Steven Roman

COLLEGE ALGEBRA AND TRIGONOMETRY

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College Algebra and Trigonometry

TO DONNA

THE ROMAN SERIES

College Algebra and Trigonometry
College Algebra
Precalculus

Preface

This book provides a thorough treatment of college algebra and trigonometry. In writing it, I have tried to maintain a "user friendly" approach to the subject by providing clear and complete explanations of the topics along with an abundance of examples and figures. At the same time, I have made every effort to maintain a level of rigor that is desirable in a mathematics textbook.

Two special features of the book are particularly noteworthy:

1. *Study Suggestions* These are exercises—about 40 per chapter—that follow most of the worked examples in the text and that can generally be solved by applying the same or similar techniques used in the corresponding examples. Thus, the Study Suggestions serve to reinforce those techniques. Answers to most of the Study Suggestions appear at the back of the book.
2. *Ideas to Remember* These are found at the end of each section. They do not represent summaries of the section, but, rather, they provide more "philosophical" remarks about the contents of the section. By remembering these ideas, the student will gain a deeper understanding of the concepts of the chapter and will be able to retain the material for a longer period of time.

College Algebra and Trigonometry is organized as follows. Chapter 1 contains a review of elementary algebra. A section on the composition of algebraic expressions is included here for two reasons. First, it gives the student another chance to sharpen his or her skills at manipulating algebraic expressions. Second, and more important, it allows the student to concentrate more on the concepts, rather than on the computations, when studying composition of functions in Chapter 3. Chapter 1 also includes a section on variation. Since variation problems are the easiest type of word problem, placing them before the applications sections of Chapter 2 gives the student a more gentle introduction to this topic. If desired, this section can be postponed until the end of Chapter 2 or 3.

Chapter 2 is devoted to solving equations. Because applications are very important here, separate sections are devoted to applications involving linear equations and quadratic equations. If desired, the material in Chapter 11 on complex numbers can be covered immediately after Section 2.3.

In Chapter 3 we begin the study of functions, emphasizing that the graph of a function reveals a great deal of information about the function itself.

Chapter 4 includes a brief discussion of the conic sections. The emphasis is on graphing; but we also discuss the geometric definition of each conic section as the locus of points satisfying certain properties.

Chapter 5 begins with division of polynomials, followed by the Remainder and Factor Theorems and by a discussion of rational roots of a polynomial. The chapter ends with an optional section on partial fractions.

Chapter 6 covers the exponential and logarithmic functions, including a discussion of the logarithmic scale. The emphasis in this chapter is on applications.

Chapters 7–9 are devoted to trigonometry. We define the trigonometric functions using the unit circle, since that is the most useful for future study. Chapter 8 contains a discussion of triangle trigonometry, including the law of sines and cosines, and an introduction to polar coordinates and vectors in the plane.

Chapter 10 contains a discussion of systems of linear and nonlinear equations, as well as systems of inequalities. Determinants, Cramer's Rule, and matrix algebra are also covered in this chapter.

Chapter 11 is an introduction to the topic of complex numbers. The first three sections of this chapter can be covered much earlier in the book (for example, after Section 2.3).

The final chapter contains a discussion of various topics: mathematical induction, the binomial formula, sequences and series, permutations, combinations, and probability.

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A Note on Calculators

To the Student

We strongly suggest that all students obtain a *scientific* calculator—that is, a calculator with keys for performing at least the following functions; y^x, $\sqrt[x]{y}$, e^x, $\log x$, $\ln x$, $\sin x$, $\cos x$, $\tan x$, $\sin^{-1} x$, $\cos^{-1} x$ and $\tan^{-1} x$. (Sometimes the last three keys are replaced by a key labeled *inv* or *arc*.) Also, we strongly suggest that you read the instruction manual that accompanies your calculator very carefully.

Let us make two important points about the use of calculators. First, since a calculator stores (and displays) only a few digits for each number entered, it generally gives only *approximations* rather than exact values. As a simple example, if you perform the operation $1 \div 3$, the display on a scientific calculator will show the number 0.333333333, which is only an approximation to 1/3.

Most scientific calculators round off answers to either eight or ten places, and while it may be true that for most applications these approximations are satisfactory, you should be aware that they are *only* approximations. To emphasize the effect of errors due to rounding, many calculators will display a small but *nonzero* number when asked to compute

$$(1 \div 3) \times 3 - 1$$

which, of course, is equal to 0.

