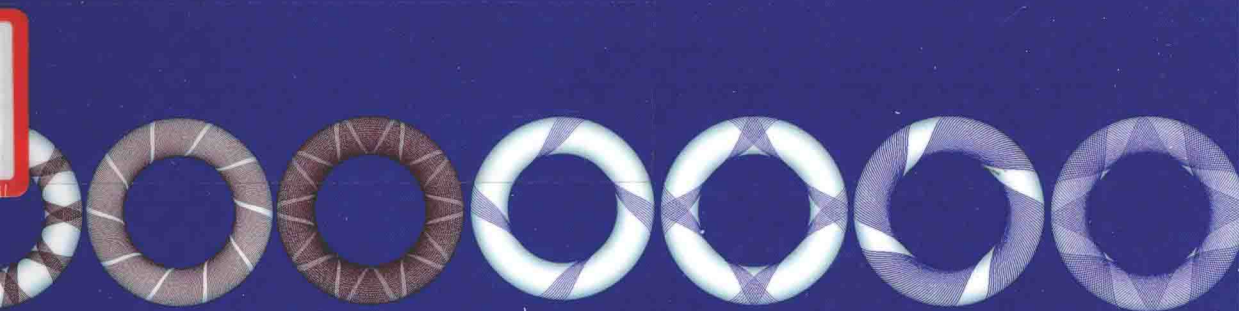
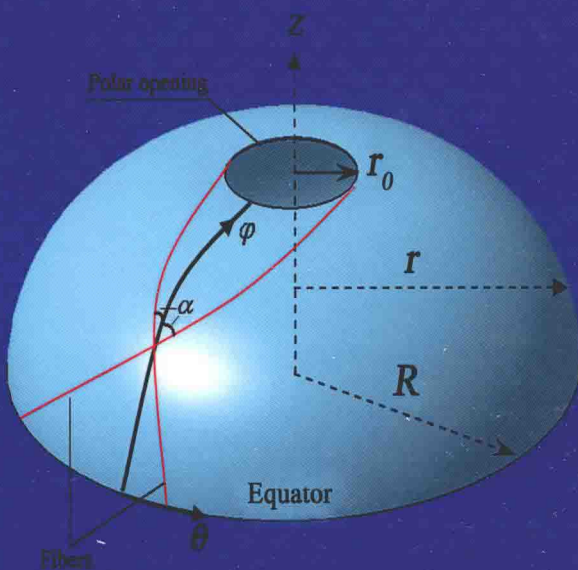


Design and Optimization of Filament Wound Composite Pressure Vessels

Lei Zu



DESIGN AND OPTIMIZATION OF FILAMENT WOUND COMPOSITE PRESSURE VESSELS

Proefschrift

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Summary

One of the most important issues for the design of filament-wound pressure vessels reflects on the determination of the most efficient meridian profiles and related fiber architectures, leading to optimal structural performance. To better understand the design and optimization of filament-wound pressure vessels, in this dissertation we present an overview and comprehensive treatment for toroidal and domed pressure vessels. Since the geodesic winding has severe boundary conditions that confine the layup optimization, the non-geodesic trajectories are here extensively applied to enlarge the design space.

Designing optimal laminate layup is not the only issue; the fibers must be stable on the mandrel and be exactly placed along trajectories as predetermined by structural design. To obtain a stable fiber trajectory, the stability-ensuring conditions are formulated in terms of both fiber slippage and bridging tendencies; these conditions provide the basic criteria for the subsequent design of various pressure vessels. The mathematical description of the geodesics and non-geodesics on a generic shell of revolution is briefly presented.

A generalized optimality criterion that is adapted to various optimal design problems for pressure vessels is elaborated. This condition originates from the idea that the optimal pressure vessels are governed by the condition of equal shell strains, or equivalently, zero shear stress at lamina level. The specific equations and the feasible intervals of the optimality condition are also given for several types of laminations.

The basic equations of the netting analysis and their applications to the design of circular toroidal pressure vessels are here outlined. The influence of the fiber layup and the geometry of the toroid on the stability of netting-dictated fiber trajectories are evaluated.

