

The cover art features a dark green, textured background. In the lower-left corner, a portion of a black Texas Instruments BA-85 calculator is visible, showing its LCD screen and various function keys like '2nd', 'CPT', 'DUE', 'PMT', 'PV', 'FV', 'ON/C', 'OFF', 'N', '%I', 'FRQ', 'Σ+', '1/x', 'y^x', '√x', and numerical keys '7', '8', '9'. Scattered across the upper and middle sections are several translucent glass spheres of different sizes, some of which are reflecting light, creating bright highlights and soft shadows on the green surface.

Contemporary Business Mathematics

with Canadian Applications

4th Edition

S.A. HUMMELBRUNNER

Fourth Edition

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PRENTICE HALL CANADA INC.
SCARBOROUGH, ONTARIO

Canadian Cataloguing in Publication Data

Hummelbrunner, S. A. (Siegfried August)
Contemporary business mathematics with
Canadian applications

4th ed.

ISBN 0-13-287582-9

1. Business mathematics. I. Title.

HF5691.H85 1994 650'.01'513 C93-095092-5

© 1994, 1990, 1986, 1982

Prentice-Hall Canada Inc., Scarborough, Ontario

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Prentice-Hall, Inc., Englewood Cliffs, New Jersey
Prentice-Hall International (UK) Limited, London
Prentice-Hall of Australia, Pty. Limited, Sydney
Prentice-Hall Hispanoamericana, S.A., Mexico City
Prentice-Hall of India Private Limited, New Delhi
Prentice-Hall of Japan, Inc., Tokyo
Simon & Schuster Asia Private Limited, Singapore
Editora Prentice-Hall do Brasil, Ltda., Rio de Janeiro

ISBN 0-13-287582-9

Acquisitions Editor: Jacqueline Wood

Developmental Editor: Maurice Esses

Copy Editor: Mia London

Production Editor: Valerie Adams

Production Coordinator: Anna Orodí

Cover Design & Image: Pronk & Associates

Page Layout: Compeer Typographic Services Ltd.

1 2 3 4 5 RRD 97 96 95 94

Printed and bound in USA.

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Preface

Contemporary Business Mathematics is intended for use in introductory mathematics of finance courses in business administration programs. In more general application it also provides a comprehensive basis for those who wish to review and extend their understanding of business mathematics.

The primary objective of the text is to increase the student's knowledge and skill in the solution of practical financial and mathematical problems encountered in the business community. It also provides a supportive base for mathematical topics in finance, accounting and marketing.

Contemporary Business Mathematics is essentially a teaching text using the objectives approach. The systematic and sequential development of the material is supported by carefully selected and worked examples. These detailed step-by-step solutions presented in a clear and uncluttered layout are particularly helpful in allowing students, in either independent studies or in the traditional classroom setting, to carefully monitor their own progress.

Each topic in each chapter is followed by an exercise containing numerous drill questions and application problems. The review exercise and self-test at the end of each chapter and the case studies are designed to assist in the integration of the material studied. The computer programs contained in the Solutions Manual provide samples of those text problems most suited to solution by computer.

The first four chapters and Appendix I (Review of basic algebra) are intended for students with little or no background in algebra and provide an opportunity to review arithmetic and algebraic processes.

The text is based on Canadian practice, and reflects current trends utilizing available technology — specifically the availability of reasonably priced electronic pocket calculators.

Students using this book should have access to calculating equipment having a power function and a natural logarithm function. The use of such calculators eliminates the arithmetic constraints often associated with financial problems and frees the student from reliance on financial tables.

The power function and the natural logarithm function are often needed to determine values which will be used for further computation. Such values should not be rounded and all available digits should be retained. The student is encouraged to use the memory to retain such values.

When using the memory the student needs to be aware that the number of digits retained in the registers of the calculator is greater than the number of digits displayed. Depending on whether the memory or the displayed digits are used, slight differences may occur. Such differences will undoubtedly be encountered when working the examples presented in the text. However, they are insignificant and should not be of concern. In most cases the final answers will agree, whichever method is used.

