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Gustav Kramer

Theory of Jets in Electron-Positron Annihilation

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With 86 Figures

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Preface

In the last ten years our understanding of the nature of the strong interaction force has changed considerably. To a large extent this has its origin in our belief that with quantum chromodynamics (QCD), the gauge field theory of quarks and gluons, we have now available a solid, well defined, theory. In the sixties the quark model played already a great role for classifying hadron states but it lacked this theoretical foundation which, we think, now exists with QCD. If we try to remember, which experimental discoveries smoothed the way for QCD, presumably we would consider: (i) the discovery of the scaling behaviour of deep inelastic lepton-nucleon scattering and its interpretation through the parton model, (ii) the discovery of additional quark flavours, the charm and the bottom quark, and the interpretation of corresponding hadron states in terms of simple potential models based on the confinement hypothesis and (iii) the discovery of quark and gluon jets in electron-positron annihilation into hadrons. On the theoretical side it was important to recognize that non-abelian gauge theories are renormalizable and that quantum chromodynamics is asymptotically free. These two properties opened the road for the application of perturbative theory which was so successful in quantum electrodynamics. Furthermore the originally apparent contradiction, the quark-gluon interaction being very strong, i.e. so strong that quarks and gluons are always confined, on one side, and the appearance of almost free quarks and gluons inside hadrons, as revealed in deep inelastic scattering experiments, on the other side, could be resolved.

In the meantime we have learned how to apply perturbative QCD to various reactions at high energies: e^+e^- annihilation, two-photon processes, deep inelastic lepton-nucleon scattering, $\mu^+\mu^-$ production in hadron-hadron collisions (Drell-Yan process) and production of particles with high transverse momenta and of jets in hadron-hadron collisions.

We can distinguish two fields for applying QCD perturbation theory which are only indirectly connected. One is predicting the Q^2 evolution of structure functions of hadrons (or decay functions of quarks and gluons) in deep-inelastic lepton-nucleon scattering and in hadron-hadron collisions. This Q^2 evolution essentially follows from summed perturbation theory in the leading logarithm approximation. The other field consists of the theoretical analysis of jet phenomena in terms of QCD perturbation theory at fixed order. For the first subject, the theory of structure and decay functions many reviews exist. We mention a few: /Altarelli, 1982; Buras, 1980; Dokshitser, D'yakonov, and Troyan, 1980; Ellis and Sachrajda, 1979; Field, 1978; Marciano and Pagels, 1978; Petermann, 1979; Pennington, 1983; Politzer, 1974; Reya, 1981; Ross, 1981; Sachrajda, 1982; Söding and Wolf, 1981/. In some of these reviews also jet phenomena are considered.

It seems generally agreed upon that e^+e^- annihilation into hadrons is the best laboratory to investigate the production of hadron jets. In e^+e^- annihilation we have a well defined, hadron free, initial state which allows us to study the final state undisturbed from effects of initial state hadrons. In deep inelastic lepton-nucleon scattering and even more so in hadron-hadron processes we always have hadrons in the initial state (with more or less known structure functions) producing beam and target jets which overlap with the perturbative QCD jets we are interested in. So far most of the empirical information on jets comes from e^+e^- annihilation experiments. But also on the theoretical side the analysis of jets in e^+e^- annihilation appears to be very much advanced. Here, many results on higher orders in QCD perturbation theory have been obtained in the last three years. Therefore, in this review we shall restrict ourselves to a representation of the theoretical considerations for the analysis of hadron jets produced in e^+e^- annihilation. We hope that the procedures outlined for this particular process may be useful also for the interpretation of jet phenomena in the other more complicated reactions mentioned above. Some earlier reviews of QCD jets are /Hoyer, 1980; Kramer, 1980; Schierholz, 1979,1981; Walsh, 1980/.

