ADVANCED THEORY OF CONSTRAINT AND MOTION ANALYSIS FOR ROBOT MECHANISMS

JINGSHAN ZHAO
ZHIJING FENG
FULEI CHU
NING MA

Advanced Theory of Constraint and Motion for Robot chanisms

Jingshan Zhao
Zhijing Feng
Fulei Chu
Ning Ma

ELSEVIER

AMSTERDAM • BOSTON • HEIDELBERG • LONDON
NEW YORK • OXFORD • PARIS • SAN DIEGO
SAN FRANCISCO • SINGAPORE • SYDNEY • TOKYO

Academic Press is an Imprint of Elsevier

Academic Press is an imprint of Elsevier
The Boulevard, Langford Lane, Kidlington, Oxford, OX5 1GB
225 Wyman Street, Waltham, MA 02451, USA

First published 2014

British Library Cataloguing in Publication Data
A catalogue record for this book is available from the British Library

Library of Congress Cataloguing in Publication Data
A catalog record for this book is available from the Library of Congress

ISBN: 978-0-12-420162-0

For information on all Academic Press publications
visit our website at store.elsevier.com

Printed and bound in the United States

14 15 16 17 10 9 8 7 6 5 4 3 2 1

Preface

All machines are made of mechanisms. Before design, the degree of freedom of the end effector of a robot mechanism should first be determined, and then the actuation scheme can be investigated. Thus, it is the fundamental basis and prerequisite for structural analysis and synthesis to analyze the degree of freedom of the end effector of a mechanism. Structure analysis of a mechanism is focusing on proposing the structure principle of mechanisms, the classification of mechanisms, and their free motions. Meanwhile, structure analysis of mechanisms is an important aspect of kinematics, dynamics analysis and mechanism synthesis. The rapid development of science and technology calls for the creation and design of better mechanisms. Therefore, it is essential to conduct further theoretical research into the structure, type, and kinematic characteristics of mechanisms. Structure analysis presents a basis to determine the number of the actuators and carry out kinematics and dynamics research. Therefore, it is one of the most important concepts in mechanism. It offers a guarantee for innovation and structure synthesis, and lays a foundation for further analyzing the driving, and kinematics and dynamics performances of mechanisms.

This book studies the constraint and motion analysis for spatial robot mechanisms based on the unified mathematical representation of its kinematic geometry constraints. Chapter 1 roughly surveys the development and problem in the structure analysis of mechanisms in the past centuries. In order to limit the mathematical prerequisites to the knowledge of elementary screw algebra and matrix calculus, it was felt necessary to present in Chapter 2 an introduction to screw theory. The material covered is limited to that necessary for an understanding of the book, and no attempt has been made to study thoroughly such topics. It describes the primary knowledge about the screw algebra from aspects such as the Plücker column vector of a line, motion representation of rigid bodies, the screw description of the motion and force, reciprocal product of the screw and its geometry significance, the linear combination of screws and the principal screws in a screw system.

In Chapter 3, the motion screw system of kinematical chains is established through the screw representation of the free motions of kinematic pairs; the terminal constraints of the kinematic chain are derived according to the reciprocal relationship between twists and wrenches. The equivalent substitution and synthesis of kinematic chains under the conditions of the same terminal constraints are studied. The investigation into the mechanical equivalent substitution theorem lays a necessary foundation for the selection and optimization of the kinematic pairs of the spatial mechanism which has purely serial kinematic chains, the research into the idle degrees of freedom of kinematic chains, especially the complex spatial kinematic chains, and the free motion analysis of the end effector in a mechanism and the analysis of the actuation scheme. According to reciprocal screw theory, terminal constraints of the kinematic chain depend entirely on the largest linearly independent set of the kinematic screws. Therefore, the structural optimization problem of the kinematic chain can be addressed through the algebraic analysis to the specified terminal constraints; and it can also be resolved through algebra analysis of the screws. In fact, the free motions of the end effector are the results of the combination of branch kinematic chains. Therefore, the study of spatial serial mechanisms will focus on the constraints of kinematic chains to the end effector. How to determine the types of various kinematic chains and how to complete the design with the simplest and economical structure arrangement have become key issues in the synthesis of kinematic chains. This is also one of the key issues in this chapter and subsequent chapters.