Students are encouraged to use preprogrammed financial calculators though this is not essential. The use of preprogrammed calculators facilitates the solving of most financial problems and is demonstrated extensively in chapters 9 to 17.

The fourth edition of the text has taken into account the suggestions, comments and criticisms received from reviewers, publisher's representatives and users of the first three editions including students. In this regard I wish to give special thanks to all those who responded to a questionnaire about ways to improve the previous edition. I would also like to thank those instructors who participated in a special focus group for this edition: Elaine Hales, Georgian College; Sharyn Jeffries, Centennial College; Colleen Quinn, Seneca College; Ishwar Dean, George Brown College; Rick Law, Humber College; Tom Quinn, Humber College; and Debbie Kirwin, Sheridan College (Brampton). Thanks are also due to Jim Gowland of Conestoga College who prepared the case studies.

SIEG HUMMELBRUNNER

*Mississauga
December 1993*

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PART ONE

Mathematics fundamentals

1 *Review of arithmetic*

Introduction

Electronic calculators are now in general use in business to perform arithmetic computations. The fundamental operations of addition, subtraction, multiplication, and division with whole numbers and decimal fractions can be done with any model on the market. However, to use a calculator as a tool in solving problems requires skill in using the basic order of operations, converting mixed numbers and common fractions into decimal form, and rounding answers to the desired number of decimal positions.

Objectives

Upon completing this chapter, you will be able to

1. simplify arithmetic expressions involving the basic order of operations;
2. solve basic problems, including arithmetic averages, involving the fundamental operations;
3. determine gross earnings for employees remunerated by the payment of salaries, hourly wages, or commissions.

1.1 *Basics of arithmetic*

A. The basic order of operations

To ensure that operations are performed consistently, we must follow the *order of operations*.

If an arithmetic expression contains brackets as well as any or all of powers, multiplication, division, addition, and subtraction, we use the following procedure:

1. perform all operations *inside* a bracket first (the operations inside the bracket must be performed in proper order);
2. perform powers;

3. perform multiplications and divisions in order;
4. perform addition and subtraction.

Example 1.1a

- (i) $9 - 4 \times 2 = 9 - 8 = 1$ ← do multiplication before subtraction
- (ii) $(9 - 4) \times 2 = 5 \times 2 = 10$ ← work inside the bracket first
- (iii) $(13 + 5) \div 6 - 3 = 18 \div 6 - 3$ ← work inside the bracket first, then
 $= 3 - 3$ do division before subtraction
 $= 0$
- (iv) $18 \div 6 + 3 \times 2 = 3 + 6 = 9$ ← do multiplication and division before adding
- (v) $18 \div (6 + 3) \times 2 = 18 \div 9 \times 2$ ← work inside the bracket first, then
 $= 2 \times 2$ do division and multiplication in order
 $= 4$
- (vi) $18 \div (3 \times 2) + 3 = 18 \div 6 + 3$ ← work inside bracket first, then
 $= 3 + 3$ divide before adding
 $= 6$
- (vii) $8(9 - 4) - 4(12 - 5) = 8(5) - 4(7)$ ← work inside brackets first, then
 $= 40 - 28$ multiply before subtracting
 $= 12$
- (viii) $\frac{12 - 4}{6 - 2} = (12 - 4) \div (6 - 2)$ ← the fraction line indicates brackets as well as division
 $= 8 \div 4$
 $= 2$

B. Converting common fractions into decimal fractions

Common fractions are converted into decimal fractions by performing the indicated division to the desired number of decimal places or until the decimal terminates.

Example 1.1b

- (i) $\frac{9}{8} = 9 \div 8 = 1.125$
- (ii) $\frac{1}{3} = 1 \div 3 = 0.33333 \dots$
- (iii) $\frac{7}{6} = 7 \div 6 = 1.16666 \dots$