Hamburg, Februar 1984 *G. Kramer*

Contents

1. Introduction

1.1 Quarks with Colour

Since Gell-Mann /1964/ and Zweig /1964/ introduced quarks as the elementary building blocks of all hadrons our understanding of the complex hadronic world of protons, neutrons, pions, kaons and all the other strongly interacting particles has increased remarkably. Quarks are spin 1/2 particles, so that a quark q and an antiquark $\bar{q}$ build mesons ($q\bar{q}$) and three quarks make baryons (qqq).

To explain the whole hadron spectrum as it exists today we need five quarks of different flavour: u, d, s, c and b quarks. They differ in their masses and in their properties concerning electromagnetic and weak interactions. Their quantum numbers I, I_3, s, c, b, Q and B are listed in Table 1.1. The u, d quarks transform as a doublet under an almost exact SU(2) flavour group, the u, d, s transform as a triplet under an approximate SU(3) flavour group, u, d, s, c as a quartet under a broken SU(4) flavour group and so on. The quark flavours, charges and baryon numbers determine the flavour of all hadrons, their isospin quantum numbers I, I_3 their strangeness s, their charm c, their bottomness b, their charge Q and their baryon number B.

The masses of u, d quarks are approximately 10 MeV, the mass of the s quark is near 150 MeV, of the c quark 1200 MeV and of the b quark 5000 MeV. These masses are not very well known. They are just parameters which quarks would have as masses if they could be produced as free particles. Since this is not the case, the masses cannot be measured directly and their values depend somewhat on the more indirect definition of the mass parameters.

In addition every quark u, d, s, c and b appears in three distinct states, a red a green and a blue quark – a property we call colour /Greenberg, 1964; Han and Nambu, 1965; Gell-Mann, 1972/. That quarks must have another degree of freedom in addition to spin and flavour was revealed by the symmetry problem of baryon ground state made out of quarks. For example, the lowest mass, spin 3/2, states of three apparently identical quarks, three u's, d's or s quarks depending whether one considers the Δ^{++}, Δ^- or Ω^- baryon, can be totally symmetric in their spin, spatial and flavour

Table 1.1. The five quarks and their flavour quantum numbers: isospin, strangeness, charm, bottomness, charge and baryon number

Quarks	I	I_3	s	c	b	Q	B
u	1/2	1/2	0	0	0	2/3	1/3
d	1/2	-1/2	0	0	0	-1/3	1/3
s	0	0	-1	0	0	-1/3	1/3
c	0	0	0	1	0	2/3	1/3
b	0	0	0	0	-1	-1/3	1/3

properties, as one expects for the ground state and yet satisfy the Pauli principle, i.e. obey Fermi-Dirac statistics. The antisymmetry of the wave function comes from the colour wave function which is the antisymmetric combination of a red, green and blue quark of the particular flavour u, d, or s. A meson is a linear superposition of red-antired, green-antigreen and blue-antiblue states. So, hadrons, being observable states, are colour singlets although each is built of coloured quarks. This construction explains why mesons, being colour singlets, behave as if made from just one $q\bar{q}$ pair and baryons as if made from a qqq configuration. Further hints for the colour of quarks come from the observed decay rate for $\pi^0 \to \gamma\gamma$ and the cross section for e^+e^- annihilation into hadrons which will be discussed in detail in Chap.2. These physical observables count the number N_c of quarks of each flavour and the experimental data tell us that this number N_c is equal to three.

That the idea of quarks is more than a tool for constructing hadrons was made apparent by the experiments on deep inelastic lepton scattering which started at the Stanford Linear Accelerator. In these experiments, a high momentum probe, a virtual photon or weak virtual quanta $W^\pm$ or Z, hits a nucleon (Fig.1.1). If its momentum is high enough its wavelength is smaller than the size of the nucleon and we expect it to probe the constituents of the nucleon. This is what the scattering experiments really have shown. The scattering of high energy leptons occurs in such a way as if

