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Frank Kneip

Iterative Learning Control for Nonlinear Systems

An Operating Regime Based Approach

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Wir verlangen, das Leben müsse einen Sinn haben –
aber es hat nur genauso viel Sinn, als wir selber ihm zu geben im Stande sind.

Hermann Hesse

To Timo

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I dedicate this thesis to my deceased brother Timo, who will always be a part of my life – within my heart and my thoughts.

Contents

Chapter 1

Introduction

Iterative learning control (ILC) is a method for improving the tracking performance of systems or processes that operate repetitively over a fixed finite time interval. A tracking problem considers the task, that a desired output signal of the system under consideration is given, and one tries to determine suitable inputs or controls to that system, such that the desired output is reproduced or tracked either perfectly or 'as well as possible'.

The idea of ILC is to learn from the output errors of the previous iteration, i. e. the difference between the desired output signal and the output signal which was obtained by applying an input signal to the system. Learning means, that one modifies the input applied to the system in the previous iteration according to an input update law, which makes use of the knowledge of the input as well as of the resulting output error of that previous iteration, in order to obtain a new input, which is applied to the system in the next iteration. In other words, in time domain based ILC, we derive the input trajectory $\mathbf{u}_{k+1}(t)$, $t \in \mathcal{T}$, where $\mathcal{T}$ denotes the finite time interval, in the $(k+1)$–th iteration according to an update law of the general form

$$\mathbf{u}_{k+1}(t) = F(\mathbf{u}_k(t), \mathbf{e}_k(t), \dot{\mathbf{e}}_k(t))$$

for continuous time systems, resp. an update law of the general form

$$\mathbf{u}_{k+1}(t) = F(\mathbf{u}_k(t), \mathbf{e}_k(t), \mathbf{e}_k(t+1))$$

for discrete time systems. Thereby $\mathbf{u}_k(\cdot)$ denotes the input and $\mathbf{e}_k(\cdot)$ denotes the output error signal of the k–th iteration. Some common input update laws, which are also considered in the underlying thesis, are so–called P–type, D–type and PD–type input update laws. We refer to chapter 3 for more details.

Besides time domain based ILC schemes, frequency domain related ILC schemes are considered in iterative learning control related literature (see e. g. [Goh94],

[PS90], [A^+96], [dR96], [GY96], [Nor00], [NG02], [JVS03]). In the underlying thesis we will be concerned with the development and analysis of a new specific time domain based iterative learning control scheme.

The natural approach of iterative learning differs from other control schemes, e. g. such as PID or $\mathcal{H}_\infty$ feedback control, which are not able to take the history of the output error over the iterations into account. The feedback approaches, apart form adaptive control schemes, will derive the same tracking quality in each iteration, but they are not able to improve their performance over the iterations. Besides being a technique for improving the tracking performance, iterative learning control is also used to improve the transient response properties of repetitively executing systems.

These advantages have led to an increasing popularity of iterative learning control over the last twenty years. According to most of the existing publications related to ILC, the origin of iterative learning control lies in 1984, where Arimoto, Kawamura and Miyazaki published [AKM84a] and [AKM84b]. In some publications, as e. g. in [Moo93] and in particular in [Ari98], Uchiyama's article [Uch78], which was published in Japanese in 1978, is denoted as the origin of ILC. Since 1984, ILC has gained considerable attention and a variety of publications concerning the theory and the application are available. We cannot cite all the various publications, but we want to point out some specific ones, namely [Moo92], [Moo93], [BX98], [CW99], [Moo99], [Lon00], [PLM00], [AS02], [XT03], which might give a good overview of the research on iterative learning control over the past twenty years. The recent publications of some books, e. g. [BX98], [CW99], [AS02], [XT03], and pre–master, master, diploma and PhD–thesis, e. g. [Eic96], [Sch96], [Leh97], [Hua99], [Nor00], [Sch00], [Sca01], [Egl02], [Hin02], [Hät04], as well as the fact that the *International Journal of Control* published a special issue on iterative learning control in volume 73 in the year 2000, shows that some vital research in the field of iterative learning control is ongoing.

Typical fields of application for ILC schemes are, among many others, e. g. robotic and motion systems, ([AKM84b], [BCG88], [KLN92], [LLB93], [NG99] [TCH00], [CH00], [dRB00], [Sch00], [Sca01]), piezoelectric tube scanners ([Hin02]), and test rigs for durability testing in automotive industry ([J^+98], [JV02], [JVS03]).

In durability testing in the automotive industry, and in particular in durability testing of a full car, a half car, or a quarter car, one mounts the corresponding object onto a durability test rig and one tries to 'shake' the object in such a way, that the resulting forces, which are measured at some specific points at the wheels, are similar to some forces one recorded e. g. during a test drive on a rough road. In other words, one tries to track the forces measured during the test ride and therefore one has to determine suitable inputs for the test rig, which, depending on the specific test rig under consideration, can be e. g. displacements, forces or voltages. Some automotive test rigs are shown in figure 1, and we refer to [DP01] and [DW04] for more details related to the test rigs.

