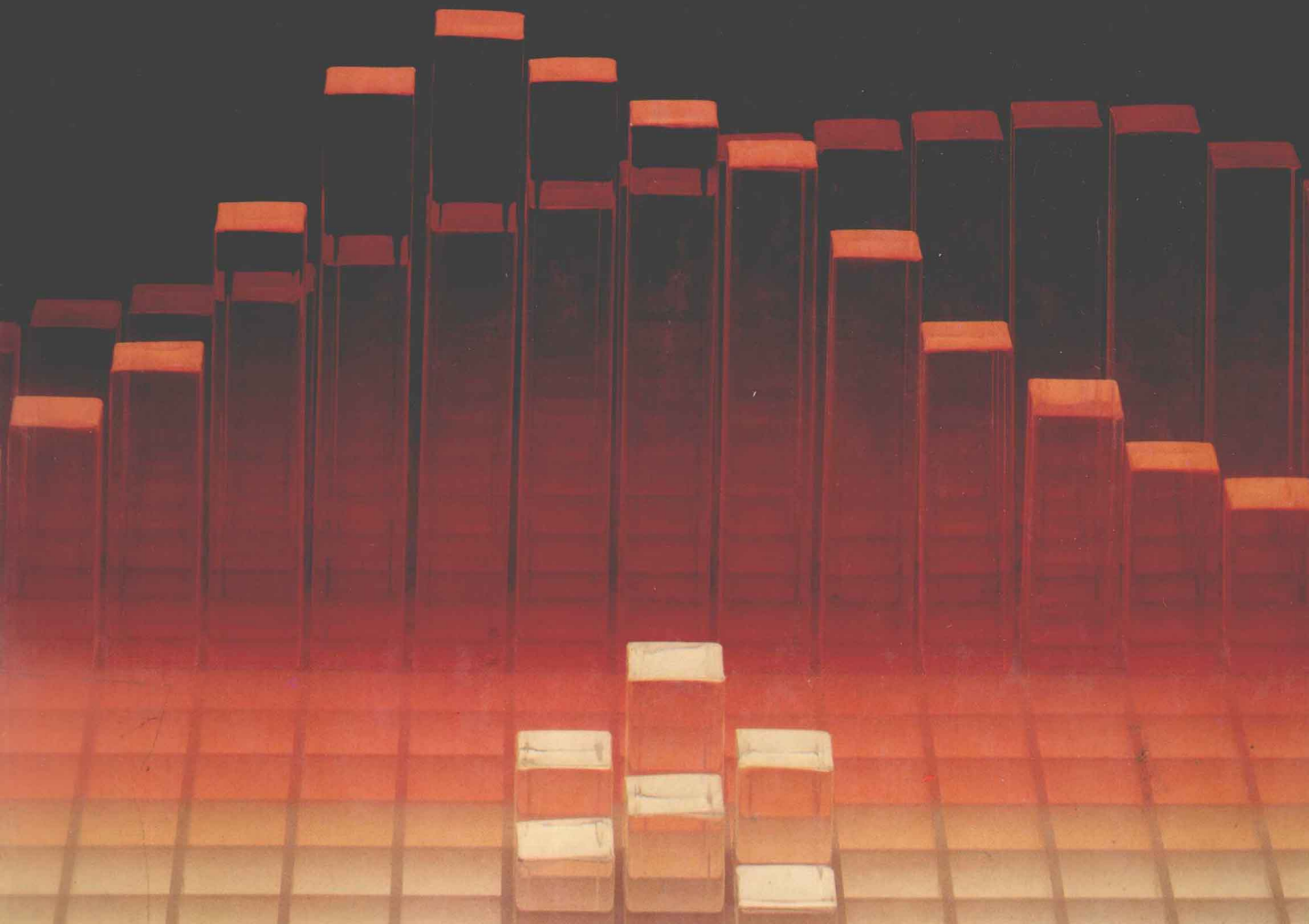
College Mathematics and Calculus

WITH APPLICATIONS TO MANAGEMENT, LIFE AND SOCIAL SCIENCES

KARL J. SMITH



College Mathematics and Calculus

WITH APPLICATIONS TO MANAGEMENT, LIFE AND SOCIAL SCIENCES

Karl J. Smith

Brooks/Cole Publishing Company
Pacific Grove, California

Brooks/Cole Publishing Company A Division of Wadsworth, Inc.

