BOUNDARY ELEMENT METHODS IN CREEP AND FRACTURE

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To Krishna, who makes the wheels turn

Preface

This book is concerned with the applications of the boundary element method to problems of time-dependent inelastic deformation and fracture of metallic media. Such problems are of interest in many fields including energy, turbomachinery and aerospace.

Although the boundary element method is rooted in classical integral equation formulation of problems, numerical implementation of the method is of fairly recent vintage. Roughly speaking, the method has been used to obtain numerical solutions to linear problems in solid mechanics in the 1960s and problems with material nonlinearities in the 1970s. I and my coworkers have been intimately involved with applications of the method to solve nonlinear problems in solid mechanics, particularly in viscoplasticity and fracture, for the last seven years.

This book is divided into ten chapters. The first four chapters set the stage for applications to specific classes of problems. These are followed by four chapters dealing with the applications to planar, axisymmetric, torsion and plate bending problems. The last two chapters are concerned with inelastic fracture mechanics.

The strength or weakness of a numerical method can only be judged through obtaining numerical solutions to specific problems. With this in view, many numerical applications of the method are presented in the book. The results obtained by the boundary element method are compared to those obtained by other methods whenever possible. In particular, the more widely used finite element method is given special attention and comparisons of results from the boundary element and finite element methods, with regard to accuracy and computational efficiency, have been carried out in many cases. Both these are powerful general-purpose methods and, as can be expected, the boundary element method appears superior in some cases while the finite element method performs better in others. In many applications, both methods appear to be equally efficient.

This book is directed towards researchers, practising engineers and scientists, and postgraduates. The reader is expected to be familiar with the general area of solid mechanics and with the basic techniques of applied mathematics and numerical methods.

I wish to thank a number of people and organizations who have contributed in various ways to make this book possible. First, my principal research collaborators, my former PhD students Dr V. Kumar and Dr M. Morjaria, who have made significant contributions to much of our group's research that has been described in this book. Next, my current students V. Sarihan and V. Banthia, for their contributions to the work on axisymmetric deformation and inelastic fracture, respectively, and A. Chandra and S. Ghosh for their help during the writing of this book. I am indebted to my colleagues at Cornell, Professors Hart, Li, Conway, Moon and Pao, for their encouragement, and to Professors Banerjee and Shaw of Buffalo and Rizzo and Shippy of Kentucky for several useful technical discussions. I greatly appreciate the terrific artwork of Mrs Jane Jorgensen who was never a day late, and the excellent typing of Ms Delores Hart who typed so many long equations accurately and cheerfully. Finally, I appreciate the effort of Ms Hillary Rettig who helped finish typing the manuscript on time.

The research performed by our group, which has been presented in this book, was supported by a contract from the US Department of Energy and a grant from the National Science Foundation.

SUBRATA MUKHERJEE

Contents

CHAPTER 1

Introduction

The boundary element method (BEM—also called the boundary integral equation method) is a powerful general-purpose procedure for the solution of boundary value problems in many branches of science and engineering. This method is based on an integral equation formulation of a given problem and has been used to solve problems in finite regions of arbitrary geometry as well as in infinite regions. The method was initially applied to linear problems but several nonlinear problems have been attacked by the method in recent years.

The boundary element method has several potential advantages over the widely used finite element method (FEM) for the solution of boundary value problems. One advantage is that the number of unknowns in resultant algebraic systems depends only on the boundary (or surface) discretization in BEM rather than on the discretization of the entire domain of the body as in FEM. The resultant matrices from the BEM are fully populated but tend to be numerically well conditioned. This arises from the fact that the singular kernels in the integral equations weigh the unknown quantities near a singular point more heavily than those far from it, thus causing the dominant components in a coefficient matrix to lie on or near its diagonal.

Another advantage of the BEM is that physical quantities obtained by differentiation of the primary variables, for example stresses or curvatures obtained from displacements, are determined pointwise inside and on the body, so that discontinuities in these variables across element boundaries cannot arise. This can be very important in problems of inelastic deformation as will be discussed later in the book. A third important advantage is that problems in infinite regions can be solved as easily as those in bounded regions. Finally, in problems where internal discretization is necessary in order to evaluate integrals with known integrands over the domain of a body, the topology of the internal cells is much simpler than for the FEM. In fact, the internal cells need not even cover the whole domain in some examples, as is the case with inelastic fracture mechanics problems discussed in the last two chapters of the book.

An important limitation of the straightforward BEM formulation is that the singular kernels of the integral equations must be known in an appropriate infinite body having the same material properties as the finite body being considered in a given problem. While these fundamental solutions are easily available in the applied mathematics literature for certain differential operators in homogeneous media, they might be difficult to obtain for a body composed of a general nonhomogeneous material. It is possible, however, to solve certain problems in nonhomogeneous media, by iterative methods using the kernel(s) for the homogeneous medium.

