VCE MATHEMATICS UNITS 1 AND 2

MATHS Quest 11

Mathematical Methods CAS

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Jennifer **NOLAN**
Geoff **PHILLIPS** Ross **ALLEN**
David **PHILLIPS** Caroline **DENNEY**

VCE MATHEMATICS UNITS 1 AND 2

MATHS Quest 11

Mathematical Methods CAS

Jennifer **Nolan** • Geoff **Phillips** • Ross **Allen**
David **Phillips** • Caroline **Denney**

Contributing authors
George Dimitriadis • Nicolaos Karanikolas • Howard Liston
Gayle Roberts • Douglas Scott

Support material
John Dowsey • Dennis Fitzgerald • Emily Hui • Caroline Mews
Vinod Narayan • Peter Swain • David Tynan • Ian Younger
Wayne Youngs • Simone Richardson • Dina Antoniou • Norrene Hill

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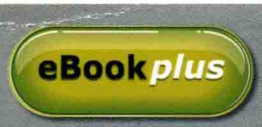

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- a student textbook with accompanying eBookPLUS and
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Career profiles and *History of mathematics* place mathematical concepts in context.

Investigations, often suggesting the use of technology, provide further discovery learning opportunities.

Each chapter concludes with a *summary* and *chapter review* exercise containing examination style questions (multiple choice, short answer and analysis) which help consolidate students' learning of new concepts. *Technology-free questions* covering all aspects of the course appear at appropriate intervals to prepare students for the Year 12 technology-free examination.

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The accompanying eBookPLUS contains the entire student textbook plus additional exercises. Students may use the eBookPLUS on laptops, school or home computers, and cut and paste material for revision or assignments.

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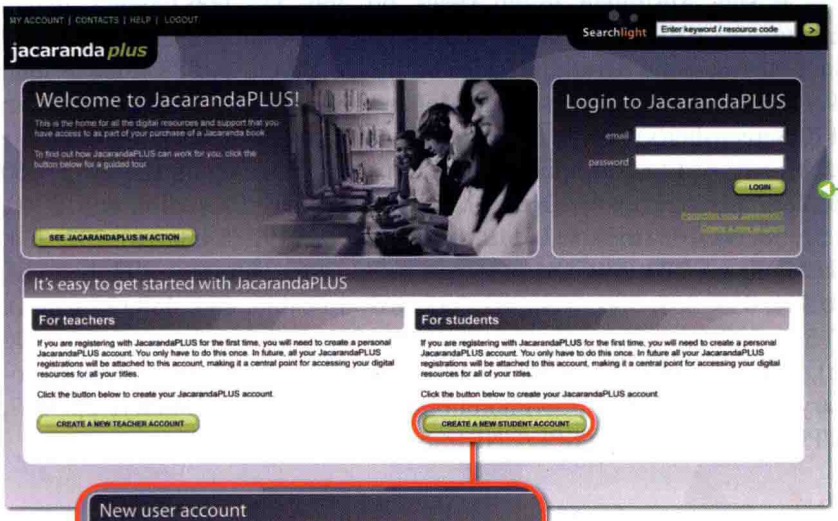

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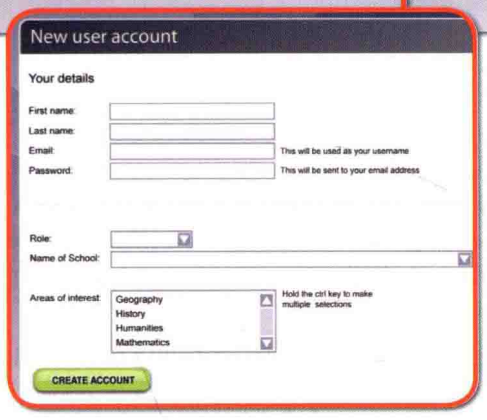