© 1988 by Wadsworth, Inc., Belmont, California 94002. All rights reserved. No part of this book may be reproduced, stored in a retrieval system, or transcribed, in any form or by any means—electronic, mechanical, photocopying, recording, or otherwise—without the prior written permission of the publisher, Brooks/Cole Publishing Company, Pacific Grove, California 93950, a division of Wadsworth, Inc.

Printed in the United States of America

10 9 8 7 6 5 4

Library of Congress Cataloging-in-Publication Data

Smith, Karl J.
College mathematics and calculus with applications to management, life and social sciences/Karl J. Smith.
p. cm.
Includes index.
ISBN 0-534-08910-0
1. Mathematics—1961– I. Title.
QA37.2.S576 1988 87-19403 CIP

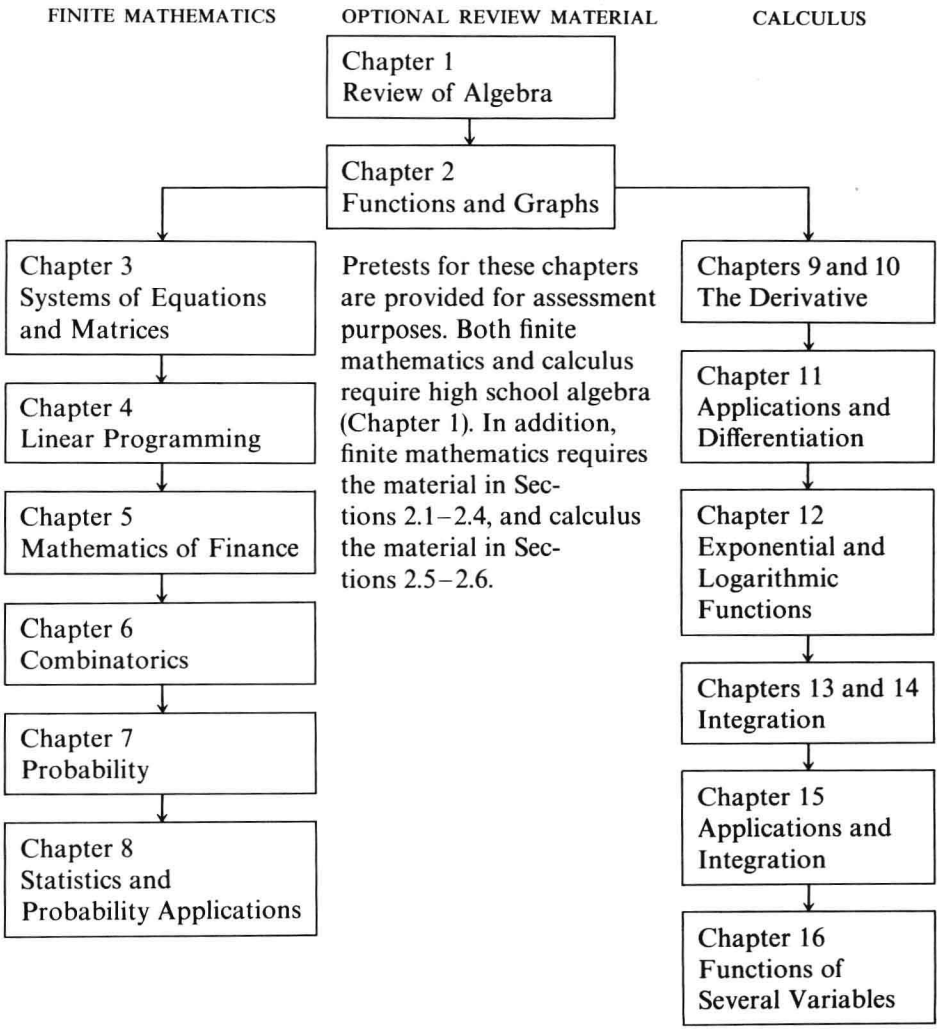
510—dc19

Sponsoring Editor: Jeremy Hayhurst
Editorial Assistant: Maxine Westby
Production Services Coordinator: Joan Marsh
Production: Cece Munson, The Cooper Company
Manuscript Editor: Betty Berenson
Interior Design: Jamie Sue Brooks
Cover Design: Katherine Minerva
Cover Photo: Lee Hocker
Interior Illustration: Carl Brown
Typesetting: Polyglot Pte Ltd, Singapore
Cover Printing: Phoenix Color Corp.
Printing and Binding: R. R. Donnelley & Sons, Harrisonburg, Virginia

This book is dedicated, with love, to my wife,
Linda Ann Smith

Preface

College Mathematics and Calculus is a textbook for a two-semester or three-quarter course to provide the mathematics background necessary for students in business, management, life sciences, or social sciences. The text is divided into two parts, Finite Mathematics in Chapters 3–8 and Calculus in Chapters 9–16.



此为试读,需要完整PDF请访问: www.ertongbook.com

The emphasis in this course is not on algebraic manipulation but rather on an understanding of the modeling process and using mathematics to make statements about real world applications. The prerequisite for this course is intermediate algebra.

It seems as if every new book claims innovation, state-of-the-art production, supplementary materials, readability, abundant problems, and relevant applications. How, then, is one book chosen over another? More specifically, how does this book differ from other books for this type of course, and what factors were taken into consideration as it was being written?