The heart of the boundary element method, as mentioned earlier, is the integral equation formulation of a given boundary value problem. The mathematical basis of this approach, of course, is classical and numerous applications of Green's functions have been reported in the literature. The earliest computer applications of the method date back about two decades and include the areas of potential theory,[1,2] fluid mechanics[3,4] and wave scattering.[5] These papers might be said to have heralded the modern era of the method.

Applications of the method have gained considerable momentum in recent years, in step, it appears, with the rapid improvements in computers. In the general area of solid mechanics, the method has been applied to a large class of linear problems. This research spans most of the important subject areas within solid mechanics, namely, linear elasticity,[6,7] linear viscoelasticity,[8] thermoelasticity,[9] linear elastic fracture mechanics,[10,11] elastic torsion,[12] bending of elastic plates,[13] shell theory[14] and wave propagation in elastic media.[15-17] This reference list is by no means complete, but is intended to be a collection of titles of some of the early papers in each subject area.

Applications of the BEM to nonlinear problems in solid mechanics is of more recent vintage, with the first formulation for time-independent plasticity being that due to Swedlow and Cruse.[18] This was followed by a numerical implementation by Riccardella.[19] Another formulation for plasticity using an equivalent body force based on initial stress is due to Banerjee and Mustoe.[20] The author of this book, together with his coauthors, have been active in the applications of the BEM to nonlinear problems of time-dependent inelastic deformation. A BEM formulation for viscoplastic problems[21] was followed by a numerical implementation for planar problems.[22] Other applications include inelastic torsion of prismatic shafts,[23] inelastic bending of plates,[24] and, very recently, inelastic fracture mechanics.[25–26] Currently, there is a great deal of activity in computer applications of the BEM to a wide class of problems in engineering science. The reader is referred to three recent books[27–29] which summarize the current state of the art of BEM applications in excellent fashion.

The purpose of this book is the presentation of applications of the boundary element method to nonlinear problems of time-dependent inelastic deformation and inelastic fracture mechanics. A discussion of constitutive models for inelastic deformation (creep and combined creep–plasticity or viscoplasticity) is given in Chapter 2. General formulations for three-dimensional problems (Chapter 3) are followed by a discussion of solution strategy and time integration in Chapter 4. Time integration with automatic time-step selection plays a crucial role in the successful completion of solutions of these time-dependent problems. Specific numerical applications to planar problems (Chapter 5), axisymmetric problems (Chapter 6), inelastic torsion (Chapter 7) and bending of thin plates (Chapter 8) follow. In each of these cases, comparisons with the results of finite element method solutions, and, whenever possible, comparisons with direct solutions from finite difference type techniques, have been carried out. The last two chapters of the book are concerned with applications of the method to problems of inelastic fracture mechanics. The time-histories of stresses near the tip of a stationary crack in a plate undergoing anti-plane shear or planar creep deformation are calculated, and the results are compared with recent asymptotic analytical solutions. In essence, this book is an attempt to present a comprehensive and up-to-date account of BEM applications in time-dependent inelastic problems in solid mechanics and to demonstrate the power of the method in solving these complicated nonlinear problems.

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CHAPTER 2

Constitutive Models

Constitutive models for the description of material behavior in the inelastic regime are discussed in this chapter. The materials considered are assumed to be metallic and the displacements and strains are assumed to remain small enough so that no distinction needs to be made between initial and current configurations.

2.1 CONSTITUTIVE MODELS FOR CREEP

Conventional design and analysis of metallic structures undergoing time-dependent inelastic deformation is generally carried out by linearly decomposing the total strain ε_{ij} into elastic ($\varepsilon_{ij}^{(e)}$), creep ($\varepsilon_{ij}^{(c)}$), plastic ($\varepsilon_{ij}^{(pl)}$) and thermal ($\varepsilon_{ij}^{(T)}$) components and then using separate constitutive descriptions for each of these components. Thus

$$\varepsilon_{ij}=\varepsilon_{ij}^{(e)}+\varepsilon_{ij}^{(c)}+\varepsilon_{ij}^{(pl)}+\varepsilon_{ij}^{(T)} \tag{2.1}$$

Hooke's law is used to relate the elastic strains and stresses σ_{ij}

$$\varepsilon_{ij}^{(e)}=\frac{1}{E}\{(1+\nu)\sigma_{ij}-\nu\sigma_{kk}\delta_{ij}\} \tag{2.2}$$

where E is the Young's modulus, ν is the Poisson's ratio, δ_{ij} is the Kronecker delta and the summation convention is used over the repeated index k. The thermal strain is generally written as

$$\varepsilon_{ij}^{(T)}=\alpha T\delta_{ij} \tag{2.3}$$

in terms of the temperature (above some base temperature) T and the coefficient of linear thermal expansion α. The plastic strain is generally described in terms of a yield criterion, flow rule and hardening law (see,