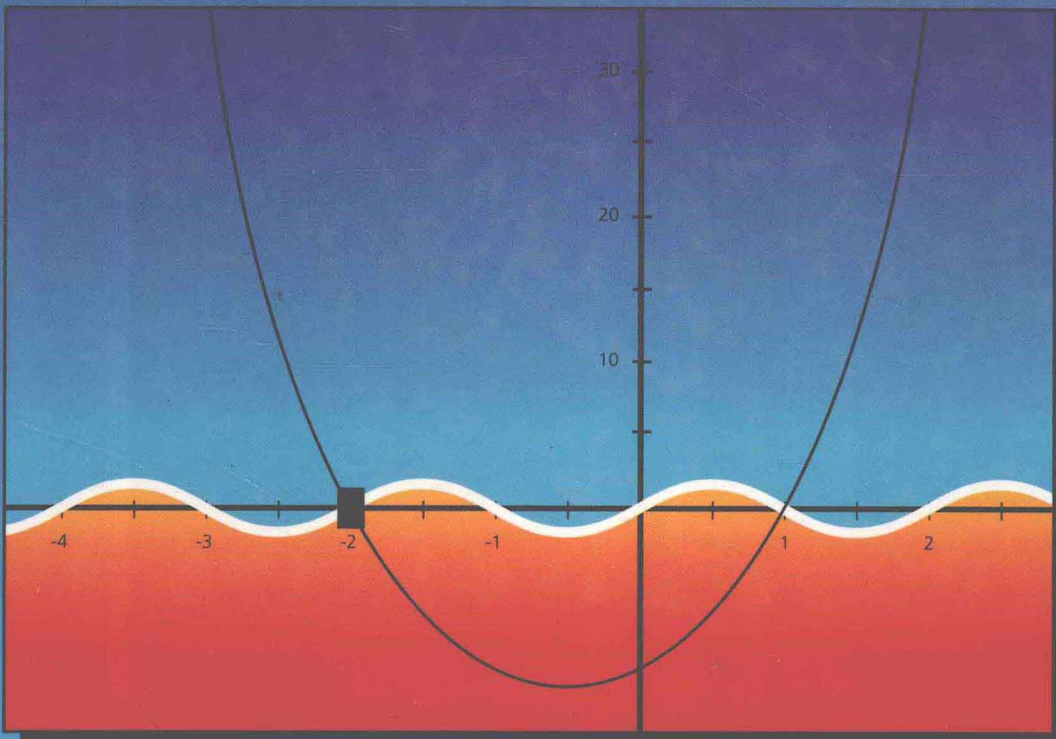
5TH EDITION

PRECALCULUS

with Graphing and Problem Solving



Karl J. Smith

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PRECALCULUS
WITH GRAPHING AND PROBLEM SOLVING

Karl J. Smith, Ph.D

Brooks/Cole Publishing Company
Pacific Grove, California

This book is dedicated to my daughter, Melissa Ann Smith. Missy graduated from UCLA and now teaches elementary school in Southern California. I am very proud of her, and dedicate this book to her with love.

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PREFACE

Preparation for Calculus

The selection of topics in this book is, of course, designed to prepare the student for calculus. Precalculus is a difficult subject, and there is no magic key to success. You will need to read the book, work the examples in the book using your own pencil and paper, and you will need to make a commitment to do your mathematics homework on a daily basis. I have written this book so that it will be easy for you to know what is important:

- Important terms are presented in **boldface type**.
- Important ideas are enclosed in boxes.
- Common pitfalls, helpful hints, and explanations are shown in *italics*.
- ⊗ Warnings are given to call your attention to common mistakes.
- Color is used in a functional way to help you "see" what to do next.

In addition to these pedagogical aids, I have included entire sections to provide built-in redundancy and help you review for exams:

- The goals of each section are listed as objectives at the end of each chapter.
- Review problems are provided for each chapter objective.
- Answers to the odd-numbered problems are provided to let you know whether you are on the right track; many solutions and hints are also provided and more than 350 graphs are presented in the answer section alone to provide additional examples.
- Problems are graded and presented in sets so that you can work some of the simpler problems before progressing to the more challenging ones.
- Cumulative reviews are given as practice tests.