The second, and perhaps more important, point is that a calculator is not meant to take the place of learning. As an example, in Chapter 1 we will discuss fractional exponents and what it means to raise a number to a fractional power. For instance, we will see that, by definition, $27^{2/3} = (\sqrt[3]{27})^2 = 3^2 = 9$. Now, you could determine the value of $27^{2/3}$ directly by using a calculator, which will give you the correct value. However, if you do this, you will have missed the entire point of the section on fractional exponents.

The goal of this book is to help you learn the basic concepts of algebra and how to apply them, not how to operate a calculator. Therefore, we urge you to reflect a moment before automatically reaching for your calculator.

Contents

1

THE TECHNIQUES OF ELEMENTARY ALGEBRA

1.1 Number Systems

Let us begin our study of algebra by discussing the basic number systems that we will use throughout the book. Our first number system is the **natural number system**

$$\mathbf{N} = \{1, 2, 3, \ldots\}$$

The braces { and } are used to denote the fact that **N** is a set, and the notation "...", called an *ellipsis,* means that the pattern established before the dots continues on forever.

The next number system is the **integers**

$$\mathbf{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$$

which includes the natural numbers, the negative natural numbers, and zero. Here we have used two ellipses to indicate that the integers contain numbers greater than 3 and less than -3.

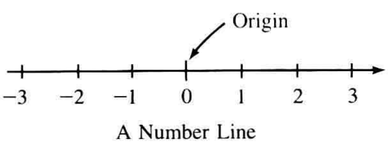

FIGURE 1.1

The integers can be represented in a very useful way as points on a line, as shown in Figure 1.1. Notice that the positive integers are to the right of 0, and the negative integers are to the left. The point corresponding to 0 is called the **origin.** When a line is used to represent numbers in this way, it is called a **number line.**

Our next number system consists of the **rational numbers,** which include all possible fractions, both positive and negative, such as

$$\frac{1}{2}, \ \frac{4}{8}, \ \frac{-7}{5}, \ \frac{122}{122}, \ \frac{-145}{-37}, \ \frac{0}{5}, \ \frac{-3}{1}$$

The only restriction that we must place on these fractions is that we cannot allow a 0 in the denominator. Thus, the rational number system can be described as the set

$$\mathbf{Q} = \left\{ \frac{p}{q} \;\middle|\; p \text{ and } q \text{ are integers and } q \neq 0 \right\}$$

In words, **Q** is the set of all numbers of the form p/q where p and q are *integers*, and q cannot be equal to 0.

It is important to keep in mind that integers are also rational numbers. For example, the integer 7 is the same as the rational number 7/1.

The rational numbers can also be placed on the number line. The location of some rational numbers is shown in Figure 1.2.

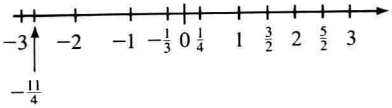

FIGURE 1.2

Try Study Suggestion 1.1.

▶ **Study Suggestion 1.1:** Draw a number line and plot the points 4, 2, −3, 1/2 and −17/3. (*Hint:* whenever you plot points on a number line, you should first plot the origin, to use as a point of reference, and the point corresponding to 1, to set the scale.) ◀

Although it might seem at first that the rational number system is "large" enough to meet all of our needs, it turns out that this is not the case. An example of the inadequacy of the rational number system is provided by the fact that, using just the rational numbers, it is not possible to measure all possible lengths.

For instance, in Figure 1.3 we have placed a square whose sides have length 1 over a number line. Then we have rotated the diagonal of the square onto the line, and labeled the point where the end of the diagonal meets the line with the letter a. Now, it so happens that the point a does *not* correspond to any rational number.

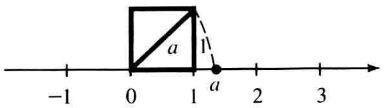

FIGURE 1.3

We can look at this a little more algebraically as follows. The letter a represents the length of the diagonal of the square. Thus, according to the Pythagorean Theorem, we have

$$a^2 = 1^2 + 1^2$$

or

$$a^2 = 2$$

But it is possible to prove that *no* rational number can have the property that its square is equal to 2, and so the point labeled a cannot correspond to any rational number. (We outline one way to prove this fact in Exercises 28–30.) Thus we see that the rational number system does not contain enough numbers to measure the length of this particular diagonal.

Figure 1.4 illustrates another case where the rational number system proves inadequate for measuring lengths. In this case, we have a circle with diameter equal to 1, sitting on top of a number line. If we "unwrap" the circle and lay it along the number line, starting at 0, the other end will fall at a point on the line that does not correspond to any rational number. The length of this line segment, that is, the circumference of the circle, is equal to π, and it is also possible to show that π is not a rational number (although that is rather difficult to do).