Content

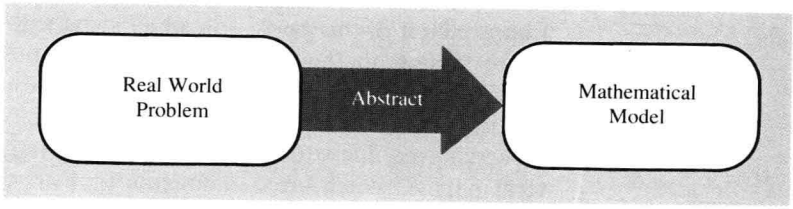
First, every book must cover the appropriate topics, and hopefully in the right order. Finite mathematics has evolved and changed considerably since it was first introduced in the 1960s. Today's course focuses on matrices to solve both linear systems of equations and inequalities (linear programming), sets, combinatorics, and probability, and some supplementary topics such as Markov chains, game theory, and mathematics of finance. New recommendations regarding discrete mathematics are also influencing finite mathematics courses, so I have included appendixes on logic and mathematical induction.

The calculus in Chapters 9–16 is presented by using a variety of real world applications. In 1986, the Sloan Foundation sponsored a four-day conference to study the calculus curriculum. The conference's work was summarized in the following statement: "We propose a calculus syllabus which emphasizes intuition and conceptual understanding by giving priority to numerical and geometrical methods and approximation techniques." The development of integration that I used in this book follows the recommendations of this conference. The use of tables and numerical integration is emphasized and we do not spend a great deal of time on integration techniques.

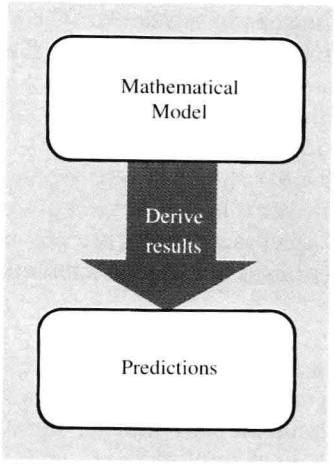
Mathematical Models

When a real life situation is analyzed, it usually does not easily lend itself to mathematical analysis because the real world is far too complicated to be precisely and mathematically defined. It is therefore necessary to develop what is known as a **mathematical model**. This is a body of mathematics based on certain assumptions about the real world, which is modified by experimentation and accumulation of data, and is then used to predict some future occurrence in the real world. A mathematical model is not static or unchanging but is continually being revised and modified as additional relevant information becomes known.