You will notice that this book is directly oriented to your success in calculus. Many of the topics (such as factoring) are topics that are not new, but when you look at the examples and problems presented, you will find them to be different from those you found in algebra. The reason is that I have included many of the problems *just as you will see them in calculus*. In addition, at the beginning of each chapter I have included not only a chapter overview but also a chapter perspective. This perspective points out how the techniques learned in that chapter will be used in calculus. The formulas summarized on the inside front and back covers are those ideas presented in this book that will be needed in calculus.

The main concept of this book is the one that is most needed for the study of calculus—namely, the **notion of a function**. This concept is defined and discussed in Chapter 2 and is then used as the unifying idea for the rest of the material. Although the presentation of material assumes high-school algebra, the central ideas from algebra (factoring, solving equations, simplifying algebraic expressions, the laws of exponents) are integrated into the text where appropriate since these topics are often the very ones that cause difficulty for the beginning calculus student. It is not necessary for students to have had trigonometry, since an entire trigonometry course is presented in Chapters 6, 7, and 8. I do not believe that the right-triangle approach is the best way to introduce trigonometry to students who are about to take calculus. Therefore, I use reference angles and the ratio definition for the trigonometric functions, and quickly introduce radian measure of angles.

In addition to the usual precalculus material, I have included problems that are not typical in precalculus courses, but that will be particularly useful in the study of calculus.

Techniques for Success

Success in this course is a joint effort by the student, the instructor, and the author. The student must be willing to attend class and devote time to the course *on a daily basis*. There is no substitute for working problems in mathematics. The problem sets in this book are divided into A, B, and C problems according to the level of difficulty. A word of warning is necessary here. The C problems are generally extraordinary problems and should be considered difficult and beyond the scope of the usual precalculus course. The *Student's Solution Manual* shows the complete solution to all of the odd-numbered problems.

I do not believe that students completely learn the material by working a particular type of problem only once. However, as a college instructor, I am well aware of time constraints and the amount of material that must be covered. I have therefore developed the idea of uniform problem sets for this book, which means that the instructor can make "standard assignments" consisting of the same problems being assigned from section to section. This makes it easy for the instructor and student to work problems from more than one section at a time. For example, a typical spiral assignment would be

First day: 5–60 multiples of 5 from Problem Set x

Second day: 5–60 multiples of 5 from Problem Set $(x + 1)$
7–28 multiples of 7 from Problem Set x

Third day: 5–60 multiples of 5 from Problem Set $(x + 2)$
7–28 multiples of 7 from Problem Set $(x + 1)$
6–24 multiples of 6 from Problem Set x

This means that the standard assignment would be 5–60 multiples of 5, multiples of 7 from the previous set, and multiples of 6 from the assignment two class meetings before. On the other hand, if you do not wish to work spiral assignments, you can still have the same assignment from section to section:

Standard 1–2 hour assignment: Problems 5–50, multiples of 5

Standard 2–3 hour assignment: Problems 3–60, multiples of 3

If you make any of these standard assignments, or spiral assignments, you will practice all of the important ideas in this book. These standard assignments do not apply to the review problems at the end of each chapter.

In addition to a large number of drill problems, there are extensive groups of application problems, including applications from agriculture, archaeology, architecture, astronomy, aviation, ballistics, business, chemistry, earth science, economics, engineering, medicine, navigation, physics, police science, psychology, social science, space science, sports, surveying, as well as consumer and general-interest applications.

Finally, at the end of each part of the book I have included an extended application introduced by a news clipping that examines a topic in more than the usual depth—population growth, solar power, and planetary orbits.

Accuracy of the Material

It is extremely important to present the material accurately. Not only has this book been reviewed by the usual number of persons, but all of the problems have been checked and rechecked by the following persons: Gary Gislason of the University of Alaska, Audrey Rose of Portland Community College, Karen Sharp of Mott Community College, Donna Szott of Allegheny Community College, Stephen P. Simonds of Portland Community College, Mary Ellen White of Portland Community College, and Jean Woody of Tulsa Junior College. Finally, just prior to publication, the following persons checked the book's accuracy: Nancy Angle, Michael Ecker, Eunice Everett, Michael Friedberg, Diana Gerardi, Mark Greenhalgh, Scott Holm, Diane Koenig, Katherine McClain, and Janis Phillip. If you find *any* errors in this book, I want you to call me at home. My phone number is (707) 829-0606.