A new possibility to improve the vessel performance can be offered by the application of adapted cross-sectional shapes instead of the conventional shapes. The isotenoid design, which leads to equal fiber tension throughout the whole structure, is conducted to determine the netting-based optimal cross-sectional shapes. The governing equations for determining

Summary

geodesic and non-geodesic isotenoids are respectively derived and their feasible intervals are also determined. In addition, a simplified method for designing isotenoid pressure vessels with unequal polar opening is also outlined, with the aid of non-geodesic trajectories.

The optimal design, based on orthotropic plate theory, is divided into two basic approaches: numerical and semi-analytical methods. A numerical optimization method is specially designed for determining the optimal meridian profiles of bellow-shaped pressure vessels. An integral design method is proposed for circular toroidal pressure vessels, with emphasis on the determination of the optimal non-geodesic trajectories and winding patterns. Based on the previously-obtained (generalized) optimality condition, semi-analytical design methods are presented for the determination of the optimal meridian profiles for continuum-based domes and toroids, respectively. The optimal cross sectional shapes lead to significantly improved vessel performance.

An extensive study of the manufacturing of filament wound toroidal pressure vessels is conducted. We here emphasize the importance of suitable winding patterns for obtaining an optimal pressure vessel, and we accordingly derived the "Diophantine"-like pattern equations that produced such patterns. The main objective of the method presented here is to match the structure-dictated number of wound circuits to the solution of the pattern equations for determining the proper winding velocities of the mandrel and the feed eye. In addition, depending on the aimed lathe machine configuration, the underlying geometric model of the new-fashioned toroidal winder is outlined and the kinematic solutions for coupling the motion of the mandrel and the feed eye are also given. Simulations of geodesic and non-geodesic trajectories are performed for winding toroidal pressure vessels.

Last but not least, since ultra-high pressure vessels require thick-walled designs, this dissertation is also extended to three-dimensional problems where the through-thickness stress gradient is taken into account. A three-dimensional (3D) elasticity analysis on multi-layered thick-walled pressure vessels is here addressed. In order to better understand the design approaches of thick-walled composite cylinders and find ways to improve their structural performance, a review is devoted to 3D elasticity approaches for obtaining the exact solutions of the stresses and strains induced by internal pressure, and the effects of hygrothermal loading and twisting. The 3D effective elastic constants and most frequently used failure criteria for cylindrically anisotropic materials are also presented.

Samenvatting

Eén van de belangrijkste aspecten van het ontwerp van gewikkelde drukvaten is de bepaling van de meest efficiënte meridiaanprofielen en gerelateerde vezelarchitecturen die leiden tot de optimale constructieve prestatie. Voor een beter begrip van het ontwerp en de optimalisatie van gewikkelde drukvaten presenteert deze dissertatie een uitvoerige behandeling van toroïde- en koepelvormige drukvaten. Vanwege het feit dat geodetische banen strikte grenswaardes met zich meebrengen die de optimalisatie van de laminaatopbouw sterk beperken, worden niet-geodetische banen veelvuldig toegepast om de ontwerpruimte te vergroten.

Het ontwerp van de optimale laminaatopbouw is niet het enige aspect dat in ogenschouw moet worden genomen. Het vezelmateriaal moet tevens stabiel op de mal gepositioneerd zijn en bovendien exact geplaatst zijn langs de uit het constructieve ontwerp afgeleide banen. Om een stabiele vezelbaan te verkrijgen, worden condities geformuleerd waardoor de stabiliteit gegarandeerd is als functie van zowel vezelslip als overbruggingsneigingen; deze condities leveren de basiscriteria voor het daaropvolgende ontwerp van diverse drukvaten. Een korte mathematische beschrijving van geodeten en niet-geodeten op een algemeen roterende schaal wordt kort gepresenteerd.

Een veralgemeniseerd optimalisatie criterium wordt uitgewerkt en toegepast op diverse ontwerp problemen. Dit criterium komt voort uit het idee dat optimale drukvaten worden gedefinieerd door de conditie van gelijke schaalrekken of - hieraan gelijkstaand - de afwezigheid van schuifspanningen op laminaatniveau. De specifieke vergelijkingen en de uitvoerbare intervallen van het optimalisatie criterium worden ook behandeld voor verschillende typen laminaten.