In Chapter 4, terminal constraints of robot end effectors from kinematic chains and the free motions under these constraints are discussed. The analytical theory of constraint space of the end effector of parallel robot mechanism is studied according to the terminal constraint space of its kinematic chains. The degree of freedom of the end effector of a parallel robot should have the three attributes of *quantity*, *type*, and *direction*. The degree of freedom of the end effector of a robot is different from the actuations to control the mechanism. However, the traditional methods did not make any distinction between the end effector and the entire mechanical system. This chapter is dedicated to establishing an analytical theory for the constrained space of the end effectors. On this basis, the equivalent description of the complex kinematic chain is discussed, which provides a theoretical support for the equivalent description of the complex kinematic chains to pure serial kinematic chains. It allows us to transform the parallel kinematic chains into serial chains with the equivalent substitution for the compound kinematic chains. This lays a foundation for the study of actuation of the prescribed end effector.

In Chapter 5, the search for reachable workspace of the spatial robot mechanism and its symmetric homomorphism with the mechanism itself is discussed. It is proved that the structural symmetry of the mechanism itself will inevitably lead to the symmetry of the reachable workspace and the dexterous workspace of its end effector. The corresponding relation between the reachable workspace and the dexterous workspace is homomorphism mapping. A quantitative description of such symmetry relations provides certain theoretical basis for the analysis, synthesis, and structure innovative design of robot mechanism. Based on this, the coordinate transformation method for resolving the workspace based on free motion analysis of the end effector of spatial parallel mechanism is also covered in this chapter, and the synthesis of the dexterous workspace is discussed with the Schoenflies-type spatial parallel mechanism as an example. By analyzing the end effectors' degrees of freedom, a minimum of variables can be chosen to establish the kinematic constraint equations and thus the objective function can be built up. By searching the desired dexterous workspace, the optimal length of linkages in kinematic chains can be obtained. Application examples show that the method can be widely applied to the synthesis of the dexterous workspace of various parallel robot mechanisms.

In Chapter 6, the singularity of the spatial robot mechanism in the workspace is studied and it is proved that the structural symmetry of the mechanisms themselves will inevitably lead, not only to the symmetry of reachable workspace and dexterous workspace of its end effector, but also to that of singular workspace. A quantitative description of such a symmetric relation provides some theoretical basis for the analysis and synthesis of workspace of symmetrical mechanisms. With screw theory, a group of balance equations which can be used to analyze the singularity of mechanisms have been obtained and the conditions for singular mechanism are derived. The order of the coefficient matrix of final equations is identical to the number of degrees of freedom of the end effector, which constitutes great advantages for the singularity analysis of the end effector of the mechanism with less than six degrees of freedom. The fewer degrees of freedom the end effector has, the lower the order of the finally converted coefficient matrix will be.

In Chapter 7, kinematics modeling of the spatial parallel robot mechanism is discussed. On the basis of the natural coordinate method, the sufficiency and necessity have been proved that a type of kinematics model spatial parallel mechanism can be completely established with only four non-collinear points with Cartesian coordinates. This method has a significant advantage in dealing with kinematics issues of spatial parallel mechanisms with more than three degrees of freedom. And the established kinematics model shares the same advantages of general natural coordinate method and can also minimize the number of independent variables. As this model only introduces the Cartesian coordinates, and the elements of the derivative matrix merely contain the linear items or even the constant items, while the elements of the derivative matrix obtained through the modeling method based on the rotation matrix transformation are often non-linear, and even contain transcendental functions. Therefore, these advantages of the four-point coordinate method have brought great convenience to kinematics and dynamics analysis.

In Chapter 8, the kinematics and static forces of robot mechanisms are discussed. Based on the analysis of terminal constraints of the kinematic chain in Chapter 4, detailed analysis is given to the motion between the end effector and actuation and the rigid body statics. Through the analysis of the velocity of mechanism end effector, the basic conditions for the feasible motion of robot end effector have been discussed and the forward and inverse solutions of velocity are also investigated. On this basis, robot static equilibrium conditions and the forward and inverse solutions of statics are derived according to virtual power principle.