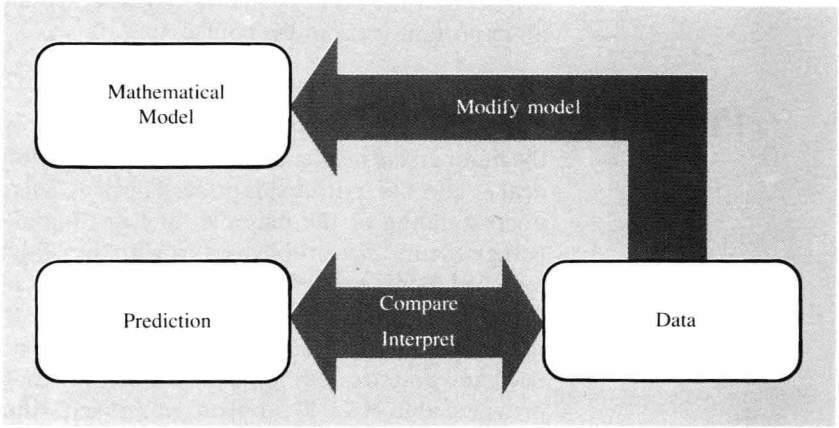
Some mathematical models are quite accurate, particularly in the physical sciences. For example, the path of a projectile, the distance that an object falls in a vacuum, or the time of sunrise tomorrow all have mathematical models that provide very accurate predictions about future occurrences. On the other hand, in the fields of management, life sciences, and social sciences, the models provide much less accurate predictions because they deal with situations that involve many unknowns or undefined variables that are often random in character. How can we go about constructing a model in management, life sciences, or social sciences? You need to observe a real world problem and make assumptions about influencing factors, a process called *abstraction*.



You must know enough about the mechanics of mathematics to *derive results* from the model.



The next step is to gather data. Does the prediction fit all the known data? If not, you will use this data to *modify* the assumptions used in the model. This is an ongoing process.



The primary skill addressed in this book is that of problem solving and building a mathematical model. Most books pay lip service to building models but rarely *develop* the skill. True model building cannot be learned in a single lesson, or even

in a single course. Learning this skill must be a gradual process, and that is how I approach it in this book—in small steps with realistic applications. I devote an entire section to the nature of linear programming and how to go about the *formulation* of an appropriate model (Section 4.2). In addition, the modeling applications at the end of each chapter are included to illustrate the *model-building process* in real life situations. These applications are presented as suggestions for term papers, which are assignments that are open-ended and require a model-building approach for their development. Even though they are designed to be used as enrichment or for supplementary study, they are also designed to illustrate mathematical modeling. One essay written in response to each of these modeling applications is given in its entirety in the *Student Solutions Manual* in order to show *how* the model building can be developed. These model-building applications, even if not assigned, serve to illustrate, in a very real way, how the material developed in the rest of the chapter can be used to answer some nontrivial questions (see, for example, the Modeling Application on the Cobb-Douglas Production Function at the end of Chapter 16).

The text is divided into sections of nearly equal size that each take about one class day to develop. Many sections are marked optional in the Table of Contents. This does not mean that I feel these topics are not important; it simply means that the material in these sections is not used in subsequent sections.

Style

The author's writing style is another factor that distinguishes one textbook from another. My writing style is informal, and I always write with the student in mind. I offer study hints along the way and let the students know what is important. Frequent and abundant examples are provided so that the student can understand each step before proceeding to the next. A second color is used to highlight the important steps or particular parts of an equation or formula. The chapter reviews list important terms and provide a sample test. A listing of the chapter objectives as well as from 50 to 100 additional practice problems keyed to the stated objectives are provided in the *Student Solutions Manual* so that the student has a clear grasp of all important ideas in the course.

