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Euclid and His Modern Rivals

LEWIS CARROLL

Euclid and His Modern Rivals

Lewis Carroll

(Charles L. Dodgson, M.A.)

With a new Introduction by
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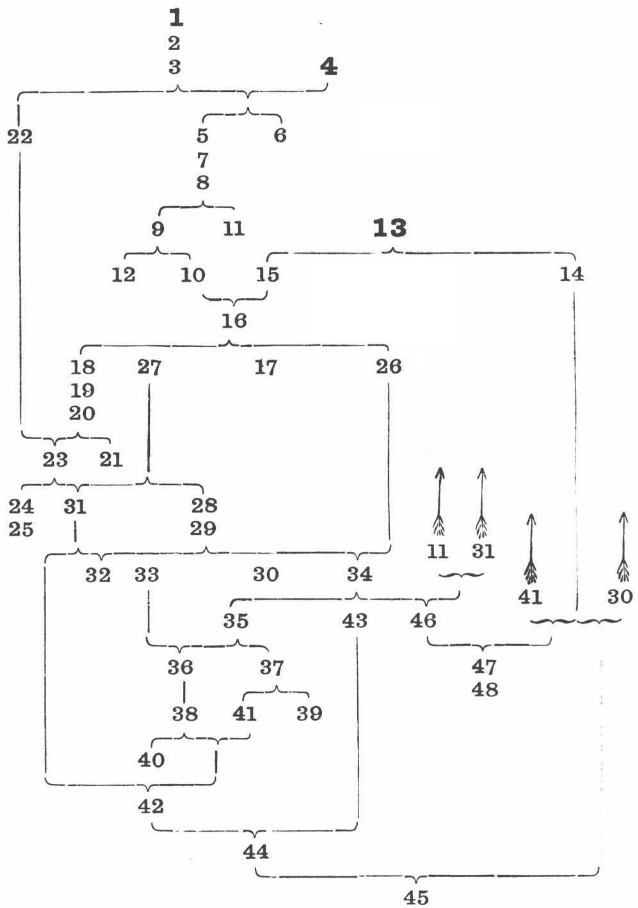
EUCLID, BOOK I.

Arranged in Logical Sequence.



Dedicated

to

the memory

of

Euclid

INTRODUCTION
TO THE DOVER EDITION

ECHOING Gerolamo Saccheri, whose *Euclides ab omni naevo vindicatus* was published in 1733, Dodgson writes (in the Preface to First Edition) "of the great cause which I have at heart—the vindication of Euclid's masterpiece." To make this project entertaining, he expresses it as a drama in which Euclid converses with Minos and Rhadamanthus, two of the three judges in Hades. Herr Niemand, "the Phantasm of a German Professor," appears on page 17, as spokesman for the 13 "modern rivals" who were trying to make the treatment of geometry more rigorous, or more palatable, by revising the definitions and rearranging the propositions. One by one, these rivals are ridiculed and put to shame. Time has justified Dodgson's scorn, for all save Legendre and Peirce have been forgotten. A. M. Legendre's *Eléments de Géométrie,* "though well suited for advanced students, is not so for beginners" (p. 59). According to Professor Benjamin Peirce (spelled "Pierce" in the text) of Harvard (pp. 144-147), "Parallel Lines are straight Lines which have the same Direction." And how did he define Direction? "The Direction of a Line in any part is the direction of a point at that part from the next preceding point of the Line."

Were Dodgson still alive, he would be equally indignant about a new generation of textbook writers, who try to make geometry easier by introducing redundant "postulates" or "assumptions." On page 57 we read, ". . . it is a generally admitted principle that, at least in dealing with beginners, we ought not to take as axiomatic any Theorem which can be proved by the Axioms we already possess." There is much to be said for his standpoint that the degree of rigor in Euclid's *Elements* is just right for high school: a modern axiomatic treatment (such as H. G. Forder, *The Foundations of Euclidean Geometry*, Dover, 1958) should be left for mature students in universities.

In *The summing-up* (p. 225) the departing ghost of Euclid pleads: "Let me carry with me the hope that I have convinced you of the importance, if not the necessity, of retaining my order and numbering, and my method of treating straight Lines, angles, right angles, and (most especially) Parallels." And again (p. 11): "The Propositions have been known by those numbers for two thousand years; . . . and some of them, I.5 and I.47 for instance— 'the Asses' Bridge' and 'the Windmill'— are now historical characters, and their nicknames are 'familiar as household words.' "

The book is by no means all Dodgson the serious teacher: every now and then Lewis Carroll appears with some unexpected analogy or outrageous pun. (See, for instance, pages 48 and 119.) And there are many quotable remarks, such as ". . . analogies give to Geometry much of its beauty" (p. 221). (For some further observations on these aspects of his work, see P. L. Heath's delightful article "Carroll, Lewis," in the *Encyclopaedia* *of Philosophy.*)

On pages 28–36 we find "TABLE I, *Containing twenty Propositions, of which some are undisputed Axioms, and the rest real and valid Theorems, deducible from undisputed Axioms*" and "TABLE II, *Containing eighteen Propositions of which no one is an undisputed Axiom, but all are real and valid Theorems, which, though not deducible from undisputed Axioms, are such that, if any one be admitted as an Axiom, the rest can be proved.*" A few years later, Dodgson would doubtless have included, in the latter table, a nineteenth item (see "A New Theory of Parallels," which is Part I of *Curiosa Mathematica,* 3rd ed., London, 1890, p. 14): *In every circle, the inscribed equilateral tetragon is greater than any one of the segments which lie outside it.*