In Chapter 9, the basic way to analyze the degree of freedom is studied from the theoretical basis for the analysis model. Through investigating the theoretical basis of the degree of freedom of the end effector and the configuration degree of freedom of a mechanism, the key for analyzing the mechanism is revealed: it is a necessary condition to distinguish the degree of freedom of the end effector and the independent actuators required to control the end effector. Application examples of spatial mechanisms demonstrate that it is the necessary requirement for correctly analyzing a mechanism, and it satisfies the current needs of engineering as well.

In Chapter 10, by analyzing the calculation of configuration degrees of freedom of spatial robot mechanism under a single Cartesian coordinate system, the basic motion characteristics of the robot mechanism in the workspace are further investigated. Based on the analysis of free motion and terminal constraint of the kinematic chain in Chapter 4, specific studies have been performed to analyze the degree of freedom of the end effectors within a single Cartesian coordinate system; the configuration degrees of freedom describing the dynamic drive control of mechanisms are investigated by studying the dynamic degree of freedom of the end effector under different actuation schemes. On the basis of the unified mathematical description of kinematic geometry constraints, the analysis theory for the degree of freedom of spatial robot mechanisms has also been discussed. The theory consists of two main parts: the first is the analysis theory targeted at the end effector; the second is the analysis theory for the configuration degree of freedom mechanism with the designated end effectors. The concept of configuration degrees of freedom means that singularity analysis of complex parallel mechanisms are naturally included in the analysis of the configuration degrees of freedom; at the same time, the approach for analyzing the structural stability of mechanisms is also provided: only when the mechanism's motion inputs remain consistent with its configuration degree of freedom in the space at any time, can the entire mechanism system be stably controlled.

In Chapter 11, the mechanism theory of spatial deployable structure based on the scissor-like-element (SLE) is dealt with. To begin with, the motion analysis of the simple planar linkage, the characteristics of the deployable structure and its elements are studied step-by-step. Through in-depth analysis of the double rotating hinge connected by two SLEs, the mechanism theory in the constitution of elements of the deployable structure has been revealed. Through the analysis of elements of the planar deployable structure, the cylindrical deployable structure, and the spherical deployable structure, the constitution mechanism of spatial deployable structures based on the SLE has been investigated. This has important practical value for the promotion of deployable structures in mechanical engineering, construction engineering, and robot engineering.

In Chapter 12, application of the analytical theory in structure synthesis and innovation of mechanisms is studied. Through presenting general steps in mechanism synthesis, the basic method for the innovative design of mechanism with the universal significance is also discussed. With a comprehensive case study for the innovative design of the lower limb of a biped humanoid robot and the independent suspension of the vehicle, applications of the theory in the innovative design of mechanisms are investigated.

In Chapter 13, a new algorithm is proposed to optimize the design of a spatial parallel manipulator for the purpose of obtaining a desired dexterous workspace rather than the whole reachable workspace. With the analysis of the degree of freedom of a manipulator, we can select the least number of variables to depict the kinematic constraints of each leg of a manipulator. The optimum parameters can be obtained by searching the extreme values of the objective functions with the specified adroit workspace. An example is utilized to demonstrate the significant advantages of this method in the dexterous workspace synthesis. In applications, this method can be widely used to synthesize, optimize, and create all kinds of new spatial parallel manipulators with the desired dexterous workspace.

Regarding kinematics, many works have addressed this problem in the past. In Chapter 14, the natural coordinates are adopted to deal with the kinematics model for different spatial mechanisms. With these constraint equations in natural coordinates, we can conduct the kinematics analysis of the mechanism and then build the dynamics model with the Newton–Euler formulation.

I would like to express my gratitude for the contributions of my coauthors Zhijing Feng, Fulei Chu, and Ning Ma. In addition, I am grateful for the inspiration of Prof. Yunqing Zhang, Prof. Jian S. Dai, Prof. Yuefa Fang, Prof. J. Michael McCarthy, Dr. Ketao Zhang, Dr. Guowu Wei, Prof. Ligang Yao, Prof. Daniel Martins, and Dr. Wenxiu Lu. Last, but not least, I gratefully acknowledge the support of the Natural Science Foundation of China, the Natural Science Foundation of Beijing, the Foundation for the Author of National Excellent Doctoral Dissertation of China, and the Program for New Century Excellent Talents in the University of Education Ministry of China, and the support of the State Key Laboratory of Tribology in Tsinghua University.