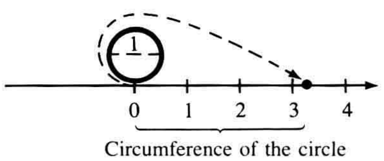

FIGURE 1.4

In view of these examples, we find ourselves in the position of needing a still larger number system—one that includes all of the rational numbers, as well as enough additional "numbers" so that we can measure all possible lengths, or to put it another way, so that we can "fill in" the rest of the number line.

The simplest way to extend the rational number system is to "invent" a new number for each of the points on the line that do not correspond to a rational number. These new numbers are called, appropriately enough, **irrational numbers.** Thus, both $\sqrt{2}$ and π are irrational numbers.

If we put the irrational numbers together with the rational numbers, we get a system known as the **real number system,** which we will denote by **R.** This number system is completely adequate for measuring lengths (as well as areas, volumes, and so on). In fact, we can say that

> **every point on the line corresponds to one and only one real number, and every real number corresponds to one and only one point on the line.**

When real numbers are assigned to points on the line in this way, we refer to the line as a **real number line.**

Let us summarize by listing the various number systems that we have discussed.

- The Natural Numbers

$$\mathbf{N} = \{1, 2, 3, \ldots\}$$

- The Integers

$$\mathbf{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$$

- The Rational Numbers

$$\mathbf{Q} = \left\{\frac{p}{q} \;\middle|\; p \text{ and } q \text{ are integers and } q \neq 0\right\}$$

- The Irrational Numbers

I = {numbers that correspond to points on the number line not associated with rational numbers}

- The Real Numbers

R = {rational and irrational numbers}

▶ **Study Suggestion 1.2:** Indicate to which number system or systems each of the following numbers belong (for example, the number 1/2 belongs to both **Q** and **R**): 0, $-2/3$, 5, -1, 6/3, $\sqrt{2}$, π ◀

Try Study Suggestion 1.2.

Let us conclude this section by briefly discussing the role of decimal expansions. Any real number can be represented in decimal form. For example, we have

$$\frac{1}{2} = 0.5, \qquad \frac{4}{3} = 1.\overline{3}, \qquad \frac{1}{7} = 0.\overline{142857},$$

$$\pi = 3.14159\cdots, \qquad \text{and} \qquad \sqrt{2} = 1.41421\cdots$$

Here we have used a bar to denote repeating digits. Those digits that lie under the bar are repeated forever in the decimal expansion.

As you probably know, there are two types of decimal expansions. **Repeating decimals** are those in which a certain pattern of digits repeats forever. These are the type of decimals that correspond to rational numbers. For example, the rational number 1/7 corresponds to the repeating decimal $0.\overline{142857}$. Similarly, the rational number 1/2 corresponds to the decimal 0.5, which can also be written as a repeating decimal $0.5\overline{0}$, where the 0 repeats forever.

Decimal numbers with no repeating pattern are called **nonrepeating decimals.** These decimals correspond to the irrational numbers. For example, the decimal expansions of π and $\sqrt{2}$ are nonrepeating decimals.

Ideas to Remember

The rational number system is not adequate for measuring all possible lengths. However, the real number system is adequate for measuring lengths, as well as areas, volumes, intervals of time, and so on.

EXERCISES

In Exercises 1–6, plot the given numbers on a number line.

1. $-1, 0, 1, 1/4, -1/2$

2. $1, 3/2, -\sqrt{2}, 0, \sqrt{3}, 2$

3. $0, 2, -2, 0.5, \pi$

4. $2, 4, -2, -4, \sqrt{5}, \pi/2$

5. $0, -5, -\sqrt{5}, \sqrt{21}, \pi/\sqrt{2}$

6. $3/4, 125/3, -67/4, 8/3, 17/5$

In Exercises 7–26, indicate to which number systems the given number belongs. For example, the number 1/2 belongs to both **Q** and **R**.

7. 0 **8.** 10

9. 1/4 **10.** -12

11. 0.01 **12.** $-7/2$

13. -0.75 **14.** $\sqrt{2}$

15. $-12/5$ **16.** $\sqrt{1}$

17. $0.\overline{714285}$ **18.** $1.\overline{3}$

19. $-19/(-2)$ **20.** 3.14159

21. $\sqrt{3}$ **22.** $10.\overline{0}$

23. $-12.\overline{0}$ **24.** $\sqrt{2}/\sqrt{2}$

25. $\sqrt{2} + 1$ **26.** $\sqrt{8}/\sqrt{2}$

27. If your calculator has a *pi* key, use it to compute $\pi - 22/7$. What would you tell a person who thought that π and 22/7 were the same number?