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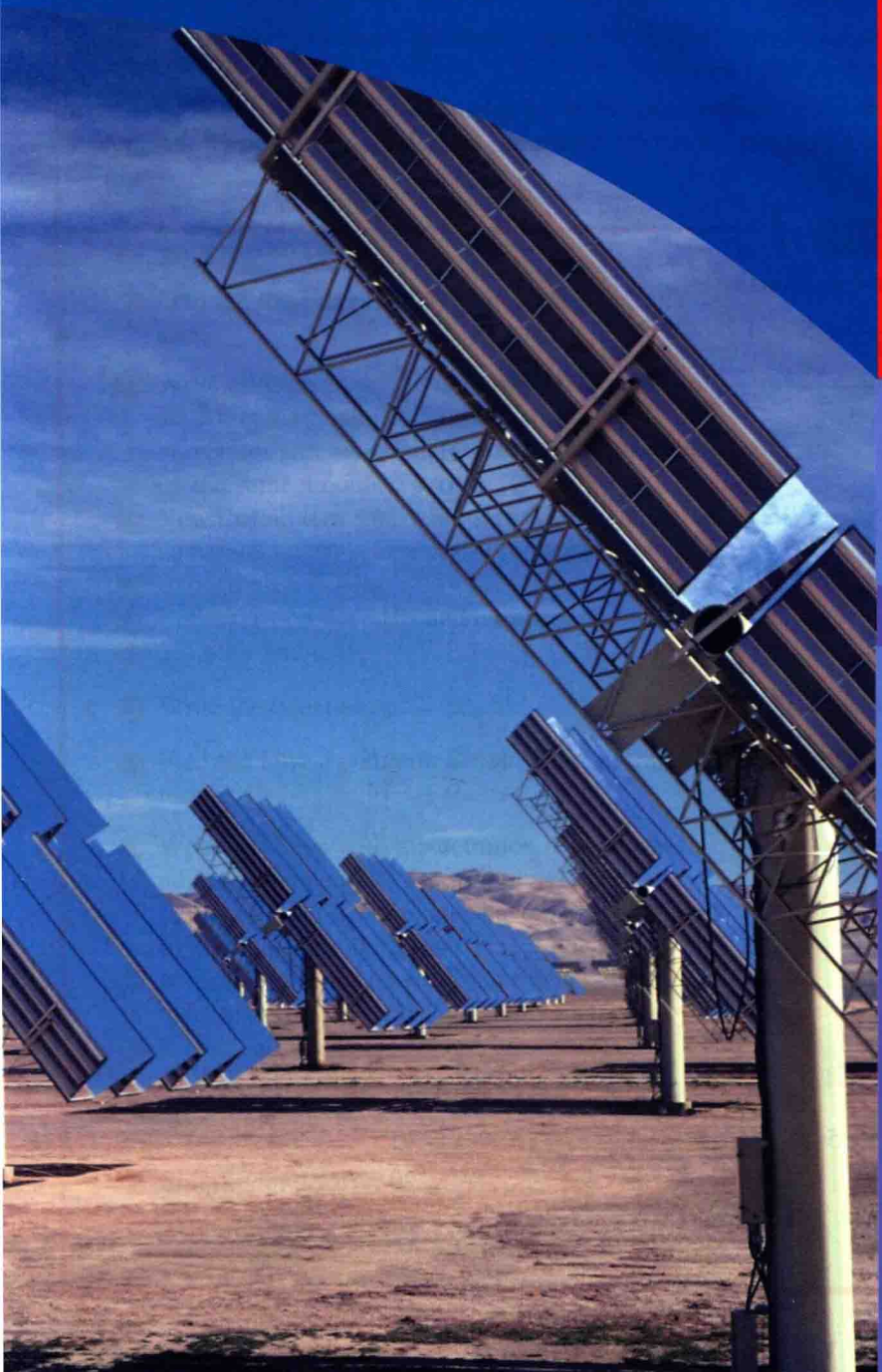
1 Linear functions