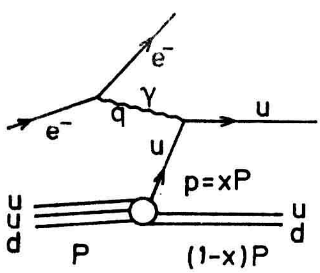

Fig.1.1. Parton model diagram for inelastic electron-proton scattering $e^- + P \to e^- + u\text{-quark} + (ud)$-diquark

there are constituents — partons — inside the hadron which are freely moving, point-like objects /Bjorken, 1967; Feynman, 1969; Bjorken and Paschos, 1969/. These partons are found to have spin 1/2 and all the other properties of quarks. Therefore the lepton scattering can be described such that the virtual photon, $W^{\pm}$ or Z with squared $q^2 < 0$ scatter on the quasi-free quarks bound in the nucleon. The final state consists then of two jets, the current jet which is the quark (q) jet and the target jet equal to the diquark (qq) jet. But, however hard the quarks inside the nucleon are hit, the quarks never appear asymptotically as free particles. The quark and the diquark jet must fragment into hadrons which then are observed in the detector.

The fact, that quarks never seem to come out as free particles, whereas hadrons do, is in accord with the colour assignments discussed above. Only colour neutrals, i.e. hadrons, are asymptotic states, All colour non-singlets, i.e. quarks, diquarks etc., cannot appear asymptotically. They always must be bound into hadrons, i.e. they are confined. This means that quarks are strongly bound inside hadrons. What is responsible for this very strong binding? Evidence that a nucleon contains not just three quarks came from the experimental fact that in deep inelastic lepton scattering the charged constituents of a nucleon carry only half of its momentum. An electrically neutral parton carries the rest. This is identified with the gluon. Quarks are assumed bound by exchanging gluons. For example a red quark interacts with a greer one by exchanging a red-antigreen gluon. These gluons are flavour neutral and do not participate in the electro-weak interactions.

Since quarks have three colours, there are nine types of gluon. All except one — the singlet gluon — mix under colour transformations. The singlet gluon may have a coupling to quarks of strength independent of the other eight. This is set to zero. The remaining eight gluons transform as the adjoint representation of colour SU(3). Gluons are assumed to have spin one. This has the effect that the force between $q\bar{q}$ is attractive, as it is needed for binding in a meson, but repulsive between qq. This leads us directly to quantum chromodynamics, the gauge theory of quarks and gluons.

1.2 The Lagrangian of Quantum Chromodynamics

Quantum chromodynamics (QCD) /Fritzsch and Gell-Mann, 1972/ is the theory which describes the interaction of a triplet of coloured quarks with an octet of vector gluons by a Yang-Mills gauge theory /Yang and Mills, 1954/. The quark fields are spinors $q_c(x)$ which transforming as the fundamental representation of SU(3) have colour quantum numbers c = 1, 2, 3. The gluon fields $A^a_\mu(x)$ transforming according to the adjoint representation have a = 1, 2, ..., 8. The SU(3) colour transformations are generated by 3×3 matrices T^a (a = 1, 2, ..., 8) ($T^a = \lambda_a/2$, where the

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λ_a's are the well known Gell-Mann matrices /Gell-Mann, 1962/). They obey the commutator relations

$$[T^a, T^b] = i f^{abc} T^c \tag{1.2.1}$$

with f^{abc} being the structure constants of SU(3). The Lagrangian density for QCD has the following form

$$\mathscr{L} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu,a} + \bar{q}(i\gamma_\mu D^\mu - m)q \tag{1.2.2}$$

wnere the field strength tensor

$$F^a_{\mu\nu} := \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu \tag{1.2.3}$$

and the covariant derivative

$$D_\mu = \partial_\mu - i g T^a A^a_\mu(x) \quad . \tag{1.2.4}$$

g is the bare coupling constant of the theory and m the bare mass of the quark field q(x). The gluons are massless. By splitting (1.2.2) into a non-interacting part and an interacting part one can read off the Feynman rules from which one can calculate quark-gluon processes perturbatively in g.