Problems

The third factor, which is one of the most important in deciding upon a textbook, is the number *and quality* of the problems presented. This is where I have spent a great deal of effort in writing this book. The problems should help to *develop* the students' understanding of the material, and not inhibit or thwart that understanding by being obscure. The problems start with the simpler ideas, presenting the problems in matched pairs so that for each new problem at least one is provided with an answer in the back and one without the answer. There are about three times more problems than are needed for assignments so that students will have the opportunity to go back and practice additional problems, both for the midterm and for the final. I have provided almost 7,000 problems in this text. After the manipulative skills have been developed, each problem set presents a large number of applications to show how the material can be used in business, management, and life and social sciences.

The types of problems include:

Drill. I have included a large number of drill problems to provide adequate practice so that the student can develop a clear understanding of each topic.

Applications. Self-contained applied problems are provided in almost every section to give relevance and practicality to the topic at hand. A list showing the extent and variety of the applications is part of each chapter opening and gives a preview of the material.

Modeling Applications. Each chapter concludes with an optional real life application that allows the material to be applied to a *real* (rather than a textbook) problem, but at a level of difficulty that is manageable for the student. The answers to these questions are open-ended, and a sample essay written in answer to each modeling application is included in the *Student Solutions Manual.*

CPA, CMA, and Actuary Exam Questions. Actual questions from Actuary, CPA, and CMA exams are scattered throughout the textbook. These test questions provide a link between textbook and profession.

Historical Questions. Historical notes provide insight into the humanness of mathematics, but instead of being superfluous commentaries, these are integrated into the problem sets to give students a taste of some of the ways the topics were originally developed as solutions to mathematical problems.

Supplementary Materials

The final factor often used in selecting a textbook is the type and quality of available supplements. In addition to the answers in the back of the book, I have provided several supplements:

Student Solutions Manual This provides complete solutions to every odd-numbered problem in the book. It also includes essays to illustrate modeling and lists of objectives that define the necessary skills studied in each chapter. It also provides additional practice problems to supplement those in the textbook.

Instructor's Supplement This includes answers to all of the problems in the book and additional questions to accompany the modeling applications.

Testing Program This is a computerized test bank with text editing capabilities that allows you to create an almost unlimited number of tests or retests of the material. This test bank is also available in printed form for those instructors not using a computer.

Computer Aided Finite Mathematics This is a computer supplement prepared by Chris Avery and Charles Barker. Appendix D lists the programs available in this supplement. An extensive list of other commercially available computer software is also provided in Appendix D.

As the author, I am also available to help you create any other set of supplementary materials that you feel are necessary or worthwhile for your course.

Acknowledgments

Many persons have reviewed either the entire manuscript or parts of this manuscript and have offered many valuable suggestions, and I would like to offer each and every one of them my sincere thanks:

John Alberghini
Manchester Community College
Patricia Bannantine
Marquette University
Craig Benham
University of Kentucky
Thomas Covington
Northwestern State University

Joe S. Evans
Middle Tennessee State University
James L. Forde
Concordia College
Marjorie S. Freeman
University of Houston
Matthew Gould
Vanderbilt University
Kevin Hastings
University of Delaware
Judith Hector
Walters State Community College
Lou Hoelzle
Bucks County Community College
Edwin Klein
University of Wisconsin
Ann Barber Megaw
University of Texas, Austin
Norman Mittman
Northeastern Illinois University
Wendell L. Motter
Florida A & M University
Jacqueline Payton
Virginia State University
William Ramaley
Fort Lewis College
Robert Sharpton
Miami-Dade Community College
Gordon Shilling
University of Texas, Arlington
Roland Sink
Pasadena City College
Harriette J. Stephens
State University of New York, Canton
Mary T. Teegarden
Mesa Junior College
William T. Watkins
Pan American University
Jan Wynn
Brigham Young University
Donald Zalewski
Northern Michigan University

The problem solvers were John Spellman, Ernest Ratliff, Ricardo Torrejon, and Gregory Passty of Southwest Texas State University, Donna Szott of Allegheny County Community College—South, and Pat Bannantine of Marquette University, and I thank them not only for reviewing the manuscript, but for the long hours spent working all the examples and problems. In addition, Terry Shell of Santa Rosa Junior College worked all the problems (at least twice) and offered many helpful suggestions, and I give him my heartfelt thanks. Chris Avery and Charles Barker from De Anza College were most cooperative and helpful in modifying their computer material to match the development in this book.