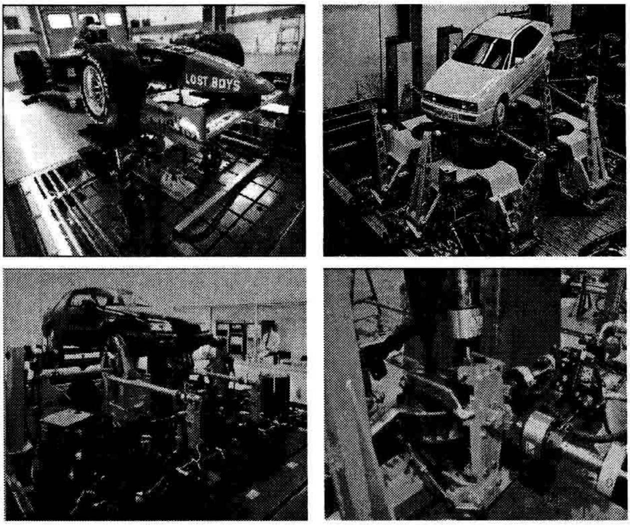

Figure 1.1: Test rigs in automotive industry

A standard, frequency domain based, iterative learning control scheme applied in industry, which is denoted as 'Time Waveform Replication' or shorthand 'TWR', is described in [J$^+$98] and [JVS03]. It is based on a nonparametric linear model of the system, which is given in terms of a truncated frequency response function (FRF), denoted by $\widehat{\mathbf{G}}(j\omega)$, that is calculated over a finite set of frequencies $\omega \in \{\omega_{min}, \ldots, \omega_{max}\}$. We refer to [JVS03] for more details. The input update is calculated in frequency domain, using the FRF and the discrete Fourier transform (DFT), according to the update law

$$\mathbf{U}_{k+1}(j\omega) = \mathbf{U}_k(j\omega) + Q_k \widehat{\mathbf{G}}(j\omega)^{-1} \mathbf{E}_k(j\omega),$$

where Q_k is a diagonal scaling matrix with diagonal elements in $(0,1)$. We refer to [JVS03] for a detailed description of the algorithm.

The TWR algorithm is based on a linear model, and possesses some robustness properties with respect to model uncertainties and possible nonlinearities of the system under consideration. If the system's nonlinearities exceed the robustness bounds, then the TWR scheme might diverge, i. e. the output error might increase over the iterations. This effect is illustrated in chapter 5 of the underlying thesis. If divergence of linear model based TWR schemes occurs, then one possible way to overcome this problem is to move the operating point in which the FRF was derived, and then to consider another linear model, represented by the new FRF obtained in the new operating point. But the TWR scheme, based on the new linear model, might also diverge and the currently applied procedure of finding a suitable operating point, which might not even be guaranteed to exist, is, at least up to now, rather of a trial and error type and only experienced engineers are able to determine suitable operating points, provided that such suitable operating points exists.

If one cannot overcome the divergence problem by an adjustment of the operating point resp. the FRF, then one has to modify resp. extend the model class under consideration, i. e. one has to consider nonlinear models instead of linear ones. But in general, deriving adequate nonlinear physical models for the complex systems under consideration is likely to be too time consuming and too expensive, in particular for a commercially oriented company. Moreover, in general, the derived nonlinear model cannot describe the system exactly, and a model error is likely to remain. On the other hand, the majority of the existing research results for iterative learning control applied to nonlinear systems are restricted in the sense that the system resp. the model has to be of a specific structure (see also remark 2.8 in chapter 2), and the derived convergence conditions depend on the exact knowledge of the output equation resp. the partial derivatives related to the output equation (see e. g. [H^+92], [ACK93], [JCA95], [SW99]). This is in particular a problem, if the model does not describe the system exactly and differences with respect to the partial derivatives between the system and the model occur.

The approach chosen in the underlying thesis has the potential to overcome these difficulties. The class of nonlinear models considered in the underlying thesis is known as the class of operating regime based local model networks, or shorthand as the class of local model networks (LMN). In a local model network, local models of the system, which are valid only within some subset of the systems entire operating space, are combined to form a global nonlinear model in order to cover the entire operating space. We restrict ourselves to the typical case, i. e. the consideration of linear local models. This modelling approach can be viewed as a rather new topic in literature related to modelling of nonlinear systems, and main contributions to this field, which are important for the underlying thesis, are e. g. the PhD–thesis of Johansen ([Joh94] and Angelis ([Ang01]), which were published in 1994 resp. in 2001. Combining different local linear models to a global nonlinear model is likely to be acceptable from an industrial point of view, as the experienced engineer, as noted above, already gained some knowledge in the sense that linear models, derived in different operating points, may deliver satisfactory results. Moreover, besides the acceptable modelling procedure, the stability resp. convergence conditions, that will be derived in the underlying thesis, will not depend on any derivative related to the system's output equation, and we will be able to develop robust synthesis procedures for the free parameters in the input update law.

The organization and the contributions of the thesis, in which we combine the two rather young approaches of iterative learning control and operating regime based local model networks, are as follows:

In chapter 2, we introduce the concept of local model networks and we investigate the approximation properties of these models. Moreover, some specific classes of local model networks are pointed out.