The Lagrangian (1.2.2) is invariant under the infinitesimal local gauge transformation defined by $\Theta^a(x)$

$$\begin{aligned} q(x) &\to q(x) + i T^a \Theta^a(x) q(x) \\ \bar{q}(x) &\to \bar{q}(x) - i \bar{q}(x) T^a \Theta^a(x) \\ A^a_\mu(x) &\to A^a_\mu(x) - f^{abc} \Theta^b(x) A^c_\mu(x) + \frac{1}{g} \partial_\mu \Theta^a(x) \quad . \end{aligned} \tag{1.2.5}$$

The requirement of local gauge invariance leads to the unique Lagrangian (1.2.2) which severely restricts the otherwise possible interaction terms between quarks and gluons. This gauge invariance is also crucial to make the theory renormalizable /t'Hooft, 1971/ and so yields sensible predictions for physical processes at high energies.

Locally gauge invariant theories like QCD are difficult to quantize because the fields $A^a_\mu(x)$ are gauge quantities and therefore exhibit extra non-physical degrees of freedom which must be dealt with. The most convenient procedure for quantization is Feynman's path integral formalism. For review of this topic see Abers and Lee

/1973/, Zinn-Justin /1975/, Becher, Böhm and Joos /1981/ and Itzykson and Zuber /1980/.

The structure of QCD is similar to that of Quantum Electrodynamics (QED), the only successful field theory we have. In QED, which is an abelian gauge theory, the right hand side of (1.2.1) vanishes and the charged matter fields transform under gauge transformations by simple phase transformations, i.e. U(1) transformations. In QCD, being a non-abelian generalization, the quarks transform under the more complicated SU(3) colour group and the vector bosons, the gluons, now carry colour charge too.

The Lagrangian (1.2.2) is written in terms of the so-called unrenormalized fields $q(x)$ and $A_\mu^a(x)$. The calculation of scattering matrix elements or other physical quantities yield finite results only if the theory is renormalized. This means, the infinities of the theory are absorbed into the basic constants of the theory such as coupling constants and masses which are renormalized to their finite physical values. Therefore these coupling constants and masses must be given and cannot be calculated in this theory. The technique for renormalizing perturbative QCD is well known from QED. The fields are multiplicatively renormalized, i.e. one defines renormalized fields q_r and $A_{\mu,r}^a$

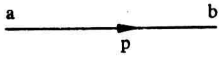
$$\begin{aligned} q &= Z_2^{1/2} q_r \\ A_\mu^a &= Z_3^{1/2} A_{\mu,r}^a \quad . \end{aligned} \tag{1.2.6}$$

Z_2 and Z_3 are renormalization constants of the quark and the gluon field respectively. In terms of the renormalized fields the QCD Lagrangian has the following form

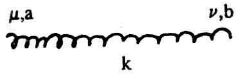
$$\begin{aligned} \mathscr{L} = &- \frac{1}{4} Z_3(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a)_r^2 - \frac{\kappa}{2} Z_3(\partial^\mu A_\mu^a)_r^2 \\ &+ \tilde{Z}_3(\eta^{a^+} \partial^2 \eta^a)_r - Z_3^{3/2} g f^{abc} (A_\mu^a A_\nu^b \partial^\mu A^{\nu,c})_r \\ &- \frac{1}{4} Z_3^2 g^2 f^{abc} f^{ab'c'} (A_\mu^b A_\nu^c A^{\mu,b'} A^{\nu,c'})_r + \tilde{Z}_3 Z_3^{1/2} g f^{abc} [\eta^{a^+} \partial^\mu (A_\mu^b \eta^c)]_r \\ &+ Z_2(\bar{q}(i\gamma^\mu \partial_\mu - m) q)_r + g Z_2 Z_3^{1/2} (\bar{q} T^a \gamma^\mu A_\mu^a q)_r \quad . \end{aligned} \tag{1.2.7}$$

