MECHANICS
OF
STRUCTURES

[A TEXT-BOOK FOR UNIVERSITY STUDENTS]

By

S. B. JUNNARKAR, M.B.E.,
B.A., B.Sc. Hons. (Eng.), London
Formerly,
Principal, Birla Vishvakarma Mahavidyalaya, Vallabh Vidyanagar,
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Sometime, Dean of the Faculty of Technology including Engineering,
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Vol. II

[WITH 333 DIAGRAMS AND NUMEROUS EXERCISES]

SIXTH REVISED AND ENLARGED EDITION: 1974

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MECHANICS OF STRUCTURES

[IN M.K.S. UNITS AND SI UNITS]

By the same author:

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ENGINEERING MECHANICS

[For Diploma Students]

•

SI UNITS

To

My children

The good Suman

Hemendra

and

Urmilla

PREFACE

This volume attempts to cover the portion of the subject which is usually dealt with in the final year of the Degree courses in most of the Indian universities. In presenting the subject-matter, the main purpose viz., to explain the basic principles as lucidly as possible and to illustrate them with a number of worked-out examples, has been steadily kept in view. It is hoped that the treatment of the methods of moment distribution and slope-deflection will appeal to students who wish to acquire an insight into these useful methods. It was thought desirable to include a chapter on the Elements of Soil Mechanics which has now acquired an important place in the Science of Structural Engineering. I am indebted to Mr. V. B. Priyani, B.E., A.M.I.E., of the Birla Vishvakarma Mahavidyalaya, who has contributed this chapter.

I am grateful to Mr. R. S. Dighe, B.E., A.M.I.E., Reader in Structural Engineering at the Maharaja Sayajirao University of Baroda, for checking the numerical work of the first eight chapters. My thanks are due to Messrs. D. B. Bhatt, N. K. Kaushik and R. N. Vakil, senior students of the College for checking the numerical work of the remaining chapters. I am grateful to Mr. L. D. Bhatt who prepared an excellent typescript for the press and to Mr. N. D. Bhatt, D.M.E., who prepared all the sketches required for the blocks. I am specially indebted to Mr. R. C. Patel of the Charotar Book Stall, Anand, Mr. V. B. Priyani, B.E., A.M.I.E. and to Mr. N. D. Bhatt, D.M.E., who have taken considerable pains in correcting the proofs. My thanks are due to the Prabhat Process Studio, Ahmedabad, for preparing the blocks used in the book.

I should like to express my keen appreciation of the excellent work done by N. Hernandez, S. J., and his staff of the Anand Press, Anand, in the printing and get-up of this volume.

Birla Vishvakarma Mahavidyalaya,
Vallabh Vidyanagar, Anand
June, 1953

S. B. JUNNARKAR

FIFTH EDITION

This revised and enlarged edition brings the book in line with its companion Volume I, in presenting the text entirely in metric units. A number of typical examples have been added almost to every chapter to illustrate the text.

A chapter on Plastic Theory which was originally published in this volume and later transferred to Volume III, has now been restored.

It is hoped that the book, in its present form, will be found useful.

Poona S. B. J.
March, 1968

SIXTH EDITION

In this edition, the chapter on "Elements of Soil Mechanics", having outlived its utility, has been replaced by chapters on the "Elastic Centre", the Betti-Maxwell theorem on "Reciprocal Displacements", "Müller-Breslau Principle" etc.

Culmann's "Elastic Centre" is an ingenious device, making the solution of problems on Fixed Arches very simple. The "Reciprocal Theorem" has been generalised to illustrate the use of models in Structural Analysis. The "Müller-Breslau Principle" is a straight application of the Reciprocal Theorem and is particularly useful in the construction of influence lines for redundant reactions.

The text has been thoroughly revised and brought up-to-date. The author desires to invite particular attention to Chapter IX on "Method of Moment Distribution" in which, the first article is entirely devoted to presenting the method from first principles; the strictly logical reasoning will, it is hoped, appeal to the reader. The rest of the chapter deals with the method in the accepted conventional manner, illustrating its use with a large number of solved examples.

To make the International metric system, known as the SI, familiar to the reader, an appendix has been added, briefly describing its relevant features with a number of solved examples. Numerous examples for practice have also been added in the text, both in the SI and m.k.s. units.

468, Ganeshkhind Road S. B. J.
Poona-16
October 1973

CONTENTS

CHAPTER I

ROLLING LOADS

1. Rolling loads: When loads move across a girder, as in the case of axle-loads of a locomotive crossing a bridge, every cross-section of the girder will be subjected to a Shear Force and a Bending Moment, their magnitudes changing, as the load position changes. For any *given* cross-section of the girder, the problem will be to find the load position for the maximum S.F. or the maximum B.M. For *every* cross-section, we can then work out the maximum S.F. and maximum B.M., placing the load in its appropriate position. These can then be plotted for all sections of the girder from one end to the other and we obtain the maximum S.F. and maximum B.M. diagrams.