Calculator Usage

We are now seeing a new generation of calculators that are able to do symbolic manipulation and graphing. These calculators are characterized by the Texas Instruments TI-81, Hewlett Packard 28S, Sharp EL-5200, and Casio fx 8000G. Even though many students still do not have access to these calculators, they are beginning to revolutionize the way precalculus and calculus are taught. Nearly every change I made in this edition was the direct result of the graphing calculator. For example, solving linear and quadratic equations and inequalities can be enhanced by the use of a graphing calculator. For this reason, I have introduced two-dimensional coordinate systems as early as possible (Section 1.3). With these calculators, students can now see a *graphical* description of a process as well as an algebraic one. Consider the problem of optimization (which is considered in Section 3.3). We can not only illustrate how to find the maximum or minimum by completing the square, but can also *show* how it is the *turning point on a grap*h. Although I do not assume that students (or instructors) using this book have a graphing calculator, I have included many references to graphing calculators throughout the book. There are CALCULATOR COMMENT boxes to show how I used my graphing calculator to do a graph or enhance the concept at hand.

In the general text of this book, I assume only that the student has an inexpensive scientific calculator. Procedures for using inexpensive calculators are presented as they occur in a natural way without any special designation. This is in recognition of the assumption that most students will have such calculators. For example, the computation aspects of the logarithmic function in Chapter 5 have been minimized, and the introduction to the evaluation of the trigonometric functions in Chapter 6 assumes that each student has a calculator. In writing this book, I used a Sharp EL-531A (for which I paid $9.95) and a Texas Instruments TI-81 for graphing and matrix applications (for which I paid $125). You should always consult the owner's manual for the calculator you own. In the previous editions of this book I showed keystrokes for both algebraic and RPN logic. However, with the new generation of calculators, I now see much more diversity in possible keystrokes for a particular process. For that reason, I have removed specific keystrokes and instead have included only general remarks about process or procedure.

For the Instructor

A great deal of flexibility is possible in the instructor's selection of topics presented in this book. I have written the book so that each section represents approximately one day of class material. I have also provided more material than can be used in a single semester or quarter so that you can select material appropriate for your school or class. Some of the material that I have included may not be considered typical. Included is a chapter on graphing techniques to allow a consideration of graphing, not only of polynomial and rational functions, but also radical and absolute functions; a complete discussion of the ambiguous case in solving triangles; De Moivre's theorem; the Fundamental Theorem of Algebra; linear programming; partial fractions; translations and rotations of the conic sections; three-dimensional coordinate systems; vectors (in two and three dimensions); parametric equations; polar coordinates; and intersection of polar-form curves.

The following changes were made in the Fifth Edition:

- Added emphasis in illustrating each idea graphically, as well as algebraically. This will make it easy for those who have access to a graphing calculator to use it with this book.
- New sections on algebraic expressions, problem solving, operations of functions, cubic and quadratic functions, real roots of rational and radical equations, and inverse matrices
- Rewrote and expanded the section on inverse functions
- Split the chapter on polynomial and rational functions into two chapters; included new material on radical functions
- Deleted specific calculator keystrokes and added generic directions. We have provided a calculator supplement to discuss specific calculator keystrokes.

Acknowledgments

I would like to thank the reviewers of the previous editions of this book:

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By the time a book reaches a fifth edition, it seems inadequate to simply thank my wife, Linda, and to acknowledge the lost family time while I was working on this book. I must thank her for a lifetime of love and support.

Karl J. Smith
Sebastopol, California

CONTENTS

J. J. Sylvester

Leonhard Euler

Karl Gauss

INTRODUCTION

FUNCTIONS AND GRAPHING

*Complex numbers can be omitted from this text. All parts of the book requiring the use of complex numbers will be marked as optional.

Amalie (Emmy) Noether

John Napier

Hipparchus

Copernicus


Benjamin Banneker


Gottfried Leibniz


Isaac Newton


Charlotte Scott

APPENDIXES

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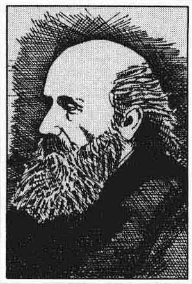


J. J. Sylvester (1814–1897)

There are three ruling ideas, three so to say, spheres of thought, which pervade the whole body of mathematical science, to some one or other of which, or to two or all three of them combined, every mathematical truth admits of being referred; these are the three cardinal notions, of Number, Space, and Order. Arithmetic has for its object the properties of number in the abstract. In algebra, viewed as a science of operations, order is the predominating idea. The business of geometry is with the evolution of the properties of space, or of bodies viewed as existing in space.