VCE coverage

Areas of study

Unit 1 • Functions and graphs
• Algebra

In this chapter

1A Solving linear equations and inequalities
1B Rearrangement and substitution
1C Gradient of a straight line
1D Equations of the form $y = mx + c$
1E Sketching linear graphs using intercepts
1F Simultaneous equations
1G Perpendicular lines
1H Finding the equation of a straight line
1I Distance between two points
1J Midpoint of a segment
1K Linear modelling

Solving linear equations and inequalities

A linear equation is one that involves a pronumeral raised to the power of 1 only. Recall that x^1 is the same as x, so a linear equation involving x would contain only x's, and not $x^{\frac{1}{2}}$, x^2, x^3, x^{-1}, x^{-2} and so on.

For example, $y = 7x - 3$, $ax + by = c$ and $\frac{8}{3}x + 1 = 9$ are all linear equations.

Note that $y = \frac{1}{x}$ is not a linear equation, as the $\frac{1}{x}$ is really x^{-1}. To isolate a particular pronumeral — known as 'making x (or whatever the pronumeral is) the subject', we focus on the pronumeral by 'undoing' other terms and operations. Remember to do the same operation to both sides of an equation, in the reverse order to that originally used to make up the equation.

Though the focus of this chapter is linear equations, some other types of equation will be included for skills practice.

When there is only one pronumeral involved in an equation, we may attempt to solve and find a numerical value by rearranging to make the pronumeral the subject.

WORKED Example 1

Solve the following linear equations.

a $7x - 4 = 17$ **b** $\frac{3x-2}{4} + 5 = 1$ **c** $2\left(\frac{4x}{5} - 1\right) = 6$

THINK	WRITE
a 1 Write the equation.	**a** $7x - 4 = 17$
2 Add 4 to both sides.	$7x = 21$
3 Divide both sides by 7.	$x = \frac{21}{7}$
	$x = 3$
b 1 Write the equation.	**b** $\frac{3x-2}{4} + 5 = 1$
2 Subtract 5 from both sides.	$\frac{3x-2}{4} = -4$
3 Multiply both sides by 4.	$3x - 2 = -16$
4 Add 2 to both sides.	$3x = -14$
5 Divide both sides by 3.	$x = -\frac{14}{3}$
c 1 Write the equation.	**c** $2\left(\frac{4x}{5} - 1\right) = 6$
2 Divide both sides by 2.	$\frac{4x}{5} - 1 = 3$
3 Add 1 to both sides.	$\frac{4x}{5} = 4$
4 Multiply both sides by 5.	$4x = 20$
5 Divide both sides by 4.	$x = \frac{20}{4}$
	$x = 5$

When an equation has pronumerals on both sides, at some stage they must be gathered together on the same side of the equation.

WORKED Example 2

Solve:

a $4x - 3 = 3(6 - x)$ **b** $\frac{9x+3}{2} = \frac{13x+7}{3}$ **c** $\frac{3-x}{4} = \frac{9(x+7)}{10} + 1$