Jingshan Zhao
Tsinghua University, Beijing, China

Contents

Preface ix

1. **Introduction** **1**
   - 1.1 Review of Mechanism 1
   - 1.2 Contradiction Between Calculation and Practice of Mobility of Spatial Mechanism 11
   - 1.3 Possible Causes for Contradiction between the Calculated DOF and the Actual One 15
   - 1.4 Contents of the Book 19
   - References 23

2. **A Brief Introduction to Screw Theory** **29**
   - 2.1 Plücker Vector 29
   - 2.2 Rigid Body's Motion Expression 33
   - 2.3 Screw Expression of Motion and Force 45
   - 2.4 Reciprocal Product of Screws and its Geometric Meaning 49
   - 2.5 Linear Combinations of Screws and Principal Screws of a Screw System 51
   - 2.6 Identification of Principal Screws of a Screw System 62
   - 2.7 Conclusions 77
   - References 78

3. **Twists and Wrenches of a Kinematic Chain** **81**
   - 3.1 Free Motions and the Constraints of a Kinematic Pair 81
   - 3.2 Twists of Kinematic Chains 94
   - 3.3 Theory of Reciprocal Screws 99
   - 3.4 Conclusions 112
   - References 112

4. **Free Motion of the End Effector of a Robot Mechanism** **113**
   - 4.1 Free Motion Space and Constraint Space of Kinematic Chain 113
   - 4.2 General Steps to Analyze the Degree of Freedom of the End Effector 115

4.3 Application of the Analytical Theory of the Degree of Freedom of the End Effector 116
4.4 The Equivalent Substitutions for Hybrid Kinematic Chains 143
4.5 Conclusions 157
References 157

5. Workspace of the End Effector of a Robot Mechanism 159

5.1 Workspace Based on Mobility Analysis 160
5.2 Symmetrical Characteristics of the Workspace for Spatial Parallel Mechanisms with Symmetric Structure 179
5.3 Applications of the Symmetrical Workspace Theorem 184
5.4 Conclusions 199
References 199

6. Singularity Analysis of the End Effector of a Mechanism within Its Workspace 201

6.1 Static Equilibrium Equations of the Parallel Manipulators 202
6.2 Symmetry of Singularity Distribution within the Reachable Workspace 204
6.3 Applications and Discussion 207
6.4 Conclusions 232
References 232

7. Kinematics with Four Points' Cartesian Coordinates for Spatial Parallel Manipulator 235

7.1 Kinematics Model for Spatial Parallel Manipulators with Four Points' Cartesian Coordinates 236
7.2 Forward Kinematics of Spatial Parallel Manipulators 243
7.3 Conclusions 262
References 262

8. Kinematics and Statics of Manipulators 263

8.1 Key Problems in Kinematics and Statics Analysis 263
8.2 Kinematics and Statics of Series Manipulators 265
8.3 Kinematics and Statics of Parallel Manipulators 274
8.4 Conclusions 283
References 284

9. Fundamental Factors to Investigating the Motions and Actuations of a Mechanism 287

9.1 The Basic Steps for Calculating the DOF of the Mechanism with a Prescribed End Effector 288
9.2 Actuation Schemes to Control of the Mechanism 294
9.3 Conclusions 320
References 320

10. Motion Characteristics of a Robotic Mechanism 323

10.1 Problems Existing in the Traditional Theories of DOF 323
10.2 Theory of DOF for the Mechanism with an End Effector 328
10.3 Proof of the Theory 334
10.4 Applications and Discussions 335
10.5 Conclusions 347
References 347

11. Mechanism Theory and Application of Deployable Structures Based on Scissor-Like Elements 349

11.1 Deployable Structures 349
11.2 Brief Review of Reciprocal Screw Theory Based on Exponential Production 351
11.3 Mechanism Theory of a Deployable Structure Unit 354
11.4 Applications of Deployable Structure Units 361
11.5 Conclusions 364
References 365

