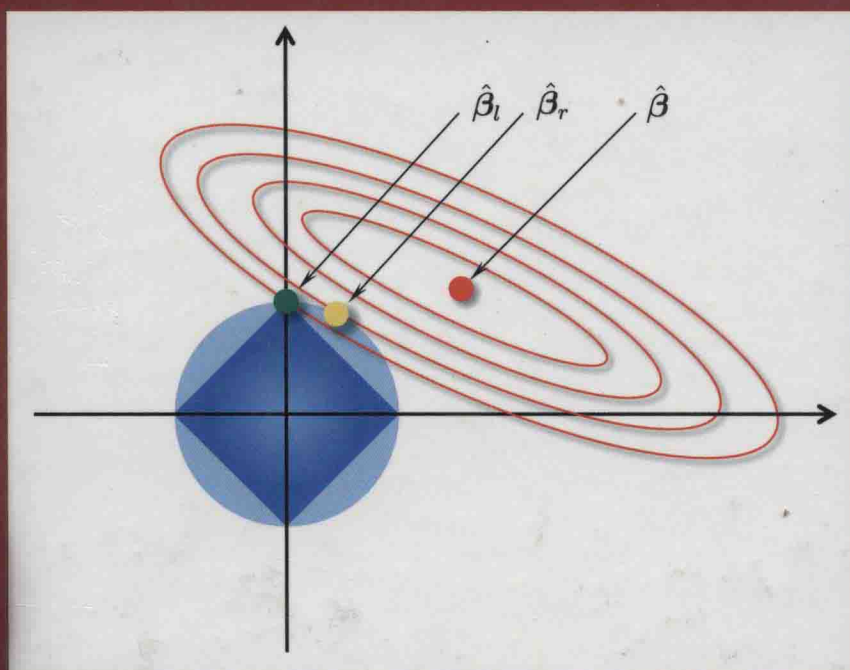


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Statistical Inference

An Integrated Approach

Second Edition



Helio S. Migon
Dani Gamerman
Francisco Louzada



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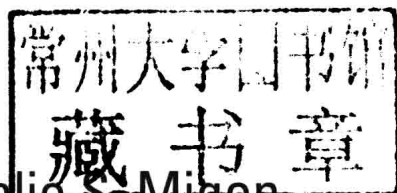
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To Mirna, our sons and grandchildren (H.S.M.)
To Bernardo (i.m.) and Violeta (D.G.)
To Gleici, Lucas, Caio, Samuel and Francisco (F.L.)

Preface to the Second Edition

It has been more than a decade since the 1st edition was released. Over this period the need for a more balanced account of the main schools of thought to statistical inference became clear. The main purpose of the book has always been a balanced and integrated presentation of these approaches to the subject. But the first edition was written only by the first two authors of this edition; both were assumedly Bayesian and mentioned that in the Preface for that edition.

Experience gathered in the meantime indicated to us some ingredients to get back to our original goal. Certainly a more detailed and comprehensive presentation of the pros and cons of each approach had to be pursued. The inclusion of a co-author who was more involved with the frequentist school was also recommended to achieve a more balanced presentation. We were fortunate to add both ingredients and present the current version of our work after these additions.

We have tried to eliminate (or reduce, more realistically) the typos present in the 1st edition. Additionally, we have included more explanation, more illustration and more exercises for the concepts already present here. More importantly, new material has been included. The main novelties are associated with a new section on empirical Bayes and penalized likelihoods and their impact on the regression models in the last chapter. Interestingly, these are topics that lie closer to the border between the two schools of thought. The content of this new material has also been connected to many other parts of the book, throughout its chapters. Additionally, we have expanded material in many sections, specially on hypothesis testing, method of moments and bias correction.

This has been an interesting journey for the authors that is far from over. Statistics is alive and growing as a discipline with an ever increasing impact of the growth in computational power. We have no illusion of having reached a complete and final text on the subject, but hope to have reached a stage where readers can grasp the main task ahead of them in their search for an integrated understanding of Statistical Inference.

There are a number of people that we would like to thank and show our gratitude. We all thank our common friend Luis Raul Pericchi for the suggestion of the 3rd author name and many other suggestions. Thanks are also due to our lifelong friend Basilio Pereira for much and relentless advice with references and perspective. There is no way to avoid acknowledging the fundamental help of Rob Calver. He has been keen on making this project viable since the release of the 1st edition. There were many many moments that he was the only one believing it. His persistence and enthusiasm was crucial to have us reached this stage. He is complemented by a number of competent colleagues at CRC who have helped us in many different ways. We also thank colleagues and students who have helped shape the work in various ways, and our families for their encouragement and support. To all of them, our sincerest gratitude. We hold none of them responsible and take full account for the views expressed in this book.

H.S.M., D.G. and F.L.

Rio de Janeiro and São Carlos, May 2014

Preface to the First Edition

This book originated from the lecture notes of a course in Statistical Inference taught at the M.Sc. programs in Statistics at UFRJ and IMPA (once). These have been used since 1987. During this period, various modifications have been introduced until we arrived at this version, judged as minimally presentable.

The motivation to prepare this book came from two different sources. The first and more obvious one for us was the lack of texts in Portuguese, dealing with statistical inference to the desired depth. This motivation led us to prepare the first draft of this book in the Portuguese language in 1993. The second, and perhaps the most attractive as a personal challenge, was the perspective adopted in this text. Although there are various good books in the literature dealing with this subject, in none of them could we find an integrated presentation of the two main schools of statistical thought: the frequentist (or classical) and the Bayesian. This second motivation led to the preparation of this English version. This version has substantial changes with respect to the Portuguese version of 1993. The most notable one is the inclusion of a whole new chapter dealing with approximation and computationally intensive methods.