We shall study a few standard cases of loading on a girder simply supported over a span l and shall begin with the simplest loading, viz., a single concentrated load W moving from left to right.

2. A single concentrated load W: Consider a section X of the girder at a distance x from the left hand support A. Let the load W roll along from A to B and consider a load position at a distance y from A.

Load in AX $(y < x)$

The reactions at the supports are $R_B = \dfrac{Wy}{l}$ and $R_A = \dfrac{W(l-y)}{l}$. The shear force at X for this load position will be given by, $F_x = +R_B = +\dfrac{Wy}{l}$. This increases as y increases, until when the load reaches the section X, the magnitude of F_x reaches the value of

$$F_{max} = +\frac{Wx}{l} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

Load in XB $(y > x)$

As soon as the load crosses the section X and enters the portion XB of the girder,

$$F_x = -R_A = -\frac{W(l-y)}{l}.$$

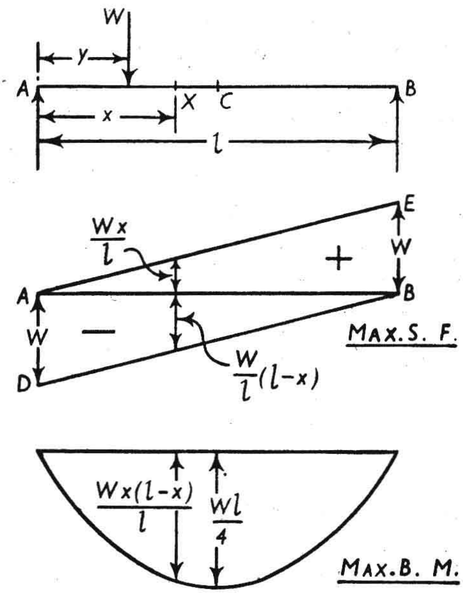

FIG. 1-1

The shear force thus changes sign. As y increases, the magnitude of the negative shear force decreases. For the maximum negative shear force at the section, the value of y must be the least, i.e., $y = x$, when

$$F_{max} = -\frac{W(l-x)}{l} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

The maximum S.F. at the cross-section, thus, occurs when the load is on the cross-section itself—in the portion AX for the positive value and in the portion XB for the negative value.

If we draw a diagram of these maximum values for all sections from $x = 0$ to $x = l$, the maximum S.F. diagram will consist of two parallel lines. For maximum positive shear, when $x = 0$, $F_{max} = 0$; when $x = l$, $F_{max} = +W$. For maximum negative shear, when $x = 0$, $F_{max} = -W$; when $x = l$, $F_{max} = 0$.

To construct the diagram, on the base AB, draw ordinates AD and BE, each equal to W to a suitable scale. Join AE and BD. The parallels AE and DB represent the maximum S.F. diagram.

Similarly, for the maximum B.M. at a given section X, consider a load position at a distance y from A.

Load in AX $(y < x)$

$$M_x = -R_B\,(l - x) = -\frac{Wy}{l}\,(l - x).$$

This increases as y increases, until when the load reaches X, $y = x$ and,

$$M_{max} = -\frac{Wx\,(l - x)}{l}.$$

Load in XB $(y > x)$

When the load enters the portion XB,

$$M_x = -R_A.x = -\frac{W(l - y)}{l}.x.$$

This decreases as y increases, the maximum value being $-\dfrac{Wx\,(l - x)}{l}$ when y is the least, i.e., when $y = x$.

The maximum B.M. at a cross-section thus occurs when the load is on the cross-section itself, the magnitude being given by,

$$M_{max} = -\frac{Wx\,(l - x)}{l} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

If we plot a diagram for M_{max} for all sections from $x = 0$ to $x = l$, the curve is evidently a parabola. The absolutely maximum bending moment anywhere in the girder occurs at the centre, its value being,

$$M_{max\ max} = -\frac{Wl}{4}.$$

3. Uniformly distributed load longer than the span: Let the uniformly distributed moving load be w per unit length and let it move from left to right. Consider a section X at a distance x from A. Let the head of the load have reached a distance y from A.