12. Structure Synthesis of Spatial Mechanisms 367

12.1 General Process of Mechanism Synthesis 368
12.2 Analysis of Structure Synthesis 371
12.3 Conclusions 395
References 395

13. Workspace Synthesis of Spatial Mechanisms 397

13.1 Theorem of Dexterous Workspace of Spatial Parallel Manipulator 397
13.2 Optimum Design of Spatial Manipulators Aiming at a Desired Dexterous Workspace 400
13.3 Conclusions 427
References 427

14. Kinematic Synthesis of Spatial Mechanisms 429

14.1 Kinematic Synthesis of a Spatial Parallel Manipulator With Three Pure Rotational Degrees of Freedom (DOFs) 429
14.2 Kinematics of a Spatial Parallel Manipulator With Two Rotational and One Translational DOF 439
14.3 Kinematic of the Suspension Mechanism with Invariable Orientation Parameters 454
14.4 Conclusions 469
References 469

Index 471

Chapter 1

Introduction

Mechanism is a branch of mechanical engineering science focusing on the study of the motion and the transmission law of mechanisms, which consists of structure analysis and synthesis [1–3]. Structure analysis of mechanism mainly investigates the mobility of the structure, and explores the trajectory, displacement, velocity, and acceleration of the mechanism [2,3]. Structure synthesis of mechanism focuses on the design of mechanisms so that they meet the kinematic requirements according to the prescribed motions. Research results of assembly principles of structure, kinematics, dynamics, and precision analysis of mechanisms provide the theories and methods for various movement generations, the kinematics and dynamics design of mechanical systems [4,5].

1.1 REVIEW OF MECHANISM

With the invention and improvement of steam engines and textile machines at the beginning of the 18th century, calculus, theoretical mechanics, and other disciplines were set up and becoming more and more sophisticated, which provided a theoretical basis and means for the design and invention of various mechanisms. At that time, mechanism was still included in the field of applied mechanics. The rapid development of mechanical industry at the second half of the 19th century proposed specific requirements for the application of mechanisms and the corresponding analysis and synthesis [1]. In the United Kingdom, scholars of mechanism, represented by R. Willis, classified mechanisms according to the characteristics of motion transformation and the speed ratio relationship, and mainly studied the gear and planetary transmission [1,2]. In Germany, scholars of mechanism, represented by F. Reuleaux and L. Burmester, introduced concepts such as kinematic pair, kinematic chain, and schematic diagram of mechanism, discussed the possible types of mechanisms from the structure principle of mechanism, and further extracted the common characteristics of mechanisms, and with graphical methods, conducted the analysis and synthesis of mechanisms according to the motion geometry principle [2,3]. In Russia, scholars, represented by П. Л. Чебышева, addressed the approximate synthesis of dimension by using algebraic methods like approximation theory of functions [4,5]. Based on these results, mechanism was developed into an independent discipline [1,2,6].

Advanced Theory of Constraint and Motion Analysis for Robot Mechanisms.
http://dx.doi.org/10.1016/B978-0-12-420162-0.00001-1

Mechanism is one of the most important fundamental disciplines on which machine design relies. The invention, innovation, and improvement of mechanisms are important tasks in machine design. The basic problems of research in mechanism can be roughly divided into two major categories: structure analysis and structure synthesis. Structure analysis focuses on investigating the mobility and constraint, kinematics and dynamics of mechanisms, so as to reveal the structure constitution, the kinematics and dynamics laws, and their inter-relationships in mechanisms, and it can provide a theoretical basis for understanding the performance and mechanical synthesis of existing mechanical systems. Mechanism synthesis is defined as the theory and method of designing new mechanisms satisfying the engineering requirements in aspects such as structure, motion, kinematics, and dynamics. It includes structure synthesis, kinematics synthesis, and dynamics synthesis. Structure synthesis of mechanism is one of the most important areas in the field of mechanism. The entire mechanism is often proposed as a multi-rigid-body system, in order to reveal the assembly laws of the structure, characteristics of the structure topology, and their intrinsic relationships with kinematics, dynamics in mechanisms, so as to provide a basis for the establishment of structure synthesis of mechanism, kinematics, and dynamics theory, and to provide a theory to design new mechanisms [7].