Tables I and II are his nearest approach to the subject of non-Euclidean geometry. In these days of enlightenment we find it difficult to realize that, 100 years ago, Professors Arthur Cayley of Cambridge and W. K. Clifford of London may well have been the only Englishmen who understood the philosophical revolution that had been instigated by Gauss, Bolyai and Lobachevsky, some 50 or 60 years earlier. One is tempted to speculate on what might have happened if Cayley or Clifford had met Dodgson and convinced him that there is a logically consistent "hyperbolic" geometry in which the "absolute" propositions in Table I still hold while all the statements in Table II are false (and the nineteenth proposition fails for any sufficiently large circle). In his *Sylvie and Bruno Concluded* (London, 1893, pp. 100–104) the real projective plane is represented as a "Purse of Fortunatus" made by sewing together three square handkerchiefs.

The same easy style and fertile imagination, applied to the infinite hyperbolic plane, would surely have produced a thrilling exploration of this new Wonderland.

H. S. M. COXETER

Toronto, Canada
March, 1973

PREFACE TO SECOND EDITION.

THE only new features, worth mentioning, in the second edition, are the substitution of words for the symbols introduced in the first edition, and one additional review—of Mr. Henrici, to whom, if it should appear to him that I have at all exceeded the limits of fair criticism, I beg to tender my sincerest apologies.

C. L. D.

Ch. Ch. 1885.

PREFACE TO FIRST EDITION.

'ridentem dicere verum
Quid vetat?'

THE object of this little book is to furnish evidence, first, that it is essential, for the purpose of teaching or examining in elementary Geometry, to employ one text-book only; secondly, that there are strong *a priori* reasons for retaining, in all its main features, and specially in its sequence and numbering of Propositions and in its treatment of Parallels, the Manual of Euclid; and thirdly, that no sufficient reasons have yet been shown for abandoning it in favour of any one of the modern Manuals which have been offered as substitutes.

It is presented in a dramatic form, partly because it seemed a better way of exhibiting in alternation the arguments on the two sides of the question; partly that I might feel myself at liberty to treat it in a rather lighter style than would have suited an essay, and thus to make it a little less tedious and a little more acceptable to unscientific readers.

In one respect this book is an experiment, and may chance to prove a failure: I mean that I have not thought it necessary to maintain throughout the gravity of style which scientific writers usually affect, and which has somehow come to be regarded as an 'inseparable accident' of scientific teaching. I never could quite see the reasonableness of this immemorial law: subjects there are, no doubt, which are in their essence too serious to admit of any lightness of treatment—but I cannot recognise Geometry as one of them. Nevertheless it will, I trust, be found that I have permitted myself a glimpse of the comic side of things only at fitting seasons, when the tired reader might well crave a moment's breathing-space, and not on any occasion where it could endanger the continuity of a line of argument.

Pitying friends have warned me of the fate upon which I am rushing: they have predicted that, in thus abandoning the dignity of a scientific writer, I shall alienate the sympathies of all true scientific readers, who will regard the book as a mere *jeu d'esprit*, and will not trouble themselves to look for any serious argument in it. But it must be borne in mind that, if there is a Scylla before me, there is also a Charybdis—and that, in my fear of being read as a jest, I may incur the darker destiny of not being read at all.

In furtherance of the great cause which I have at heart—the vindication of Euclid's masterpiece—I am content to run some risk; thinking it far better that the purchaser of this little book should *read* it, though it be with a smile, than that, with the deepest conviction of its seriousness of purpose, he should leave it unopened on the shelf.

To all the authors, who are here reviewed, I beg to tender my sincerest apologies, if I shall be found to have transgressed, in any instance, the limits of fair criticism, To Mr. Wilson especially such apology is due—partly because I have criticised his book at great length and with no sparing hand—partly because it may well be deemed an impertinence in one, whose line of study has been chiefly in the lower branches of Mathematics, to dare to pronounce any opinion at all on the work of a Senior Wrangler. Nor should I thus dare, if it entailed my following him up 'yonder mountain height' which *he* has scaled, but which *I* can only gaze at from a distance: it is only when he ceases 'to move so near the heavens,' and comes down into the lower regions of Elementary Geometry, which I have been teaching for nearly five-and-twenty years, that I feel sufficiently familiar with the matter in hand to venture to speak.

Let me take this opportunity of expressing my gratitude, first to Mr. Todhunter, for allowing me to quote *ad libitum* from the very interesting Essay on Elementary Geometry, which is included in his volume entitled 'The Conflict of Studies, and other Essays on subjects connected with Education,' and also to reproduce some of the beautiful diagrams from his edition of Euclid; secondly, to the Editor of the Athenæum, for giving me a similar permission with regard to a review of Mr. Wilson's Geometry, written by the late Professor De Morgan, which appeared in that journal, July 18, 1868.

C. L. D.

Ch. Ch. 1879.

EUCLID
and His
MODERN RIVALS

SCENE II.

[MINOS *and* EUCLID.]

§ 1. A priori *reasons for retaining Euclid's Manual.*

§ 2. *Method of procedure in examining Modern Rivals.*