The production of a textbook is a team effort. I would like to thank Jeremy Hayhurst, Maxine Westby, Joan Marsh, Katherine Minerva, Cece Munson, Jamie Sue Brooks, and Carl Brown.

And last, but not least, my thanks go to my family, Linda, Missy, and Shannon, for their love, support, and continued understanding of my involvement in writing this book.

Karl J. Smith
Sebastopol, California

Contents

CHAPTER 1 Review of Algebra (Optional) **1**

1.1 Algebra Pretest 2
1.2 Real Numbers 4
1.3 Algebraic Expressions 8
1.4 Factoring 14
1.5 Linear Equations and Inequalities 18
1.6 Quadratic Equations and Inequalities 22
1.7 Rational Expressions 27
*1.8 Summary and Review 31

CHAPTER 2 Functions and Graphs 33

2.1 Functions and Graphs Pretest 34
2.2 Functions 35
2.3 Graphs of Functions 42
2.4 Linear Functions 47
2.5 Quadratic and Polynomial Functions 58
2.6 Rational Functions 65
*2.7 Summary and Review 71

FINITE MATHEMATICS 73

CHAPTER 3 Systems of Equations and Matrices 75

3.1 Systems of Equations 76
3.2 Introduction to Matrices 84

* Optional sections

此为试读,需要完整PDF请访问: www.ertongbook.com

3.3 Gauss–Jordan Elimination 99
3.4 Inverse Matrices 113
*3.5 Leontief Models 122
*3.6 Summary and Review 127
Modeling Application 1: Gaining a Competitive Edge in Business 130

CHAPTER 4 Linear Programming 131

4.1 Systems of Linear Inequalities 132
4.2 Formulating Linear Programming Models 137
4.3 Graphical Solution of Linear Programming Problems 147
4.4 Slack Variables and the Pivot 157
4.5 Maximization by the Simplex Method 164
4.6 Nonstandard Linear Programming Problems 174
*4.7 Duality 182
*4.8 Summary and Review 191
Modeling Application 2: Air Pollution Control 194

CHAPTER 5 Mathematics of Finance 195

5.1 Difference Equations 196
5.2 Interest 204
5.3 Annuities 214
5.4 Amortization 217
*5.5 Sinking Funds 222
*5.6 Summary and Review 226
Modeling Application 3: Investment Opportunities 229

CHAPTER 6 Combinatorics 230

6.1 Sets and Set Operations 231
6.2 Permutations 242
6.3 Combinations 250
6.4 Binomial Theorem 259
*6.5 Summary and Review 264
Modeling Application 4: Earthquake Epicenter 266

CHAPTER 7 Probability 267

7.1 Probability Models 268
7.2 Probability of Equally Likely Events 273
7.3 Calculated Probabilities 280
7.4 Conditional Probability 285
7.5 Probability of Intersections and Unions 293

7.6 Mathematical Expectation 296
7.7 Stochastic Processes and Bayes' Theorem 302
*7.8 Summary and Review 308
Modeling Application 5: Medical Diagnosis 311

CHAPTER 8 Statistics and Probability Applications 312

8.1 Random Variables 313
8.2 Analysis of Data 317
8.3 The Binomial Distribution 327
8.4 The Normal Distribution 333
8.5 Normal Approximation to the Binomial 338
*8.6 Correlation and Regression 343
*8.7 Summary and Review 351
Cumulative Review Chapters 3–8 354

CALCULUS 359

CHAPTER 9 The Derivative 360

9.1 Limits 361
9.2 Continuity 371
9.3 Rates of Change 379
9.4 Definition of Derivative 390
9.5 Differentiation Techniques, Part I 395
9.6 Differentiation Techniques, Part II 400
9.7 The Chain Rule 405
*9.8 Summary and Review 408
Modeling Application 6: Instantaneous Acceleration: A Case Study of the Mazda 626 410