1.1.1 Development of Mechanism

Germany was most active in the early study of mechanism. After the 1940s, and especially after the 1960s, the United States achieved rapid development in the research of mechanism. A lot of research work has been done in areas such as structure analysis and synthesis of planar and spatial linkages, cam dynamics, dynamics of elastomeric mechanical systems, as well as mechanical design optimization. The United States leads the world in the research of many aspects of mechanism. After the 1960s, the United Kingdom accelerated its pace of research in mechanism, and achieved substantial results in cam mechanism, mechanical system dynamics, as well as areas such as spatial linkages and industry robots. Ever since the establishment of the Institute of Mechanics of the Academy of Sciences in Soviet Union during the 1930s, mechanism became one of the main directions of development in the former Soviet Union. After World War II, the Soviet Union's research in mechanism developed rapidly. The Soviet Union made important contributions in aspects like mechanical structure theory, precision analysis, lower flat pair mechanical analysis, higher pair envelope theory, spatial mechanism, and system dynamics of multiple degrees of freedom (DOFs) mechanisms. Especially from the 1950s to the1970s, the Soviet Union remained the leader in the main areas of study in mechanism in the world. Similarly, Japan studied the cam mechanism dynamics and its computer-aided design and manufacturing and the analysis and synthesis of planar linkages and so on. After the 1970s, its research in robots and manipulators developed especially fast. Significant progress had been made in the classification and performance identification of grasping mechanisms, the motion possibility, kinematic analysis, and motion stability of multi-DOF multi-joint kinematic chains. Some researchers studied the functions of fingers, arms and legs in the field of bionics in order to build artificial muscles, artificial bones, and artificial feet, and gradually formed a new discipline, biomechanical engineering, and set up the institute of bionic mechanism [8] in Japan. In recent years, there has been great progress in kinematics, ZMP and dynamics, biped walking, the generation of whole-body motion patterns, and dynamic simulation of biped humanoid robots. In 1935, Professor Xianzhou Liu, China's well-known expert in mechanical engineering at the National Tsinghua University, published *Mechanisms and Machine Theory*, the first book focused on a systematic introduction of mechanical principles. This book marked the entry of Chinese scholars into mechanism fields.

In order to strengthen the international exchange and cooperation in mechanism research, the International Federation for the Theory of Machines and Mechanisms (the IFToMM) was established in the autumn of 1969. Well-known experts in mechanism around the world formed the Executive Board and various technology development committees under the IFToMM and founded the journal *Mechanism and Machine Theory*, in which a number of important research papers on mechanism are published.

Although the research of spatial mechanism began in the late 1920s, its progress remained slow until the end of 1950s. Due to its advantages (compact structure, occupies small space and can realize various flexible and complex motions with only a small number of links), fruitful results were achieved in spatial mechanism between the 1950s and the 1990s. The emerging robot mechanisms and complex spatial mechanisms like the virtual axis machine tools all fall under the research area of spatial mechanism.

From the perspective of the development process, parallel mechanism has become one of the hot research topics in mechanism after the 1990s. In the theoretical research, various parallel mechanisms, series-parallel mechanisms, the design theory of bionic mechanisms, and performance evaluation index systems of kinematics and dynamics were all important issues in the research. In applications, people conducted research on flight simulators, virtual axis machine tools, micro-actuators, sensor frameworks, error compensators, walking robots, a variety of bio-robots, etc. Developed in the last 30 years and based on the principle of Stewart platform, the parallel mechanisms [9,10] have attracted scholars' attention both at home and abroad. It has brought about new processing equipment, a virtual axis machine tool which wins much attention from all over the world. With more than 20 years' development, it has already been profoundly understood, and it has been widely applied in many areas of mechanical engineering.

