).

EXERCISES

1. Verify that

(*a*) $(\sqrt{2} - i) - i(1 - \sqrt{2}i) = -2i$;

(*b*) $(2, -3)(-2, 1) = (-1, 8)$;

(*c*) $(3, 1)(3, -1)\left(\frac{1}{5}, \frac{1}{10}\right) = (2, 1)$.

2. Show that

(*a*) $\text{Re}(iz) = -\text{Im}\, z$;

(*b*) $\text{Im}(iz) = \text{Re}\, z$.

3. Show that $(1 + z)^2 = 1 + 2z + z^2$.

4. Verify that each of the two numbers $z = 1 \pm i$ satisfies the equation $z^2 - 2z + 2 = 0$.

5. Prove that multiplication of complex numbers is commutative, as stated at the beginning of Sec. 2.

6. Verify

(*a*) the associative law for addition of complex numbers, stated at the beginning of Sec. 2;

(*b*) the distributive law (3), Sec. 2.

7. Use the associative law for addition and the distributive law to show that

$$z(z_1 + z_2 + z_3) = zz_1 + zz_2 + zz_3.$$

8. (*a*) Write $(x, y) + (u, v) = (x, y)$ and point out how it follows that the complex number $0 = (0, 0)$ is unique as an additive identity.

(*b*) Likewise, write $(x, y)(u, v) = (x, y)$ and show that the number $1 = (1, 0)$ is a unique multiplicative identity.

9. Use $-1 = (-1, 0)$ and $z = (x, y)$ to show that $(-1)z = -z$.

10. Use $i = (0, 1)$ and $y = (y, 0)$ to verify that $-(iy) = (-i)y$. Thus show that the additive inverse of a complex number $z = x + iy$ can be written $-z = -x - iy$ without ambiguity.

11. Solve the equation $z^2 + z + 1 = 0$ for $z = (x, y)$ by writing

$$(x, y)(x, y) + (x, y) + (1, 0) = (0, 0)$$

and then solving a pair of simultaneous equations in x and y.

Suggestion: Use the fact that no real number x satisfies the given equation to show that $y \neq 0$.

Ans. $z = \left(-\frac{1}{2}, \pm\frac{\sqrt{3}}{2}\right)$.

3. FURTHER ALGEBRAIC PROPERTIES

In this section, we mention a number of other algebraic properties of addition and multiplication of complex numbers that follow from the ones already described in Sec. 2. Inasmuch as such properties continue to be anticipated because they also apply to real numbers, the reader can easily pass to Sec. 4 without serious disruption.

We begin with the observation that the existence of multiplicative inverses enables us to show that *if a product $z_1 z_2$ is zero, then so is at least one of the factors z_1 and z_2.* For suppose that $z_1 z_2 = 0$ and $z_1 \neq 0$. The inverse z_1^{-1} exists; and any complex number times zero is zero (Sec. 1). Hence

$$z_2 = z_2 \cdot 1 = z_2\left(z_1 z_1^{-1}\right) = \left(z_1^{-1} z_1\right) z_2 = z_1^{-1}(z_1 z_2) = z_1^{-1} \cdot 0 = 0.$$

That is, if $z_1 z_2 = 0$, either $z_1 = 0$ or $z_2 = 0$; or possibly both of the numbers z_1 and z_2 are zero. Another way to state this result is that *if two complex numbers z_1 and z_2 are nonzero, then so is their product $z_1 z_2$.*

Subtraction and division are defined in terms of additive and multiplicative inverses:

$$z_1 - z_2 = z_1 + (-z_2), \tag{1}$$

$$\frac{z_1}{z_2} = z_1 z_2^{-1} \qquad (z_2 \neq 0). \tag{2}$$

Thus, in view of expressions (5) and (6) in Sec. 2,

$$z_1 - z_2 = (x_1, y_1) + (-x_2, -y_2) = (x_1 - x_2, y_1 - y_2) \tag{3}$$

and

$$\frac{z_1}{z_2} = (x_1, y_1)\left(\frac{x_2}{x_2^2 + y_2^2}, \frac{-y_2}{x_2^2 + y_2^2}\right) = \left(\frac{x_1x_2 + y_1y_2}{x_2^2 + y_2^2}, \frac{y_1x_2 - x_1y_2}{x_2^2 + y_2^2}\right) \qquad (z_2 \neq 0) \tag{4}$$

when $z_1 = (x_1, y_1)$ and $z_2 = (x_2, y_2)$.

Using $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$, one can write expressions (3) and (4) here as

$$z_1 - z_2 = (x_1 - x_2) + i(y_1 - y_2) \tag{5}$$

and

$$\frac{z_1}{z_2} = \frac{x_1x_2 + y_1y_2}{x_2^2 + y_2^2} + i\frac{y_1x_2 - x_1y_2}{x_2^2 + y_2^2} \qquad (z_2 \neq 0). \tag{6}$$

Although expression (6) is not easy to remember, it can be obtained by writing (see Exercise 7)

$$\frac{z_1}{z_2} = \frac{(x_1 + iy_1)(x_2 - iy_2)}{(x_2 + iy_2)(x_2 - iy_2)}, \tag{7}$$

multiplying out the products in the numerator and denominator on the right, and then using the property

$$\frac{z_1 + z_2}{z_3} = (z_1 + z_2)z_3^{-1} = z_1z_3^{-1} + z_2z_3^{-1} = \frac{z_1}{z_3} + \frac{z_2}{z_3} \qquad (z_3 \neq 0). \tag{8}$$

The motivation for starting with equation (7) appears in Sec. 5.

EXAMPLE. The method is illustrated below:

$$\frac{4 + i}{2 - 3i} = \frac{(4 + i)(2 + 3i)}{(2 - 3i)(2 + 3i)} = \frac{5 + 14i}{13} = \frac{5}{13} + \frac{14}{13}i.$$