

INTRODUCTION TO
THE THEORY
AND PRACTICE
OF ECONOMETRICS

Second Edition

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Introduction to the Theory and Practice of Econometrics

Second Edition

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**Introduction to
the Theory and Practice
of Econometrics**

To

Lisa and Laura Judge

Mima Hill

JoAnn, Jill, David and Wendy Griffiths

Hilde and Karlheinz Lütkepohl

Nancy, Tony and Jean Lee

Preface to Second Edition

In revising the first edition of ITPE, our objective was to make changes in the content, organization, and exposition so that it is a friendly book for both the instructor and the student. The scope of the book is not greatly changed from the first edition. However, (1) by adding some new material we have made the book more self-contained, (2) by changing the organization and exposition we have tried to make the book serve wider introductory- and intermediate-level audiences, (3) by carrying through meaningful examples in each chapter we have tried to improve the interweaving of theory and practice, and (4) by providing a Computer Handbook the computer is made an important part of the teaching-learning process.

As in the first edition, the linkage between the economic process thought to underlie data generation and the statistical model reflecting the corresponding sampling process serves as a unifying theme throughout the book as we progress from the simple to the complex ways of modeling economic data. Also as in the first edition, to ensure the student fully appreciates the sampling theory approach to inference, Monte Carlo sampling procedures are introduced in the Exercises for most of the chapters to illustrate important sampling concepts.

To demonstrate how standard econometric software can be used to implement the procedures outlined in the text, a Computer Handbook containing the applications and Monte Carlo exercises in each chapter has been developed. This Computer Handbook should make possible a “hands on” experience with micro or main frame computers through the use of the widely available SHAZAM and SAS software packages. A corresponding GAUSS manual is being developed separately.

The book is more self-contained than the first edition in that it contains chapters on the basic concepts of classical and Bayesian inference and an extensive appendix on linear algebra. Therefore, introductory knowledge in these areas, although beneficial, is not necessary.

Some of the specific changes relative to the first edition are

1. The early chapters are devoted to a discussion of the basic concepts of classical and Bayesian inference. For most students, these chapters are

intended as a review or reference point for some of the basic definitions and concepts of statistical inference.

2. The discussion of the linear-statistical (regression) model is started with two unknown location parameters, and the analysis is carried through with both summation and matrix-vector notation.
3. A linear algebra appendix is included that serves the needs of each chapter (for example, the operations of vectors and matrices and the matrix algebra relevant to normal distribution theory).
4. Relative to the first edition, we have combined some of the chapters and changed the order in which some of the topics are developed. For example, we have included autocorrelation and heteroskedasticity in the chapter concerned with Aitken estimation with an unknown covariance matrix.
5. One or more applied examples are presented in each chapter that can be reproduced by the student.
6. Asymptotics have been introduced in the early chapters, and this concept is made use of throughout this book.
7. A Computer Handbook is provided that makes the computer an integral part of the teaching-learning process for each chapter. Both micro and mainframe computers are used along with SHAZAM and SAS econometric software packages. A corresponding manual for the GAUSS software package is also being developed.

As with the first edition, the objectives of the book are multiple. The first third of the book reviews statistical concepts and introduces the linear statistical model and its uses. The remainder of the book introduces the student to econometric problems that arise when we take into account that economic data are generated from a system of relations that are dynamic, stochastic, and simultaneous and that statistical procedures change as we change the statistical model, the amount and type of information used, and the measure of performance. These topics, although not treated in depth, in each case identify the general problem area and suggest one or more ways of mitigating its statistical impact. For a more in depth treatment of each problem area, the student is referred to a particular chapter in the second edition of *The Theory and Practice of Econometrics*.

As the book is designed, it may be used (1) as a one-semester/quarter course that introduces the undergraduate student to classical and Bayesian statistics and to the general linear-statistical (regression) model, (2) as a one- or two-semester/quarter course in undergraduate econometrics, (3) as a one- or two-semester/quarter course in intermediate econometrics at the graduate level, (4) as a problems course in econometrics at the undergraduate or graduate level, and (5) as a review or refresher course in statistical inference and econometrics.

The criticisms and suggestions we have received from colleagues and students who used the first edition were very helpful in eliminating errors and in determining the contents and organization of this volume. We cannot acknowledge each of the individual contributions, but we do want to indicate our debt to those behind-the-scenes colleagues and to each we offer our sincere thanks.

Ken White has not only taken the leadership in developing the Computer Handbook, but he has also contributed to the content of many of the chapters. To Ken we owe a special thanks, and we are pleased to see his name associated with the book.

In addition the following have made substantive contributions: Larry Marsh, Minbo Kim, James Chalfont and Shirley Haun.

For a book of this nature, skilled technical typists are a necessity. In this context we would like to recognize and thank members of the Giannini Foundation Word Processing Center, Jerry Rowley, Mary Jo Neathery, Shirley DeJean, Val Boland, and Marlene Youman. Partial support for this work was provided by a National Science Foundation grant.