Although its prototype, Stewart platform [9], came out as early as 1965, and the Soviet Union Novosibirsk Electro Technical Institute developed a pilot machine [11] based on this new concept of virtual axis machine tool, what attracted the world's attention was the parallel machine tool VARIAX which two American companies, Ingersoll and Giddingls & Lewis, displayed for the first time at the Chicago International Machine Tool Exhibition (IMTS94) in 1994. They brought about the sensational effect of international R&D boom. After that, at the 11th European Machine Tool Exhibition in 1995, the Swiss Geodetic company exhibited GPM1000-20S-type machine tools, and the Italian Comau company exhibited the Tricept deburring robot. Parallel structures were all applied in their actuators. At the 1997 Hanover International Machine Tool Exhibition (EMO97) in Germany, more than 10 prototype parallel machine tools were displayed, and the first milling metal work-piece performances were conducted. Research on the application of virtual axis machine tools has been of general concern to the academic community around the world. For instance, Switzerland has developed a parallel machine tool in which Hexaglide parallel mechanisms are used, and Pierrot has put forward a Hexa type of parallel mechanism. The Delta mechanism is now very widely used in the three-DOFs parallel mechanisms [12]. For the latest development of the virtual axis machine tools at home and abroad, refer to the literature [13,14]. China has also made important progress in the theoretical research into the structure of planar and spatial linkages, and has reached world-class level in the inverse kinematics of serial robots, workspace analysis, kinematic analysis of the parallel multi-loop spatial linkages, as well as the synthesis of linkages.

In the theoretical study of mechanism, the use of vectors to describe the rigid body motion is one of the most basic traditional analysis methods. In spatial mechanism, the right-handed Cartesian coordinate of vector is often used. Along with that coordinate, regular expressions such as vector inner product and outer product, and vector differential method are involved in studying the mechanical kinematics and dynamics problems such as the speed and acceleration. In 1959, Shigley [15] carried out systematic research on the vector. Hartenberg and Denavit [16] first proposed the idea of using the matrix transformation of four structural size parameters and position parameters to describe projection relations of the point coordinates in the absolute coordinate system, i.e. using matrix operations of kinematic pairs to describe the position and orientation of mechanisms. Hartenberg and Denavit also conducted analysis into and synthesis of several commonly used spatial four-bar linkages, now known as the DH transformation matrix method. In the kinematic analysis of spatial mechanism, the dual number, screw vector, and dual vector are also often used. As early as the end of 1940s, ДимАитьирг, a Soviet scholar, conducted kinematic analysis and local freedom degree analysis using the dual number and a sub-screw affine. At the same time, the majority of mechanism scholars in the Soviet Union, led by the first President of IFToMM, Academician И. И. Артоболевский, creatively built systematic geometry and algebra methods for the structure constitution, kinematic analysis, dynamic analysis, and synthesis, and they made tremendous contributions to the development of mechanism. In 1967, Hunt [17] studied the problem of kinematic analysis of spatial mechanisms using screw-axis and line geometry. In 1970, Woo and Freudenstein [18] derived the algebraic formula for the kinematic analysis of mechanisms using the principles of line geometry and developed a numerical algorithm of the screw vector. Scholars like Duffy [19,20] derived the input and output displacement equations of dual numbers for the spatial five-bar linkage, using spherical trigonometry. Duffy and Keen [21] again applied the same method to RRERR mechanisms. They viewed the plane pair (E) as a combination of 2P and 1R for discussion. In 1968, Roth [22] studied the general design principles of kinematic pairs containing revolute pair (R), prismatic pair (P), and cylindrical pair (C). In 1972, Tsai and Roth [23] discussed the synthesis of limited or unlimited separated locations containing revolute pair (R), cylinder pairs (C), spherical pair (S), prismatic pair (P) and helical pair (H).

In 1978, Hunt [24] published *Kinematic Geometry of Mechanisms*, systematically introduced kinematic geometry, an effective tool for studying mechanism, and further developed Ball's screw theory [25]. The book discussed the kinematic pairs, links, planar and spherical mechanisms, the exchange of kinematic pairs, synthesis of types, number, and sizes of mechanisms, mobility and constraints; it studied the kinematic geometry representation of the planar and spatial mechanisms; it developed the theory of dual number and screw vectors; and applied these tools to the analysis of spatial mechanisms. Phillips published *Freedom in Machinery, Volume 1: Introducing Screw Theory* [26] and *Freedom in Machinery, Volume 2: Screw Theory Exemplified* [27] in 1984 and 1990, respectively, and systematically discussed the free motions and constraints of mechanisms. He directly discussed the kinematic and static constraints between rigid bodies in the three-dimensional space. As a result, he developed the screw theory. Volume II focuses on the screw theory, and its applications in the analysis and synthesis of general mechanisms, as well as the actual design of mechanisms. Huang published *Spatial Mechanism* [28] in 1991 and systematically introduced the spherical analysis theory and the screw theory in China.