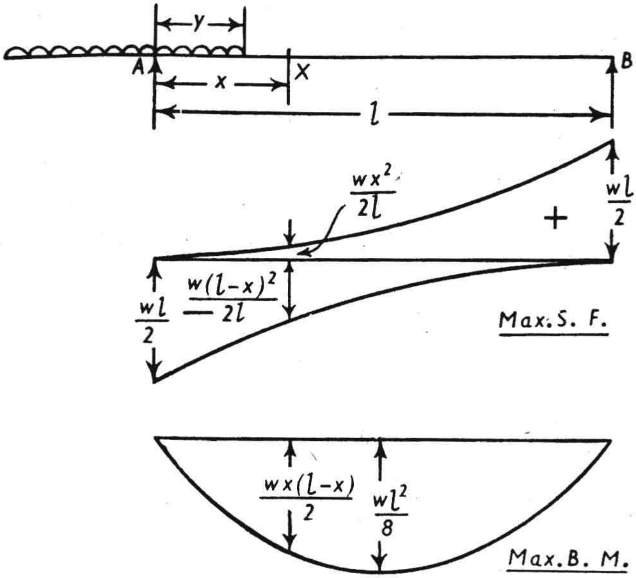

FIG. 1-2

Load in AX ($y < x$)

The reaction at B, is $R_B = \frac{wy^2}{2l}$.

The shear force at the given section X is,

$$F_x = +R_B = +\frac{wy^2}{2l}.$$

This increases as y increases until when the head of the load reaches the section X, $y = x$ and

$$F_{max} = +\frac{wx^2}{2l} \quad \dots\dots\dots\dots\dots\dots\dots\dots \quad (1)$$

As soon as the load crosses X and enters the portion XB, F_x starts diminishing. This can be easily proved. When the head of the load is at X, $R_B = \frac{wx^2}{2l}$. Let the head now move into the portion XB by a small distance δx. The reaction at B will now slightly increase by δR_B such that,

$$R_B + \delta R_B = \frac{w\,(x + \delta x)^2}{2l}.$$

At the given section X, the shear force will now be,

$$F_x = +\,(R_B + \delta R_B) - w.\delta x$$

$$= +\,\frac{w\,(x + \delta x)^2}{2l} - w.\delta x$$

$$= +\,\frac{wx^2}{2l} + \left\{wx.\frac{\delta x}{l} - w\delta x\right\},$$ neglecting small quantities of the second order. The expression inside the bracket is negative. Therefore, F_x is less than $+\frac{wx^2}{2l}$.

The maximum value of positive shear at X is $+\frac{wx^2}{2l}$, when the head of the load has reached the cross-section, in other words, when AX is fully loaded and XB is empty.

As the load advances into the portion XB, the shear force continues to diminish until, for some load position, the S.F. at the section is zero. As the load continues to advance, the S.F. changes sign and becomes negative. When the load covers the span entirely,

$$F_x = -\,R_A + wx$$

$$= -\,\frac{wl}{2} + wx,$$ and is negative for a given section in the left hand half of the span. The S.F. will remain constant at this value while the entire span is covered and this condition will obtain until the tail of the load is at A. As the load moves on, so that XB is fully loaded and AX is partially loaded,

$$F_x = +\,R_B - w\,(l - x).$$

The magnitude of R_B diminishes as the load moves off the portion AX and consequently the negative value of F_x increases, since the expression $-w\ (l-x)$ remains constant. Let the tail of the load reach the section X so that XB is fully loaded and AX is empty. We now have,

$F_x = -R_A = -\dfrac{w(l-x)^2}{2l}$. This is the maximum value of negative shear at the section, since, as the load moves still further away, the magnitude of R_A diminishes.

Therefore,

$$F_{max} = -\frac{w(l-x)^2}{2l} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \text{(2)}$$

Thus, the maximum positive sheaı at a given section is $+\dfrac{wx^2}{2l}$ when AX is loaded and XB is empty; while the maximum negative shear is $-\dfrac{w\,(l-x)^2}{2l}$ when AX is empty and XB is fully loaded.

The maximum S.F. diagram consists of two parabolas with $\dfrac{wl}{2}$ as end-ordinates as shown in fig. 1-2.

For the maximum bending moment at a given section X, let the load be in the portion AX so that $y < x$.

Now, $M_x = -R_B\,(l-x) = -\dfrac{wy^2}{2l}\,(l-x)$.

This increases to $-\dfrac{wx^2\,(l-x)}{2l}$ when the head of the load reaches the section X. As the load passes X and enters the portion XB, the B.M. continues to increase, as can be easily seen. When the head has reached X, $R_B = \dfrac{wx^2}{2l}$. If the load advances a small distance δx into XB, the reaction at the support B, now, is,

$$R_B + \delta R_B = \frac{w\,(x+\delta x)^2}{2l}.$$