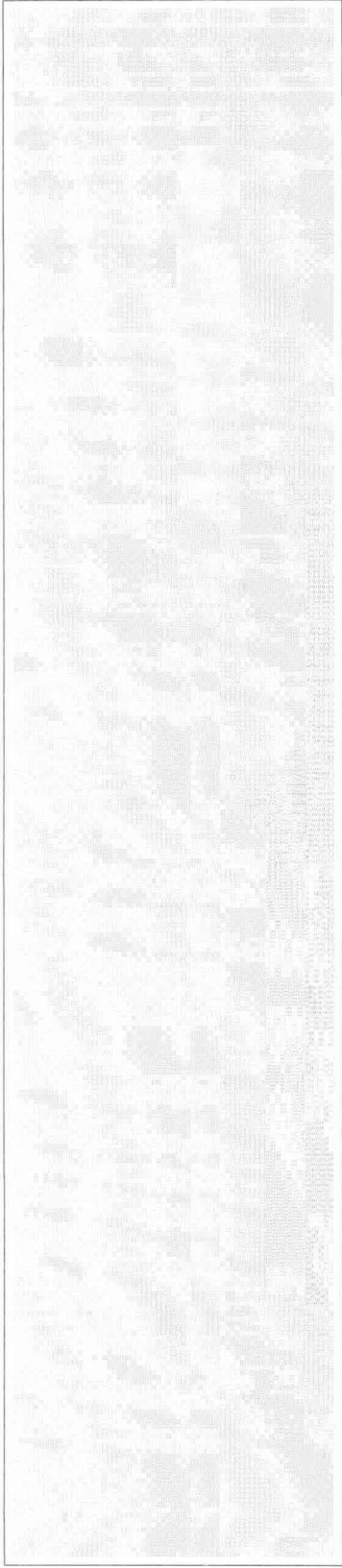


Calculus and Its Applications

■ Stanley J. Farlow and Gary M. Haggard



CALCULUS AND ITS APPLICATIONS

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To Susan

Preface

The primary goal of this text is to provide understanding and comprehension of the calculus as well as to establish sound technical proficiency. The level of presentation is easily accessible to most students, and there is a strong degree of reliance on intuition—more so than on overly formal and abstract mathematical theory. To fulfill our goal, we have used a broad, rich selection of topics, features, and motivational items in conjunction with proven pedagogical techniques for the teaching of mathematics.

Calculus and Its Applications is designed for use in a two-term course in calculus taken primarily by students majoring in business, economics, life sciences, and social sciences. The only prerequisite for studying the material in this book is three or four semesters of high school algebra or its equivalent. A companion volume, *Introduction to Calculus and Its Applications*, is composed of preliminaries and Chapters 1–7 and is suitable for a one-term course.

Pedagogical Features

Emphasis and Writing Style: In writing this book, we have used a number of features designed to enliven the text and motivate the student. We use real-world examples, historical comments, and intuitive presentations to explain the intelligent use of the calculus. Our basic approach is to present the mathematics in a humanistic manner and thereby enhance its use as a genuine aid to decision making by nonmathematicians.

Format: Major concepts and definitions are highlighted with a colored box so that they may be found easily and referred to throughout the book. All interest motivating material is set off in special boxes.

Strong Visual Program: More than 800 figures and numerous photographs convey a strong visual sense of the mathematics for ease of learning and to provide a realistic context to the applications. We have tried to provide helpful captions to all figures and photographs, either reinforcing an idea or providing additional explanation.

Realistic Applications: Over 600 realistic applications are included in the examples and exercise sets. Many of the applications will appeal to *all* students in the course, regardless of their major area of study. All the applications were chosen and developed for their pedagogical appeal and effectiveness in helping to teach mathematics.

Worked Examples: The book contains over 300 worked examples, each carefully chosen to illustrate a particular concept or technique. We collected these over the many years we have been teaching the material to our own students.

Exercises: Effective exercises are at the heart of any mathematics textbook. The more than 2500 exercises reinforce understanding as well as develop technical skills. They are graded by level of difficulty and include many challenging applied problems.

Historical Comments: To further enhance both student and instructor interest, we have included many historical comments and profiles of key historical figures in mathematics.

Chapter Epilogues: A brief epilogue at the end of each chapter relates the material to larger contemporary society.