CHAPTER 10 Additional Derivative Topics 411

10.1 Implicit Differentiation 412
10.2 Differentials 415
10.3 Business Models Using Differentiation 420
*10.4 Related Rates 426
*10.5 Summary and Review 431
Modeling Application 7: Publishing: An Economic Model 432

CHAPTER 11 Applications and Differentiation 433

11.1 First Derivatives and Graphs 434
11.2 Second Derivatives and Graphs 443

11.3 Curve Sketching—Relative Maximums and Minimums 450
11.4 Absolute Maximum and Minimum 454
*11.5 Summary and Review 461
Cumulative Review for Chapters 9–11 463

CHAPTER 12 Exponential and Logarithmic Functions 464

12.1 Exponential Functions 465
12.2 Logarithmic Functions 471
12.3 Logarithmic and Exponential Equations 476
12.4 Derivatives of Logarithmic and Exponential Functions 482
*12.5 Summary and Review 487
Modeling Application 8: World Running Records 489

CHAPTER 13 Integration 490

13.1 The Antiderivative 491
13.2 Integration by Substitution 497
13.3 The Fundamental Theorem of Calculus 503
13.4 Area between Curves 513
*13.5 Summary and Review 522
Modeling Application 9: Computers in Mathematics 524

CHAPTER 14 Additional Integration Topics 525

14.1 Integration by Parts 526
14.2 Integration by Table 530
14.3 Numerical Integration 535
*14.4 Summary and Review 544
Modeling Application 10: Modeling the Nervous System 546

CHAPTER 15 Applications and Integration 547

15.1 Business Models 548
15.2 Probability Density Functions 555
15.3 Improper Integrals 567
*15.4 Differential Equations 570
*15.5 Summary and Review 574
Cumulative Review for Chapters 12–15 576
Cumulative Review for Chapters 9–15 577

CHAPTER 16 Functions of Several Variables 579

16.1 Three Dimensional Coordinate System 580
16.2 Partial Derivatives 587

16.3 Maximum–Minimum Applications 592
16.4 Lagrange Multipliers 598
*16.5 Multiple Integrals 604
*16.6 Summary and Review 615
Modeling Application 11: The Cobb-Douglas Production Function 617

Appendixes 618

A Logic 619
B Mathematical Induction 628
C Calculators 633
D Computers 637
E Tables 641
1. Pascal's Triangle—Combinatorics 642
2. Squares and Square Roots 643
3. Binomial Probabilities 644
4. Standard Normal Cumulative Distribution 648
5. Compound Interest—Future Value and Present Value 650
6. Ordinary Annuity 652
7. Annuity Due 654
8. Present Value of an Annuity 656
9. Mortality Table 658
10. Powers of e 659
11. Common Logarithms 661

F Markov Chains 663
G Answers to Odd-Numbered Problems 681

Index 719

CHAPTER 1
Review of Algebra
(Optional)

CHAPTER CONTENTS

1.1 Algebra Pretest
1.2 Real Numbers
1.3 Algebraic Expressions
1.4 Factoring
1.5 Linear Equations and Inequalities
1.6 Quadratic Equations and Inequalities
1.7 Rational Expressions
1.8 Summary and Review
Important Terms
Sample Test

The prerequisite for the material in this book is a course in basic algebra. Since it has probably been a while since you studied algebra, this chapter includes a review of the algebraic topics you need to understand the mathematical models presented in this book. The chapter begins with an algebra pretest. Part I of the pretest reviews general algebraic concepts. If you have difficulty with these questions, take an algebra course before attempting the material in this book. Part II of the pretest reviews more advanced algebraic topics that are discussed in this chapter. If you have difficulty with these questions, review the appropriate section of the chapter. The answers to all the questions on the pretest are given in the answer section of this book.

1.1 Algebra Pretest

Part I. Basic Algebraic Concepts

Choose the best answer in Problems 1–15.