This Lagrangian is complete. It contains also the gauge fixing term proportional to κ, familiar from QED, which is required to insure a proper quantization procedure. $\kappa = 0$ being the Landau and $\kappa = 1$ is the Feynman gauge. The first two terms in (1.2.7) determine the gluon propagator. The gluon propagator, however, contains too many degrees of freedom for a physical massless vector particle. So it includes an unphysical scalar component which must be removed. Removal of these unphysical states is achieved by adding a Fadeev-Popov ghost term involving $\eta^a(x)$. These occur at all places where there are gluon loops. The propagator of the ghost field is also read off from (1.2.7) as well as the coupling of the ghost field with the gluon field. The ghost field has the renormalization constant $\tilde{Z}_3$.

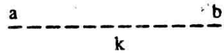
In momentum space the free propagators have the following form

quarks propagator 

$$\frac{i}{(2\pi)^4}\,\delta_{ab}\,\frac{1}{\gamma p - m}$$

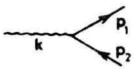
gluon propagator 

$$\frac{i}{(2\pi)^4}\,\delta_{ab}\,\frac{1}{k^2}\left(-g_{\mu\nu} + \frac{k_\mu k_\nu}{k^2} - \kappa\,\frac{k_\mu k_\nu}{k^2}\right)$$

ghost propagator 

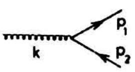
$$\frac{i}{(2\pi)^4}\,\delta_{ab}\,\frac{1}{k^2}\quad . \qquad (1.2.8)$$

The gluon and the ghost are massless. Of course the ghost field does not appear as an external particle. The vertices, i.e. the quark-quark-vertex, the three-gluon vertex, the four-gluon vertex and the ghost-ghost-gluon vertex are also obtained from (1.2.7). They are represented in momentum space in Fig.1.2 together with the

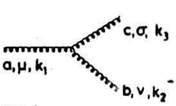


$$= ie\,\gamma_\mu (2\pi)^4 \delta(k + p_2 - p_1)$$

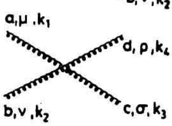
(a)



$$= ig\,\gamma_\mu T_a (2\pi)^4 \delta(k + p_2 - p_1)$$



$$= g\,f^{abc}\,(g_{\mu\nu}(k_1 - k_2)_\sigma + \text{cyclic}) \cdot (2\pi)^4\,\delta(k_1 + k_2 + k_3)$$



$$= -ig^2\,(f^{abe} f^{cde} \cdot (g_{\mu\sigma} g_{\nu\rho} - g_{\mu\rho} g_{\nu\sigma}) + \text{symmetr}) \cdot (2\pi)^4\,\delta(k_1 + k_2 + k_3 + k_4)$$

$$= g \cdot f^{abc}\,p_\mu\,(2\pi)^4\,\delta(k + p + q)$$

(b)

Fig.1.2. a) Fundamental vertex in QED. b) Fundamental vertices in QCD

fundamental vertex in QED for comparison. This figure shows that the structure of interactions in QCD is much richer than in QED. Because of the non-abelian nature of the gauge interaction even the theory without fermions has interaction, the three-gluon (ggg) and the four-gluon coupling (gggg). The quark-gluon coupling ($\bar{q}qg$) is similar to that of QED. It contains in addition only the colour matrix $T^a = \lambda^a/2$. Given this coupling by gT^a, gauge invariance requires the ggg coupling to be proportional to the commutator $g[T^a,T^b]$ and it is related to gf^{abc}. It is essential to recognize that all vertices contain the same coupling constant g. With the propagators (1.2.8) and the vertices in Fig.1.2 all Feynman diagrams of interest can be calculated.

