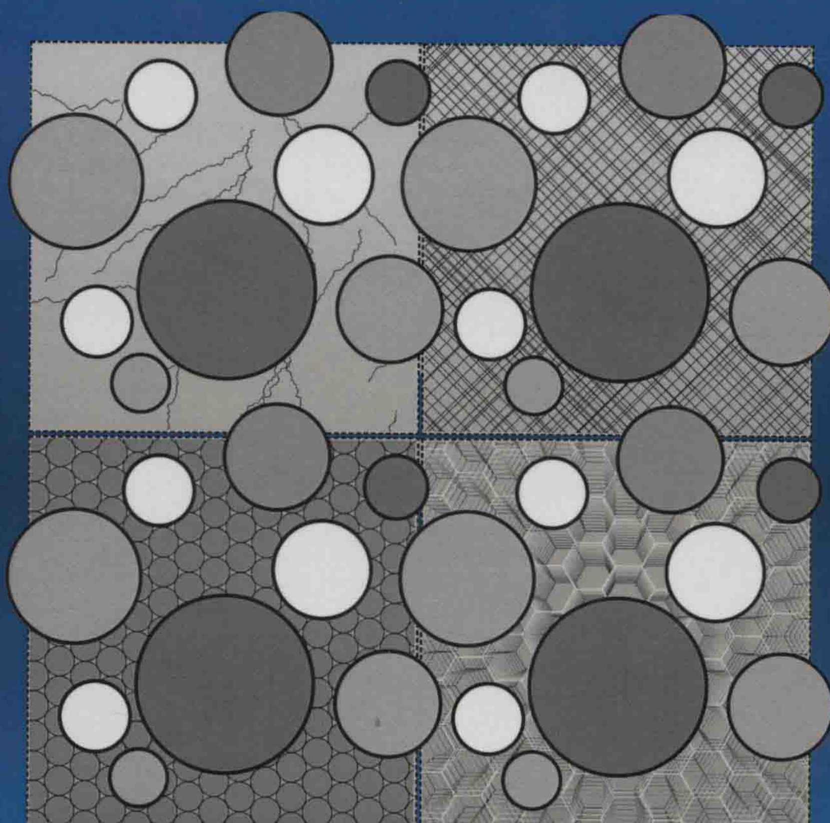
Micromechanics of Composites

Multipole Expansion Approach



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Micromechanics of Composites

Preface

The subject of this book is micromechanics being the analysis of heterogeneous materials on the level of individual constituents. Number of manmade composites is permanently increasing, in parallel with the rising need to study the "structure - properties" relationships as this knowledge enables purposeful tailoring of composite materials with superior properties by rational choice of components and composition. This explains an importance of micromechanics as a science and motivates its rapid development in recent decades.

In past years, the main effort in micromechanics was focused on the macroscopic properties of heterogeneous solids and the most work in the area has been done with aid of the single inclusion (Eshelby) model being the theoretical framework of several applied theories. These theories provide useful for practice bounds and approximations for the effective constants and so they can be grouped under the title of engineering, or *applied* micromechanics. Most of the published up to now books on micromechanics fall in this category. Certainly, applied micromechanics is easy and convenient for use—but, as always, convenience comes at a price. The latter involves low or uncertain accuracy, inability to account the microstructure of composite and, as a consequence, inapplicability to study the phenomena (e.g., damage) caused by the local fields.

Now, it is well recognized that a reliable prediction of composite's behavior must combine a realistic model of microstructure with an adequate analysis of the relevant model boundary problem. The need for an in-depth study has led to the development of *computational* micromechanics. Recent dramatic increase in computational power and available commercial FEA software made direct numerical approach accessible (not affordable for individual researcher, however) and enabled consideration of involved heterogeneous structures. The drawback of this approach is high computational effort, especially for 3D models. Another and, probably, even more substantial problem is extracting the meaningful data from a bunch of numbers generated by FEA code.

A promising alternative to computational micromechanics is the multipole expansion method also providing an efficient analysis of complex heterogeneous structures. Being mostly analytical in nature, this method constitutes a theoretical basis of high-performance computational algorithms and found numerous applications in astronomy, physics, chemistry, engineering, statistics, etc. Introduced by J.C. Maxwell in 1873 and then further developed by Lord Rayleigh in 1892, this is historically the first method of micromechanics. A substantial progress made since that time (and, especially, in recent years) in development of the multipole expansion method has been reported in numerous journal papers. However, in the author's opinion, a true value of this method for micromechanics is still underestimated and its potential in the area is not fully discovered so far. This book is the first monograph giving systematic account of the method, with application to the actual problems of micromechanics. The multipole expansion method uses a classical approach and toolkit of mathematical physics which is a compelling reason to consider it as *theoretical* micromechanics.

The book does not pretend to cover all the aspects and methods of micromechanics. Its specific aim is to describe theory and technique of the method in detail, with application to the selected actual problems of micromechanics. This is primarily the multipole expansion approach that sets this monograph apart and is alternative one to what readers would find in the other books. In the author's opinion, the following features of this work can be of particular interest for the reader.

- The multipole expansion theory and technique have been described and further developed. In this respect, the book is of interest for a wide readership including the specialists in applied mathematics, mathematical physics, engineering, and related areas dealing with heterogeneous media.
- A detailed analysis of a variety of micromechanical multi-inclusion models has been performed. The contemporary topics include the composites with imperfect and partially debonded interface, nanostructured materials, cracked solids, statistics of the local fields, brittle strength, etc. The obtained complete analytical solutions provide a clear insight into the physical nature of the problems.
- The book contains a number of tabulated data and plots for the various problems. The results of the multipole analysis are commonly considered as the most reliable and serve as a benchmark for testing applicability of approximate models and accuracy of numerical solutions.
- The considered mechanical models are readily generalized in many ways to take the specific features of real-world heterogeneous materials into account. The Fortran source codes given in the Appendix can be used by the readers as a starting point in developing their own codes.
- An important feature of the developed approach is high numerical efficiency. In contrast to computational micromechanics, the multipole expansion does not require the powerful computers and expensive software to be used and appears probably the most efficient (especially in the fast multipole version) method of micromechanics.

The book summarizes the work done by the author with colleagues for more than 20 years in development and application of the multipole expansion method and is expected to be of particular interest to researchers and professionals in applied mathematics, physics, mechanics, materials science, engineering, and related areas dealing with the heterogeneous solids. I am very grateful to all my colleagues and friends, in Ukraine and abroad, who have contributed to this work as well as inspired and supported me in many ways in conducting the research and writing this book.

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October, 2012

Volodymyr I. Kushch

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