1. In $5x^2y$, the 5 is
A. a term **B.** a binomial **C.** an exponent
D. a literal factor **E.** a numerical coefficient

2. In $6x^2y + 3z$, the 6 and 3 are
A. terms **B.** exponents **C.** binomials
D. coefficients **E.** literal factors

3. If $(-2)(-2)(-2) = x$, the value of x is
A. -8 **B.** -6 **C.** 6
D. 8 **E.** none of these

4. $21 - (-5)$ equals
A. -26 **B.** -16 **C.** 16
D. 26 **E.** none of these

5. If $a = -3$ and $b = 5$, then $a^2 - 2ab + b^2$ equals
A. -4 **B.** 2 **C.** 64
D. -2 **E.** none of these

6. The expression $8 + 2 \cdot 3 - 8 \div 2$ equals
A. 11 **B.** 10 **C.** 3
D. -3 **E.** none of these

7. $3x - 5x$ equals
A. $15x^2$ **B.** $8x$ **C.** $15x$
D. $-2x$ **E.** none of these

8. -5^2 equals
A. -10 **B.** 25 **C.** -25
D. 10 **E.** none of these

9. $(-3y^2)^3$ equals
A. $27y^6$ **B.** $-27y^5$ **C.** $-3y^6$
D. $-27y^6$ **E.** none of these

10. $\dfrac{a+b}{2}$ means
A. $a + b \cdot \frac{1}{2}$ **B.** $a + b \div 2$ **C.** $a + (b \div 2)$
D. $(a + b) \cdot \frac{1}{2}$ **E.** all of these

11. If $3x + 12 = 6$, x equals
A. 6 **B.** 2 **C.** -2
D. -6 **E.** none of these

12. If $6 + 3x = x - 4$, x equals
A. 5 **B.** $2\frac{1}{2}$ **C.** 1
D. 2 **E.** none of these

13. If $2x - 16 = 3x - 9$, x equals
A. $-\frac{7}{5}$ **B.** -7 **C.** 5
D. 25 **E.** none of these

14. $(32x^8) \div (-2x)$ equals
A. $30x^7$ **B.** $16x^8$ **C.** $34x^8$
D. $-30x^7$ **E.** none of these

15. Simplify $3(x + 2) - (x - 4y) + (x + y)$.
A. $3x + 11y$ **B.** $3x - 3y + 6$
C. $x + 5y + 6$ **D.** $3x + 5y + 6$
E. none of these

Indicate if Problems 16–30 are true or false.

16. If $x \neq 0$, then $-x$ always indicates a number that is less than zero.

17. $2(xy) = (2x)(2y)$

18. $-2^4 = 16$

19. x^2 is always positive.

20. $(a + b)^2 = a^2 + b^2$

21. $\dfrac{A + C}{B + C} = \dfrac{A}{B}$

22. $\dfrac{2 + x}{6} = \dfrac{x}{3}$

23. $\frac{0}{5}$ is not defined.

24. A natural number is both rational and real.

25. $\dfrac{A}{B} + \dfrac{C}{B} = \dfrac{1}{B}(A + C)$

26. $\dfrac{-x}{-y} = -\dfrac{x}{y}$

27. $(x - y)^2 = (y - x)^2$

28. $\dfrac{x}{y} + \dfrac{y}{x} = 1$

29. $\dfrac{1}{x} y \dfrac{1}{z} = \dfrac{y}{xz}$

30. $\sqrt{x^2 + y^2} = x + y$

Part II. Algebra Pretest

Choose the best answer for each of the following problems.

[1.2] REAL NUMBERS

1. $|2\pi - 7| =$
A. $2\pi - 7$ **B.** $7 - 2\pi$ **C.** -0.7168 **D.** 0.7168
E. The expression may have more than one value.

2. $6.23\overline{4}$ is an example of
A. a natural number **B.** a real number **C.** an irrational number
D. all the answers in parts A, B, and C **E.** none of these

此为试读,需要完整PDF请访问: www.ertongbook.com