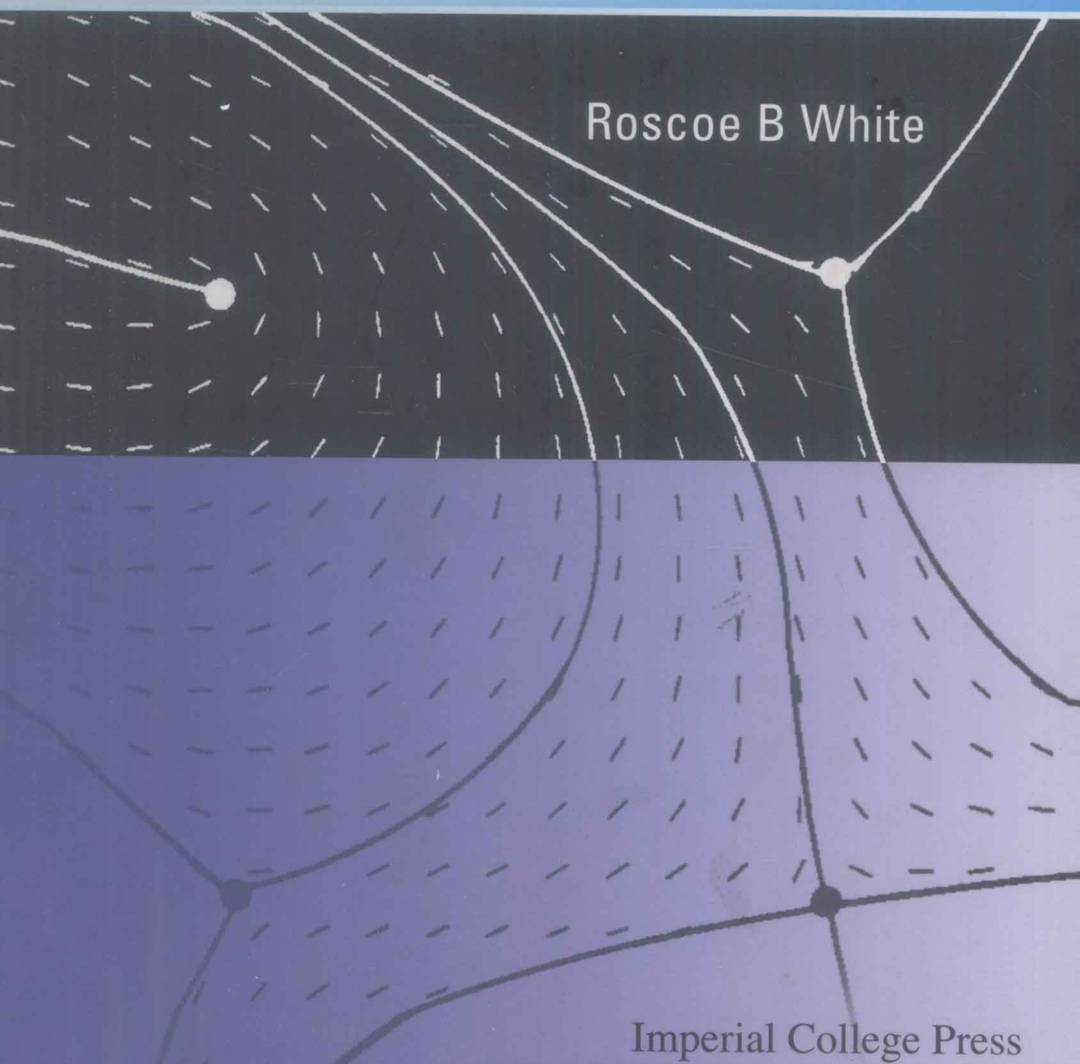
Asymptotic Analysis *of* Differential Equations