The vertex $\bar{q}T^a\gamma^\mu A^a_\mu q$ is renormalized with the renormalization constant Z_1 such that the renormalized coupling g_r is

$$g_r = Z_2 Z_3^{1/2} Z_1^{-1} g \quad . \tag{1.2.9}$$

This relation will be used later in order to obtain the renormalized coupling in a specific renormalization scheme, the so-called minimal subtraction scheme. Since we have also other couplings the renormalized coupling g_r can be defined also either with the three-gluon vertex or with the ghost-ghost-gluon vertex.

By writing the renormalization constants Z_i in the form $Z_i = 1 + (Z_i - 1)$ the terms in (1.2.7) without interaction are isolated. The terms proportional to $(Z_i - 1)$ determine the subtraction terms which cancel the ultraviolet divergent parts of the Feynman diagrams. For reviews on the renormalization of gauge theories we recommend Taylor /1976/, Zinn-Justin /1975/ and Lee /1976/.

It is generally assumed that QCD is responsible for the strong force which binds quarks and gluons in the hadrons. This must be a very strong interaction, so strong that quarks and gluons are confined in the hadronic bag. However, in deep inelastic scattering, these quarks appear as freely moving, almost non-interacting particles with a coupling which is effectively small. How this feature of QCD arises will be discussed in the next section. It is clear that only for this small coupling regime we can expect that perturbation theory is applicable.

1.3 The Coupling at High Energies

If the QCD Lagrangian (1.2.2) is evaluated in perturbation theory, i.e. by a power series expansion in g, it describes a world of coloured quarks and gluons with free quarks and gluons at $t \to \pm\infty$. Since free quarks and gluons are not observed in nature, i.e. they are always confined in colour singlet hadron states, this perturbative evaluation cannot be totally realistic. On the other hand the experiments with high energy lepton beams show that the virtual quanta with large negative q^2 are scattered on quasi-free quarks and gluons which exist inside the nucleon. In the framework of QCD the scattering of the virtual photon etc. on an almost free quark is interpreted as the zeroth order approximation (g^0) in the quark-gluon coupling constant. The next higher order in g leads to the emission of an additional gluon or to the scattering of an almost free gluon with the production of a $q\bar{q}$ pair (see Fig.1.3). How should we interpret this perturbation theory in g knowing that the coupling of quarks and gluons is so large that it produces confinement? At this point we remember that g is not uniquely defined. As we explained in the last section the coupling g must be renormalized. In a theory of massless quarks – this is the appropriate approximation for high energy processes – an arbitrary mass parameter μ appears in the definition of the renormalized coupling g_r (this point will be considered in more detail in Chap.3). This parameter μ can be chosen such that the perturbation series, for example for the process in Fig.1.3, converges best. In deep inelastic scattering this parameter is chosen $\mu^2 = q^2$, where q^2 is the squared momentum transfer. Of course this makes sense only if $g^2/4\pi = \alpha_s(q^2)$ is sufficiently small. In QCD this is the case for large enough q^2 which we shall discuss next.

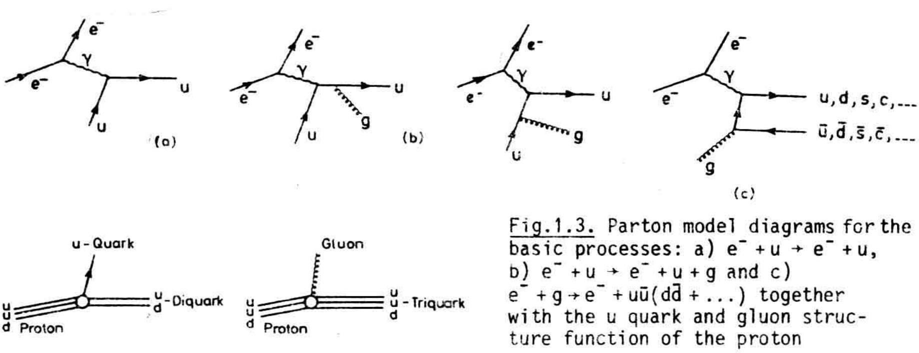


Fig.1.3. Parton model diagrams for the basic processes: a) $e^- + u \to e^- + u$, b) $e^- + u \to e^- + u + g$ and c) $e^- + g \to e^- + u\bar{u}(d\bar{d} + \ldots)$ together with the u quark and gluon structure function of the proton