Generally, statistical books follow their author's point of view, presenting at most, and in separate sections, related results from the alternative approaches. In this book, our proposal was to show, wherever possible, the parallels existing between the results given by both methodologies. *Comparative Statistical Inference* by V. D. Barnett (1973) is the book that is closest to this proposal. It does not, however, present many of the basic inference results that should be included in a text proposing a wide study of the subject. Also we wanted to be as comprehensive as possible for our aim of writing a textbook in statistical inference.

This book is organized as follows. The first chapter is an introduction, presenting the readers with the way we find most appropriate to think of Statistics: discussing the concept of information. Chapter 2 presents some basic concepts of statistics such as sufficiency, exponential family, Fisher information, permutability and likelihood functions. Another basic concept specific

to Bayesian inference is prior distribution; that is separately dealt with in Chapter 3.

Certain aspects of inference are individually presented in Chapters 4, 6, and 7. Chapter 4 deals with parameter estimation where, intentionally, point and interval estimation are presented as responses to the summarization question, and not as two unrelated procedures. The important results for the Normal distribution are presented and also serve an illustrative purpose. Chapter 6 is about hypotheses testing problems under the frequentist approach and also under the various possible forms of the Bayesian paradigm.

In between them, lies Chapter 5 where all approximation and computationally based results are gathered. The reader will find there at least a short description of the main tools used to approximately solve the relevant statistical problem for situations where an explicit analytic solution is not available. For this reason, asymptotic theory is also included in this chapter.

Chapter 7 covers prediction from both the frequentist and Bayesian points of view, and includes the linear Bayes method. Finally in Chapter 8, an introduction to Normal linear models is made. Initially the frequentist approach is presented, followed by the Bayesian one. Based upon the latter approach, generalizations are presented leading to the hierarchical and dynamic models.

We would like to alert the readers from the onset that the our preferred point of view is the Bayesian one. So, a natural emphasis is given to this approach. We tried, however, to develop a critical analysis and to present the most important results of both approaches commenting on the positive and negative aspects of both. As it has already been said, the level of this book is adequate for an M.Sc. course in Statistics, although we do not rule out the possibility of its use in an advanced undergraduate course aiming to compare the two approaches.

This book can also be useful for the more mathematically trained professionals from related areas of Science such as Economics, Mathematics, Engineering, Operations Research and Epidemiology. The basic requirements are knowledge of calculus and probability, although basic notions of linear algebra are also used. As this book is intended as a basic text in statistical inference, various exercises are included at the end of each chapter. We have also included sketched solutions to some of the exercises and a list of distributions at the end of the book, for easy reference.

There are many possible uses of this book as a textbook. The first and most obvious one is to present all the material in the order it appears in the book and without skipping sections. This may be a heavy workload for a one-semester course. In this case we suggest dropping Chapter 8 for a later

course. A second option for exclusion in a first course is Chapter 5, although we strongly recommend it for anybody interested in the modern approach to statistics, geared towards applications. The book can also be used as a text for a course that is more strongly oriented towards one of the schools of thought. For a Bayesian route, follow Chapters 1, 2, 3, Sections 4.1, 4.4.1 and 4.5, Chapter 5, Sections 6.3, 6.4, 6.5, 7.1, 7.3.1, 7.4, 8.1, 8.3, 8.4 and 8.5. For a classical route, follow Chapter 1, Sections 2.1, 2.2, 2.5, 2.6, 4.2, 4.3, 4.4.2 and 4.5, Chapter 5, Sections 6.1, 6.2, 6.4, 6.5, 7.2, 7.3.2, 7.4, 8.1 and 8.2.

This book would not have been possible without the cooperation of various people. An initial and very important impulse was the typing of the original lecture notes in TeX by Ricardo Sandes Ehlers. Further help was provided by Ana Beatriz Soares Monteiro, Carolina Gomes, Eliane Amiune Camargo, Monica Magnanini and Otávio Santos Figueiredo. Besides these, many of our former students helped with suggestions and criticism. Careful proofreading of this manuscript was made by our past M.Sc. students and present colleagues Alexandra Mello Schmidt, Hedibert Freitas Lopes and Marco Antonio Rosa Ferreira. Many useful suggestions and comments were provided at this later stage by Steve Brooks, Eduardo Gutierrez-Peña and Gabriel Huerta. We also had the stimulus of several colleagues; in particular, we would like to mention Basílio de B. Pereira. I would also like to thank Nicki Dennis for her support and encouragement throughout all the stages of preparation of this book and for making us feel at home with Arnold. Our families also played the important roles of support and understanding, especially in the weekends and late nights spent trying to meet deadlines! To all of them, our gratitude.

Finally, the subject of the book is not new and we are not claiming any originality here. We would like to think that we are presenting the subject in a way that is not favored in many textbooks and that will help the readers to have an integrated view of the subject. In our path to achieve this goal, we have had the influence of many researchers and books. We have tried to acknowledge this influence by referring to these books whenever we felt it provided a worthwhile reading description of a topic. Therefore, for every major subject presented in our book we tried to relate it to books that treated the subject in a more complete or more interesting way. In line with a textbook character, we opted to favor books rather than research papers as references. We would like to think of our book as a basis for discovery and will feel our task is accomplished whenever readers understand the subject through the book alone, its references or a combination of both.

H.S.M. & D.G.

Rio de Janeiro, December 1998

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