Therefore, mechanism, as a basic discipline in mechanical engineering, was formed in the 19th century step by step, and developed rapidly in the 20th century. The continuing development of the mechanism theory promotes the development of robot mechanism. The analysis and synthesis methods of robot mechanism are focusing in the direction of high-precision, high-speed, and visualization. Research on the types of mechanisms focuses on the overall structure of the model, the mechanism's mobility, singularity, trajectory, rigid-body guidance, drive, workspace design and planning, as well as the design optimization of multi-dimensional constraints. In the research on the mechanism structure theory, many scholars conducted systematic analysis into the structure types and the DOF of motion by using a variety of mathematical tools such as vector, tensor, matrix, quaternion, graph theory, linear algebra, analytical geometry, dual number, screw theory, etc.

The key point of study on the DOF of kinematics is how to analyze the multi-loop spatial linkages with different general constraints in different loops and how to analyze these spatial linkages with overconstraints and local DOF; and how to explore and establish structure principle of mechanisms. Structure theory of mechanism focuses on the structure principle of mechanisms, possible types, and the calculation of kinematic DOF. Structure theory of mechanism came up very early, and initially focused on the planar mechanisms. The mechanisms in the practical engineering application at that time were generally of single DOF and of closed-loop, and had only a few small links. In recent years, mechanism theory has focused on parallel robot mechanisms, dexterous hand operation, walking robots, as well as parallel robot machine tools and so on. Mechanism scholars around the world have recently conducted research on issues such as the structure design and assembly process design of the parallel mechanism [29,30], robot position analysis [31–42] kinematics [35,43–45] and dynamic analysis [46–49], the singularity [50–55] as well as workspace [56–74], the analysis and design of flexible linkages [75–77], and have made significant progress. However, how to correctly determine the mechanisms' general constraints has always been a difficult problem in the structure analysis of complex spatial mechanisms. Many scholars [78–83] have carried out extensive research in determining the general constraints in parallel mechanisms with multiple closed-loops.

The modern complex spatial mechanisms have already been applied in practice. It can be expected that in the near future, extensive applications of modern mechanisms will contribute significantly to the invention of modern new machines, promoting the rapid development of machine industry. The modern mechanism has opened an effective way for the design of micro-machines. The development of micro-machines requires the relative research on design theories and methods. In addition, the investigation into the types of bionic mechanisms will render creations of all sorts of bionic machines, so that the application of modern mechanism will be much wider and more effective. As a result, the designs and development of robots regarding mechanism type, dimension synthesis, workspace, dynamic stiffness, and control techniques should all be guided by the modern theory and method of mechanism. This has been of broad consensus among mechanism scholars around the world. In 2003, the International Federation for the Theory of Mechanism and Machine (IFToMM) published a third edition of terminology in which many nouns such as "mechatronics," "micro-electro mechanical systems (MEMS)," and "bionic mechanisms" appeared. At the same time, IFToMM changed its name from the International Federation for Theory of Mechanism and Machine to International Federation for Promotion of Mechanism and Machine Science in 2003. New theories and methods of mechanism are being systematized, and the new theoretical system relating to the overall characteristics and new inventions in mechanism are being improved step-by-step. There are inherent inter-relationships between different level units and the overall characteristics of the system. This has led to the integration of structural synthesis of mechanism, kinematics, and dynamics. The crossover of mechanism and computer science brings about computing mechanism; the crossover of mechanism and bionics has contributed to the formation of mechanism bionics; and the crossover of mechanism, biomedicine, and automatics develops into rehabilitation engineering and robotics. Undoubtedly, the innovation in designing robots, especially robots for special purposes, relies on