De basisvergelijkingen van de 'netting analysis' en de toepassing hiervan op het ontwerp van circelvormige toroïde drukvaten worden besproken. De invloed van de lay-up en de geometrie van de toroïde op de stabiliteit van de netting-gedomineerde vezelbanen worden

Samenvatting

geëvalueerd.

Er wordt een nieuwe mogelijkheid gepresenteerd ter verbetering van de prestatie van het drukvat welke berust op de toepassing van aangepaste doorsnedes in plaats van conventionele doorsnedes. Het isotensoïde ontwerp welke leidt tot gelijke vezelspanning door de gehele structuur, wordt uitgevoerd ter bepaling van de netting-gebaseerde optimale doorsnedes. De vergelijkingen voor de bepaling van geodetische en niet-geodetische isotensoïdes worden afgeleid en tevens worden hun uitvoerbare intervallen bepaald. Aanvullend wordt een vereenvoudigde methode uiteengezet voor het ontwerp van isotensoïde drukvaten met ongelijke poolopeningen door gebruikmaking van niet-geodetische banen.

Het optimale ontwerp gebaseerd op de orthotrope plaat theorie wordt verdeeld in een tweetal basis methodes: numerieke en semi-analytische methodes. Een numerieke optimalisatie methode wordt speciaal opgesteld om de optimale meridiaanprofielen van balgvormige drukvaten te bepalen. Een integrale ontwerpmethode voor circelvormige toroïde drukvaten wordt voorgesteld, met de nadruk op de bepaling van de optimale niet-geodetische banen en wikkelpatronen. Gebaseerd op de eerder verkregen (gegeneraliseerde) optimalisatie conditie worden semi-analytische ontwerpmethodes gepresenteerd ter bepaling van de optimale meridiaanprofielen voor op continuüm gebaseerde koepelvormen en toroïdes. De optimale doorsnedes leiden tot significant verbeterde drukvaten.

Er wordt een omvangrijke studie uitgevoerd naar de productie van gewikkelde toroïde drukvaten. De nadruk ligt hier op de belangrijkheid van bruikbare wikkelpatronen om optimale drukvaten te verkrijgen en dienovereenkomstig leiden we de Diophantische patroonvergelijkingen af die dergelijke wikkelpatronen tot gevolg hebben. Het belangrijkste doel van die hier gepresenteerde methode is om het door de structuur gedomineerde aantal gewikkelde circuits passend te maken met de oplossing voor de patroonvergelijkingen ter bepaling van de juiste wikkelsnelheden van de mal en het wikkeloog. Dienovereenkomstig, afhankelijk van de beoogde machine configuratie, wordt een uiteenzetting gegeven van het onderliggende geometrische model van de nieuw ontworpen toroïde wikkelmachine en tevens worden de kinematische oplossingen voor de gekoppelde beweging van mal en wikkeloog gepresenteerd. Simulaties van geodetische en niet-geodetische banen worden uitgevoerd voor het wikkelen van toroïde drukvaten.

Tenslotte, omdat zeer hoge drukvaten dikwandige ontwerpen vereisen, is deze dissertatie verder uitgebreid met driedimensionale problemen waarbij de spanningsgradiënt door de dikte ook een rol speelt. Een driedimensionale elasticiteitsanalyse op meervoudig gelaagde dikwandige drukvaten wordt hier behandeld. Om de ontwerpbenaderingen van dikwandige

composieten cilinders better te begrijpen en manieren te vinden om hun constructieve eigenschappen te verbeteren wordt een bespreking gewijd aan driedimensionale elasticiteitsbenaderingswijzen om de exacte oplossingen van de spanningen en rekken geïnduceerd door de interne druk te verkrijgen. Tevens worden de effecten van hygrothermale belasting en verdraaiing bestudeerd. Daarnaast worden de driedimensionale effectieve elastische constanten en de meest toegepaste faalcriteria voor cilindrische anisotrope materialen gepresenteerd.

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