J. J. SYLVESTER
Philosophical Magazine, vol. 24 (1844), p. 285

It is appropriate to begin our study of precalculus with a quotation from J. J. Sylvester, one of the most colorful men in the history of mathematics. This quotation of Sylvester ties together the ideas of number, space, and order. As we look at the topics of precalculus, we see that those very notions form the basis not only for this chapter, but also for the entire course.

Sylvester's writings are flowery and eloquent. He was able to make the dullest subject bright, fresh, and interesting. His enthusiasm and zest for life were evident in every line of his writings. He was, however, a perfect fit for the stereotype of an absent-minded mathematics professor.

Born and educated in England, he accepted an appointment as Professor of Mathematics at the University of Virginia in 1841. After only 3 months, he resigned because of an altercation with a student. He returned to England penniless, but in 1876 he came back to America to accept a position at Johns Hopkins University. The 7 years he spent at Johns Hopkins were the happiest and most productive in his life. During his stay there, he founded the *American Journal of Mathematics* (in 1878).

It is in the spirit of Sylvester that I write this book for you, the student. I have tried to deal with your frustration and concerns by writing a book that is bright, fresh, and interesting. I have tried to make the material easy to read and understand. If I have overlooked anything, I hope that you will not dismiss this as the work of a typical absent-minded professor; please take time to write me a letter or a postcard to communicate your thoughts.

As you begin your study of precalculus, I want to remind you that learning mathematics requires systematic and regular study. Do not expect it to come to you in a flash, and do not expect to cram just before an examination. Take it in small steps, one day at a time, and you will find success at your doorstep.

1 FUNDAMENTAL CONCEPTS

Contents

Preview

This chapter reviews many of the preliminary ideas from previous courses and sets the foundation for the remainder of this course. Numbers, linear and quadratic equations and inequalities, as well as a two-dimensional system are introduced in this chapter. This chapter has 29 objectives, which are listed at the end of the chapter on pages 62–64.

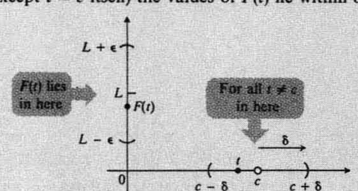
Perspective

Calculus is divided into two parts: differential and integral calculus. Both the definition of a derivative and that of an integral are dependent on a notion defined in calculus called the *limit.* The formal definition of a limit involves both an absolute value and an inequality,

$$0 < |t - c| < \delta$$

as you can see in the portion of a page from a calculus book shown below. In the middle of the page you can also see that intervals are used. We introduce interval notation and discuss absolute value inequalities in Section 1.4.

We need to require that for *every* interval about L, no matter how small, we can find an interval of numbers about c all of whose F-values lie within that interval about L. In other words, given *any* positive radius ϵ about L, there exists some positive radius δ about c such that for all t within δ units of c (except $t = c$ itself) the values of $F(t)$ lie within ϵ units of L:



Thus, the closer t stays to c without equaling c, the closer $F(t)$ must stay to L.

DEFINITION

Limit

The **limit** of $F(t)$ as t approaches c is the number L if:

Given any radius $\epsilon > 0$ about L there exists a radius $\delta > o$ about c such that for all t

$$0 < |t - c| < \delta \quad \text{implies} \quad |F(t) - L| < \epsilon. \tag{4}$$

From George Thomas and Ross Finney, *Calculus and Analytic Geometry*, 7th ed. (Reading, Mass.: Addison-Wesley), pp. 57–58.

1.1 Real Numbers

Sets of Numbers

You are, no doubt, familiar with various sets of numbers, as well as with certain properties of those numbers. Table 1.1 gives a brief summary of some of the more common subsets of the set of **real numbers.**