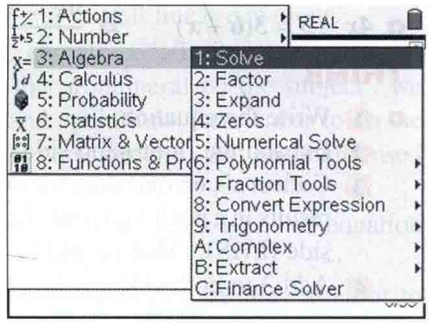
THINK	WRITE
a 1 Write the equation.	**a** $4x - 3 = 3(6 - x)$
2 Expand the right-hand side (RHS).	$4x - 3 = 18 - 3x$
3 Collect x's on one side, for example, the side which results in a positive x term, in this case, the left-hand side (LHS). (That is, add $3x$ to both sides.)	$7x - 3 = 18$
4 Add 3 to both sides.	$7x = 21$
5 Divide both sides by 7.	$x = \frac{21}{7}$
	$x = 3$
b 1 Write the equation.	**b** $\frac{9x+3}{2} = \frac{13x+7}{3}$
2 Find the lowest common denominator for both terms.	
3 Write all terms with the common denominator, adjusting numerators accordingly (so that numerator and denominator have been multiplied by the same amount).	$\frac{3(9x+3)}{6} = \frac{2(13x+7)}{6}$
4 Now that all terms have the same denominator, the numerators must be equal. (Multiply each side by 6.)	$3(9x + 3) = 2(13x + 7)$
5 Expand all brackets.	$27x + 9 = 26x + 14$
6 Collect x's on the LHS and numbers on the RHS.	$27x - 26x = 14 - 9$
7 Simplify and solve.	$x = 5$
c 1 Write the equation.	**c** $\frac{3-x}{4} = \frac{9(x+7)}{10} + 1$
2 Find the lowest common denominator for all three terms.	
3 Write all terms with the common denominator, adjusting numerators accordingly (so that numerator and denominator have been multiplied by the same amount).	$\frac{5(3-x)}{20} = \frac{2 \times 9(x+7)}{20} + \frac{20}{20}$
4 Now that all terms have the same denominator, the numerators must be equal. (Multiply each term by 20.)	$5(3 - x) = 18(x + 7) + 20$
5 Expand all brackets.	$15 - 5x = 18x + 126 + 20$
6 Collect x's on the RHS and numbers on the LHS.	$15 - 126 - 20 = 18x + 5x$
	$-131 = 23x$
7 Simplify and solve.	$-\frac{131}{23} = x$
	$x = -\frac{131}{23}$

CAS Calculator tip! Solving linear equations

A CAS calculator can be used to solve linear equations.

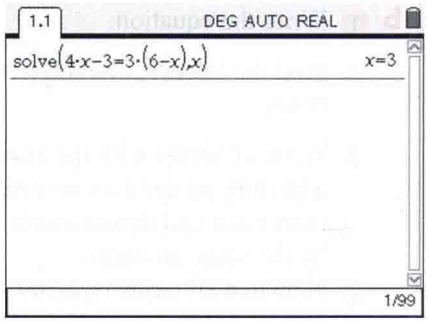
1. Consider worked example 2a.

   (a) Press (⌂), then 1:Calculator (1). Then press (menu) followed by 3:Algebra (3) and 1:Solve (1).

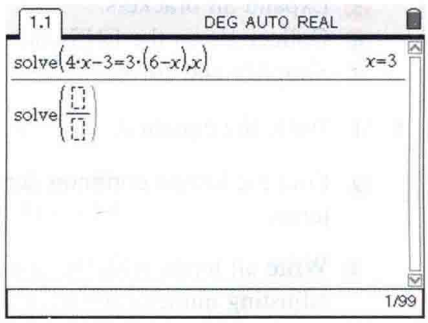


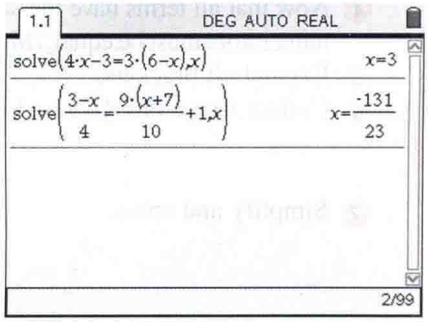
   (b) Type in the first equation, putting a comma followed by x before you close the bracket. Press (enter) for the solution.

   Note: The multiplication operation must be placed in front of the bracket $(6 - x)$.



2. Consider worked example 2c. Repeat the steps above to access the Solve application. To type the fractions, use the fraction template. To do this press (ctrl), then (÷). Type the fraction and then repeat the process until the whole equation is entered. Then press (enter).





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