Roscoe B White



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Preface

The material for this book has resulted from teaching a graduate course in ordinary differential equations at Princeton for several years. Methods of asymptotic analysis including dominant balance, the use of divergent asymptotic series, phase integral methods, asymptotic evaluation of integrals, and boundary layer analysis are covered. The construction of integral solutions and the use of analytic continuation are used in conjunction with the asymptotic analysis, to show the interrelatedness of these methods. Some of the functions of classical analysis are used as examples, to provide an introduction to their analytic and asymptotic properties, and to give derivations of some of the important identities satisfied by them, since it is my experience that students are insufficiently familiar with Whittaker and Watson. There is no attempt to give a complete presentation of all these functions. The emphasis is on the various techniques of analysis; obtaining asymptotic limits, connecting different asymptotic solutions, and obtaining integral representations. Less attention is paid to strict mathematical rigor. A sufficient number of different examples are chosen to demonstrate the various techniques and approaches. Prerequisite material consists of elementary calculus and a basic knowledge of the theory of functions of a complex variable.

In Chapters 1–9 the material in each chapter depends on the results obtained in the previous chapters. These Chapters cover the use of dominant balance for solving equations with a small parameter, a review of the techniques for obtaining exact solutions of soluble differential equations, complex variable theory and analytic continuation, classification of singular points of differential equations and the construction of local expansions to solutions, including nonconvergent asymptotic expansions, phase integral methods, perturbation theory, asymptotic evaluation of integrals, basic properties of the Euler gamma function, and methods for finding integral solutions of differential equations. Some basic results concerning the gamma function are in fact used in earlier Chapters, so some parts of this chapter should be previewed when necessary. Most of the relations proved concerning $\Gamma(z)$ however depend on results in Chapters 1–7, so it was impractical to move it to an earlier point in the book. Some of the material in the first chapters may be already known, in which case it can be quickly reviewed or skipped.

The remaining Chapters are independent of one another, depending only on the material in the first nine, and can be covered depending on personal interest. All Chapters cannot be covered in one semester, and I normally choose Chapters 11, 12, 16, and 17 after the first nine. Chapter 10 introduces the theory of expansions in series of orthogonal polynomials and derives some of their properties, and also introduces the theory of wavelets. Chapter 11 gives the theory of the Airy function, an understanding of which is essential for a good grasp of the WKB treatment of scattering and bound state problems. Chapter 12 further illustrates the use of integral representations and analytic continuation through an examination of the Bessel function. Chapter 13 does the same for the parabolic cylinder function, and Chapter 14 for the Whittaker function. Chapter 15 examines inhomogeneous equations, some arising from the theory of resistive reconnection of a magnetized plasma, and further explores the role of causality using the Green's function technique for the examination of the equations for a parametric instability. Chapter 16 on the Riemann zeta function provides a brief introduction to the distribution of prime numbers. Chapter 17 treats differential equations containing a small parameter giving rise to boundary layers.

Roscoe B. White
Princeton 2005

Acknowledgment

The development of this text is the outcome of interactions with many students and research colleagues over many years. Some of the material presented here is also present in other texts which treat asymptotic methods. Normally not covered are the use of Kruskal–Newton graphs, the derivation of Heading's rules for analytic continuation of phase integral solutions, the associated derivation of the Stokes constants, and the methods for obtaining integral solutions. These approaches have been integrated to show how they are interrelated.

The book was written using Latex, by Leslie Lamport (Addison Wesley, 1994). The figures were made using Super Mongo, written by Robert Lupton, Princeton University (rhl@astro.princeton.edu), and Patricia Monger (monger@mcmaster.ca). I am grateful to Bedros Afeyan for discussions concerning wavelets, Greg Rewoldt for help with Latex, Leonid Zakharov for help with computing, and to Tim Stoltzfus-Dueck for proofreading several chapters.

Finally, I am grateful to my wife Laura and my daughter Veronica for their patience and support during my many years of teaching and research, and to my brother Bob for encouragement and stimulation.

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