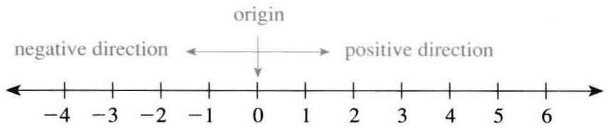
Table 1.1 Sets of numbers

NAME	SYMBOL	SET	EXAMPLES
Counting numbers or natural numbers	$\mathbb{N}$	$\{1, 2, 3, 4, \ldots\}$	$86; 1{,}986{,}412; \sqrt{16}; \sqrt{1}; \sqrt{100}; \ldots$ $\sqrt{a}$ is the nonnegative real number b so that $b^2 = a$. It is called the *principal square root* of a. Some square roots are natural numbers.
Whole numbers	$\mathbb{W}$	$\{0, 1, 2, 3, 4, \ldots\}$	$0; 86; 49; \frac{8}{4}; \frac{16{,}425}{25}; \ldots$ $\frac{a}{b}$ means $a \div b$. It is called the *quotient* of a and b. Some quotients are whole numbers.
Integers	$\mathbb{Z}$	$\{\ldots, -2, -1, 0, 1, 2, 3, \ldots\}$	$-8; 0; \frac{-16}{4}; 43{,}812; -96; -\sqrt{25}; \ldots$
Rational numbers	$\mathbb{Q}$	Numbers that can be written in the form p/q, where p and q are integers with $q \neq 0$; they are characterized as numbers whose decimal representations either terminate or repeat.	$\frac{2}{3}; \frac{4}{17}; .863214; .866\ldots; 5; 0; \frac{-19}{10}; -16; \sqrt{\frac{1}{4}};$ $3.1416; 8.\overline{6}; .1\overline{56}; \ldots$ An overbar indicates repeating decimals. Some square roots are also rational.
Irrational numbers	$\mathbb{Q}'$	Numbers whose decimal representations do not terminate and do not repeat	$4.1234567891011\ldots; 6.31331333133331\ldots;$ $\sqrt{2}; \sqrt{3}; \sqrt{5}; \pi; \frac{\pi}{2}; \ldots$ π is the ratio of the circumference of a circle to its diameter. It is sometimes approximated by 3.1416, but this is a rational approximation of an irrational number. We write $\pi \approx 3.1416$ to indicate that the numbers are *approximately* equal. Square roots of most numbers are irrational.
Real numbers	$\mathbb{R}$	Numbers that are either rational or irrational	All examples listed above are real numbers. Not all numbers are real numbers, however. Some of these will be considered in Section 1.5; they are called *complex numbers.*

One-Dimensional Coordinate System

The real numbers can most easily be visualized by using a **one-dimensional coordinate system** called a **real number line** (Figure 1.1).

FIGURE 1.1 A real number line



A **one-to-one correspondence** is established between all real numbers and all points on a real number line:

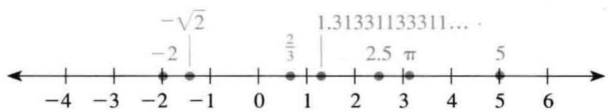
1. Every point on the line corresponds to precisely one real number.
2. Every real number corresponds to precisely one point.

A point associated with a particular number is called the **graph** of that number. Numbers associated with points to the right of the **origin** are called **positive real numbers** and those to the left are called **negative real numbers.** Numbers are called **opposites** if they are plotted an equal distance from the origin. The **opposite of a real number *a*** is denoted by $-a$. Notice that if a is positive, then $-a$ is negative; and if a is negative, then $-a$ is positive. It is a common error to think of $-a$ as a negative number; it might be negative, but it might also be positive. If, say, $a = -5$, then $-a = -(-5) = 5$, which is positive.

EXAMPLE 1 Graph the following numbers on a real number line:

$$5; \quad -2; \quad 2.5; \quad 1.31331133311\ldots; \quad \tfrac{2}{3}; \quad \pi; \quad -\sqrt{2}$$

SOLUTION When graphing, the exact positions of the points are usually approximated. ▮



Absolute Value and Distance on a Number Line

A very important idea in mathematics involves the notion of **absolute value.** If a is a real number, then its graph on a number line is some point; call it A. The distance between A and the origin is the geometric interpretation of the **absolute value of *a*.**

ABSOLUTE VALUE

The absolute value of a real number a is denoted by $|a|$ and is defined by

$$|a| = a \quad \text{if } a \geq 0$$
$$|a| = -a \quad \text{if } a < 0$$

Thus $|5| = 5$, $|-5| = 5$, $|0| = 0$, and $|-\pi| = \pi$. Notice that $|a|$ is nonnegative for all values of a.

EXAMPLE 2 $|\pi - 3| = \boldsymbol{\pi - 3}$ since $\pi - 3 > 0$; use the first part of the definition of absolute value ▮

EXAMPLE 3 $|\pi - 4| = -(\pi - 4) = \boldsymbol{4 - \pi}$ since $\pi - 4 < 0$; use the second part of the definition of absolute value ▮