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April 1987

Preface

To First Edition

The descriptive and prescriptive goals of understanding, predicting, and controlling economic processes and institutions requires that the student in economics and business makes use of an array of statistical models and measurement procedures. An operational knowledge in this area calls for (1) an understanding of the basic concepts of the calculus, linear algebra, and statistical inference; (2) having an array of statistical models that are consistent with the alternative ways in which economic data are generated; and (3) given the statistical model, a set of procedures or rules that permits the data to be used in an “optimal” way.

Many undergraduate textbooks in statistics and econometrics treat these topics in a disjoint way. Thus a student may first learn about probability and distribution theory, then about estimation and hypothesis testing from a sampling theory or Bayesian approach, and finally turn to the area of econometrics or the application of these tools to a particular subject matter area. Often this approach leaves the student without a clear understanding of the alternative approaches to statistical inference, the connection between statistical theory and econometric practice and, from a research standpoint, how one would go about producing new econometric knowledge. In this book the objective is to interweave inferential approaches and theory and practice. Therefore, for example, the basic statistical and linear algebra concepts are introduced as they are needed to give life to the statistical model under study.

Also, because there is limited opportunity to experiment in economics, most econometric applications start with a tentative theory or hypothesis, a sample of data, and the goal of learning something about the phenomena under study from the limited set of observations. Therefore, a sample of data that may be used to investigate a particular economic hypothesis is presented to motivate the analysis of each of the statistical models presented. This linkage between the economic process that is thought to have generated the data and a particular statistical model is a unifying theme throughout the book. We progress from the special case of investigating the possibilities for determining the location and scale parameters for a population from a sample of observations to investigating a complex simultaneous system of structural equations under general stochastic assumptions. To ensure that the reader understands the basic concepts and conclusions as they

relate to linear statistical models, simple special case models are evaluated, and then the analysis is repeated for the general case.

To make certain that the student fully appreciates the sampling theory approach to inference, Monte Carlo experiments and results that illustrate important sampling concepts and properties are introduced in many chapters. Exercises that improve manipulative skills and samples of data from a Monte Carlo experiment for a particular statistical model form the basis for student exercises in each chapter. In this way students can obtain hands-on experience with the computer and verify empirically the sampling characteristics that have been analytically derived.

The book is self-contained in the sense that statistical concepts and linear algebra are introduced when they are needed and are most relevant for analysis and inference. Applications of calculus are presented in such a way that the ideas are transmitted even though the underlying concepts may be unfamiliar. Introductory knowledge in these areas, although beneficial, is not necessary.

The objectives of the book are multiple. The first half of the book gives the student a solid introduction to the formulation and use of linear statistical models. The second half introduces the student to the econometric problems that arise when we take into account the facts that economic data are stochastic, dynamic, and simultaneous and that the optimal statistical procedure sometimes changes as we change the statistical model, the amount and type of information used, and the measure of performance. These topics, although not treated in great depth, identify the general problem area and suggest one or more procedures for mitigating the statistical impact of the econometric evil in question. For a more complete treatment of each of the problems covered in the last half of the book, the reader should refer to our other book, *The Theory and Practice of Econometrics* (Wiley, 1980).

As written, this book is designed to serve multiple uses by varying the sections used or the sequence of chapters.

Each instructor will, of course, emphasize different aspects of the econometric puzzle. Our purpose has been to put together a book that is rich enough in the basic ingredients to permit instructors and students to select the menu that will satisfy their individual needs.

After we completed *The Theory and Practice of Econometrics* it seemed appropriate that we should undertake the writing of an introductory text that would be the foundation for econometric practice. The resulting set of words and symbols represents the combined judgments that have come from teaching a range of mathematics, statistics, economics, and econometrics courses over the last three decades. Students' and colleagues' ideas have been very important in the organization, readability, and possible usefulness of the book. In particular we thank Albert Link, Auburn University, Malcolm Dowling, University of Colorado, and Peter

Zadrozny, New York University, for their early input and suggestions. The final product owes a great debt to the careful and detailed recommendations of Keith Johnson, New York University, E. P. Howrey, University of Michigan, and Gregg Duncan, Washington State University. Rich Esposito, the economics editor at Wiley, made substantive contributions at each stage of the project. Mary Halloran contributed her superb talents to solving the problems of style and format and the production of the book.

For a book of this nature a skilled technical typist is a necessity. Dixie Trinkle is not only a skilled typist, but her patience and emphasis on accuracy and consistency made the task of going from the handwritten copy to the final manuscript a pleasure. Others who helped with the typing include Shirley Williams, Judy Griffin, Sylvia Graves, Carline Lancaster, Linda McKellar, and Rosean Swan. Partial support of this work was provided by a National Science Foundation grant.

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