A major part of the thesis's contributions can be found in chapter 3, where the analysis of the stability and convergence properties of some iterative learning control schemes applied to nonlinear systems, based on a local model network description together with the corresponding model error, is carried out. We consider continuous time systems as well as discrete time systems, and the analysis is based on the sup–norm as well as on the λ– resp. the α–norm. The entire results derived in this chapter are new, and hence they can be viewed as a considerable contribution to the field of iterative learning control. We refer to the various comments in chapter 3 for more details about the corresponding contributions.

In chapter 4, we develop, based on the stability resp. convergence conditions derived in chapter 3, synthesis schemes for the determination of the free parameters in the various input update laws under consideration, in order to fulfill the corresponding conditions. The entire results of this chapter are new and hence comprise the second major part of the thesis' contributions to the field of iterative learning control.

In chapter 5, we investigate an example of a nonlinear spring–mass–damper system, where the standard TWR approach fails in the sense that it leads to divergence, and the iterative learning control scheme developed in the underlying thesis, which is based on an operating regime based local model network of the nonlinear system, converges.

Finally, in chapter 6, a summary and an outlook are given.

Chapter 2

An Operating Regime Based Approach to Nonlinear Modelling

Describing a dynamical system by means of a mathematical model constitutes an essential basis for the analysis and simulation of the systems's behavior, for the improvement of the underlying system by selective modification or for the design of a suitable control strategy. A variety of model structures and classes, such as linear and nonlinear or white, black and grey box models are well known. The applicability of the different model types depends on the available information, such as physical knowledge of the system and measured data, the desired application, such as simulation or controller design, and the required accuracy. Any model may, especially if the dynamical system under consideration is nonlinear, only be accurate under certain conditions, i. e. it will have a limited range of validity, which might depend e. g. on the modelling assumptions, the model structure or the experimental conditions during the data acquisition. A model which has a range of validity equal to the full operating range of the system is called a global model, as opposed to a local model, which is valid only in a so–called operating regime, a subset of the operating range.

In this chapter we introduce a modelling framework, which combines a number of local models in order to obtain a global model for a given nonlinear dynamical system. This model class is called a local model network (LMN). In the underlying work we are primarily concerned with the situation that the local models are linear models. It should be mentioned, that the underlying concept appears in the literature under many different guises. We refer to section 2.4 for a survey on related denotations and approaches.

The advantage of the LMN approach is, as we will see in the subsequent sections, that it provides a suitable, practical relevant framework for modelling complex nonlinear systems. Therefore we have chosen this model class as a basis for the development, analysis and design of iterative learning control schemes for nonlinear dynamical systems, denoted as operating regime based iterative learning control (ORBILC), which will be presented in the subsequent chapters.

The aim of the current chapter, which is mainly based on [Joh94] and [Ang01], is to give an overview of some important aspects of local model networks and to impart the intuitive feeling for this modelling approach. Thereby we omit a detailed introduction into the mathematical framework of system theory, and we assume that the reader possesses some required basic knowledge, which can be found in corresponding literature (see e. g. [Kha96], [Unb97], [Son98], [Unb98]). The organization of this chapter is a follows: In section 2.1 we will introduce the main ideas of the local model network (LMN) approach. The approximation capabilities of the LMN framework will be investigated in more detail in section 2.2. The first two sections basically will focus on the approximation of static nonlinear functions. Some specific classes of dynamical LMNs will be listed in section 2.3. Thereby the global state LMNs are of particular importance, as they provide the basis of the operating regime based iterative learning control approach. Note that the approximation properties of the dynamical LMNs will not be treated in detail, as in the ORBILC framework we will only require some information about the approximation properties of LMNs applied to the static functions describing the right hand sides of the nonlinear state space models, i. e. the functions describing the state and the output equation of the underlying nonlinear model. A literature survey on LMNs and related approaches, the modelling procedures and control strategies based on LMNs will be provided in section 2.4. A summary will be given in section 2.5.

2.1 Basis Concept of Local Model Networks

The intention of this section is to impart the concept of the LMN. At this stage, we widely omit rigorous mathematical precision concerning some terms like 'validity', 'accurate' and others, as we want to give an intuitive introduction to the LMN approach. A more detailed analysis will be given in section 2.2.

Consider a nonlinear system of the form

$$\mathbf{y}(t) = \mathbf{f}(\psi(t)), \tag{2.1}$$

where $\mathcal{T}$ is a time set , e. g. $\mathcal{T} = [0, \infty)$ in the continuous time case or $\mathcal{T} = \Delta t \cdot \mathbb{Z}_0^+$ with sampling time Δt in the discrete time case, $\psi : \mathcal{T} \to \Psi$ is an input function, where $\Psi \subseteq \mathbb{R}^d$ is called input set or operating space, and $\mathbf{f} : \Psi \to Y \subseteq \mathbb{R}^p$ is a nonlinear function. $\psi(t) \in \Psi$ is called the operating vector or information vector and