In quantum electrodynamics we are used to consider the fine structure constant $\alpha = e^2/4\pi$ as a given fixed constant. Of course this has its origin in the fact that in all calculations the same definition, i.e. the same renormalization, of the coupling e is employed. e is defined by the electron-electron-photon vertex with all three particles on the mass shall $p_1^2 = p_2^2 = m_e^2$ and $q^2 = (p_1-p_2)^2 = 0$ (see Fig.1.2a). This is only one of many possibilities to define the renormalized charge. Any other values for the momenta p_1, p_2 or q could be chosen. Suppose we are interested to work with the coupling α which is defined for $p_1^2 = p_2^2 = m_e^2$ but for arbitrary $q^2 \neq 0$. This coupling $\alpha(q^2)$ is related to the usual coupling α, which we denote $\alpha_0 \equiv \alpha(0)$, by the following expression — considering only the lowest order term in an expansion in α_0 and assuming $q^2 >> m_e^2$:

$$\alpha(q^2) = \alpha_0\left(1 + \frac{\alpha_0}{3\pi}\ln\frac{q^2}{m_e^2}\right) \quad . \tag{1.3.1}$$

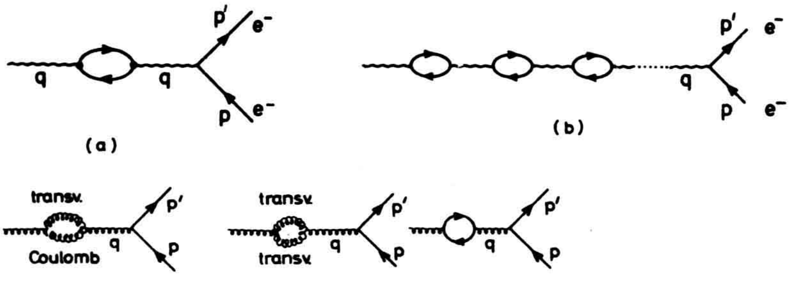


Fig.1.4. a) One-loop contribution to the photon-electron coupling in QED, b) multi-loop contribution to the coupling in QED and c) one-loop contribution to the quark-gluon coupling in QCD. "Coulomb" and "transv." denote gluons with this polarization in the Coulomb gauge

This relation is obtained from the vacuum polarization contribution to the photon propagator in Fig.1.4a. Calculating also the higher order terms in α_0, shown in Fig.1.4b, in the leading logarithm approximation, we obtain terms proportional to $[\alpha_0 \ln(q^2/m_e^2)]^n$ which can be summed up with the result

$$\alpha(q^2) = \frac{\alpha_0}{1 - \frac{\alpha_0}{3\pi}\ln\frac{q^2}{m_e^2}} \quad . \tag{1.3.2}$$

In QED the summation of the series in the form (1.3.2) is not essential since $\alpha_0 = 1/137$ is very small so that even for very large q^2 the first few terms of the series in α_0 are sufficient which, of course, are taken into account in the higher order radiative corrections. For $(\alpha_0/3\pi)\ln(q^2/m_e^2) \simeq 1$ the approximations used to derive (1.3.2) break down. Therefore no statement about the behaviour of $\alpha(q^2)$ for $q^2 \to \infty$ can be made.

In QCD the behaviour of the renormalized coupling constant $\alpha_s = g^2/4\pi$ as a function of $q^2 = (p_1-p_2)^2$ is completely different. The reason is the additional interactions of the gluon which are absent in QED. Suppose the QCD coupling has been de-

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