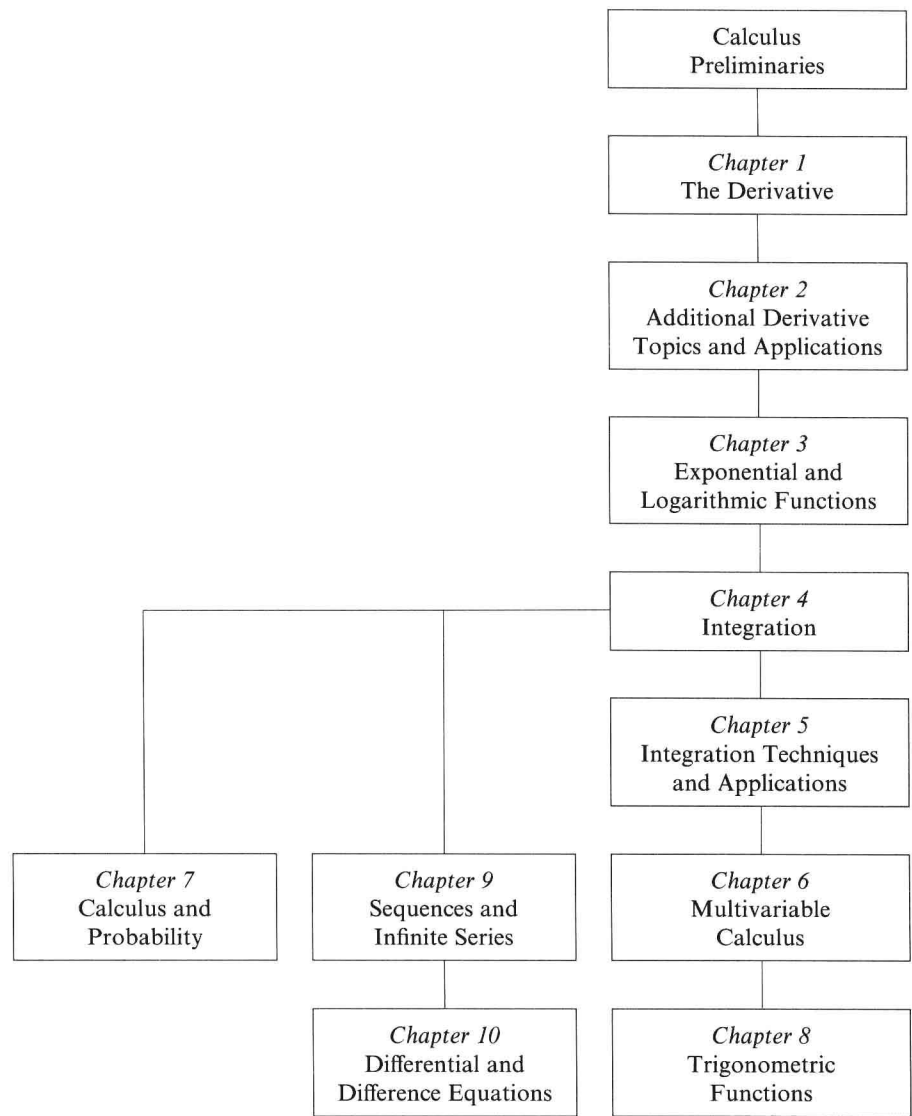
End-of-Chapter Review Material: Each chapter closes with a list of key terms, an extensive chapter review exercise set, and a brief practice test.

Projects and Problems: Special exercise sets that combine cumulative review and comprehensive projects are included at the end of Chapters 3, 6, 8, and 10. These sections should challenge the motivated student to explore a wider variety of unconventional problems. Several writing projects have been included to help instructors who are trying to develop a writing-intensive course.

Algebra Review Material: The algebra review, “Calculus Preliminaries,” is intended for students whose background may be weak in some topics in algebra. An instructor may omit the material, cover all or part of the material in class, or assign portions for students to work on their own. A practice test has been included to help instructors place students and assess the amount of review they need.

Chapter Organization

The following chart indicates how the chapters are related.



Supplements for Student and Instructor

Student Solutions Manual: This manual is available to students at a nominal cost. It contains solutions to all odd-numbered exercises in the book.

For the Instructor: An instructor's solutions manual, available to adopters, contains detailed solutions to all even-numbered exercises in the book as well as sample chapter tests, midterms, and final exams. A computerized test bank (IBM) and printout are also available for instructor use.

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All errors are the responsibility of the authors. We would appreciate having these brought to our attention. We would also appreciate any comments and suggestions from students and instructors.

Stanley J. Farlow

Gary M. Haggard

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