INTERMEDIATE ALGEBRA

Graphs and Functions

LARSON HOSTETLER NEPTUNE

SECOND EDITION

Intermediate Algebra
Graphs and Functions

Second Edition

Roland E. Larson
The Pennsylvania State University
The Behrend College

Robert P. Hostetler
The Pennsylvania State University
The Behrend College

Carolyn F. Neptune
Johnson County Community College

with the assistance of

David E. Heyd
The Pennsylvania State University
The Behrend College

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Preface

A word from the authors ...

Welcome to *Intermediate Algebra: Graphs and Functions,* Second Edition! In the revision of this early graphing and functions text, we focused on fine-tuning our student-oriented approach and incorporating the best aspects of reform in a meaningful yet easy-to-use manner. We hope you will be as excited about the Second Edition as we are after you take a look at it.

A student-oriented approach ...

Some students must take intermediate algebra more than once because they do not know how to study mathematics. The Second Edition helps students break out of this cycle by outlining a straightforward program of study with continual reinforcement and progressive confidence building.

This practical approach begins with Strategies for Success, a new feature found at the beginning of each chapter. In addition to outlining the key skills to be covered in the chapter, this checklist provides page references for the various study tools in the chapter. The Chapter Summary at the end of each chapter reinforces the Strategies for Success with a comprehensive list of the skills covered in the chapter, section references, and a correlation to the Review Exercises for guided practice.

Throughout each chapter there are many opportunities for students to assess their progress: at the end of each section (Warm-Ups and section exercises); in the middle of each chapter (Mid-Chapter Quiz); and at the end of each chapter (Review Exercises, Chapter Test, and Cumulative Test). The test items and text exercises are carefully crafted and graded in difficulty to give students a higher comfort level with algebraic skill building and problem solving.

These study tools reinforce the message that mathematics is a continuing story that requires constant synthesis and review. Along the way, students are guided by Study Tips that address special cases, expand on concepts, and help them avoid common errors.

Foremost for us, this text was written to be read and understood by students—not to be merely a source of homework assignments. We paid careful attention to the presentation—using precise mathematical language, innovative full-color design for emphasis and clarity, and a level of exposition that appeals to students—to create an effective teaching and learning tool.

... that incorporates the best aspects of reform

We wholeheartedly embrace many of the features of the mathematics reform movement. Our First Edition led the way in developing many innovative learning techniques and our Second Edition is maintaining the pace.

In the Second Edition, we have increased the coverage of technology and integrated it throughout the text at point of use. Students are encouraged to use a graphing utility as a tool for exploration, discovery, and problem solving. Our reviewers were pleased to notice that technology is always introduced in support of the concepts, rather than being the central focus of the text.

We introduce graphing and functions early and integrate them throughout, always stressing visualization. We have increased the emphasis on real-life applications, problem solving, conceptual exercises (for example, Think About It and Section Projects), and motivational features (for example, Explorations and Group Activities). In addition, we added three new sections on modeling data: Section 3.2, Modeling Data with Linear Functions; Section 6.6, Modeling Data with Quadratic Functions; and Section 9.6, Modeling Data (with exponential and logarithmic functions). This new material gives students the opportunity to focus on generating, exploring, and analyzing data.

We hope you will enjoy the Second Edition. It is a readable text that incorporates the best aspects of reform. The straightforward approach and effective study tools should appeal to your students.

Roland E. Larson

Robert P. Hostetler

Carolyn F. Neptune

Features of the Text

Chapter Openers NEW

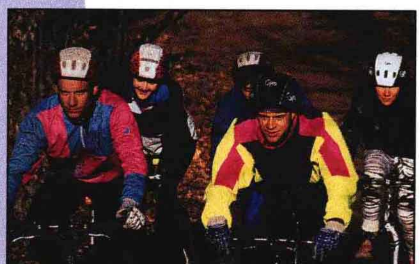
Each chapter opens with *Strategies for Success.* This new checklist outlines the key skills to be covered in the chapter and gives a list of the study tools in the chapter—with page references that will help students master the key skills. Each chapter opener also contains a list of the section topics, as well as a photo referring students to an interesting exercise in the chapter.

Introduction to Graphs and Functions **2**

Strategies for Success

SKILLS *When you have completed this chapter, make sure you are able to:*
- Create graphs that represent real-life data
- Sketch graphs of equations and find the x- and y-intercepts
- Sketch graphs of lines using slopes and y-intercepts
- Evaluate functions and find their domains
- Identify and sketch transformations of graphs of functions

TOOLS *Use these study tools to master the skills above:*



In Exercise 79 of Section 2.1, you will graphically represent the number of male and female sports participants in the United States.

Section Topics

Each section begins with a list of important topics that are covered in that section. These topics are also the subsection titles and can be used for easy reference and review by students.

Definitions, Rules, and Formulas

All of the important definitions, rules, and formulas are highlighted for emphasis. Each is also titled for easy reference.

SECTION 5.2 Rational Exponents and Radicals 357

5.2 Rational Exponents and Radicals

Roots and Radicals • Rational Exponents • Radicals and Calculators • Radical Functions

Roots and Radicals

In Section 1.1, you reviewed the use of radical notation to represent nth roots of real numbers. Recall from that section that b is called an nth root of a if $a = b^n$. Also recall that the principal nth root of a real number is defined as follows.

Principal nth Root of a Number

Let a be a real number that has at least one (real number) nth root. The **principal nth root of a** is the nth root that has the same sign as a, and it is denoted by the **radical symbol**

$\sqrt[n]{a}$. Principal nth root

The positive integer n is the **index** of the radical, and the number a is the **radicand.** If $n = 2$, omit the index and write $\sqrt{a}$ rather than $\sqrt[2]{a}$.

NOTE "Having the same sign" means that the principal nth root of a is positive if a is positive and negative if a is negative. For example, $\sqrt{4} = 2$ and $\sqrt[3]{-8} = -2$.

Therefore, $\sqrt{49} = 7$, $\sqrt[3]{1000} = 10$, and $\sqrt[5]{-32} = -2$.

You need to be aware of the following properties of nth roots. (Remember that for nth roots, n is an integer that is greater than or equal to 2.)

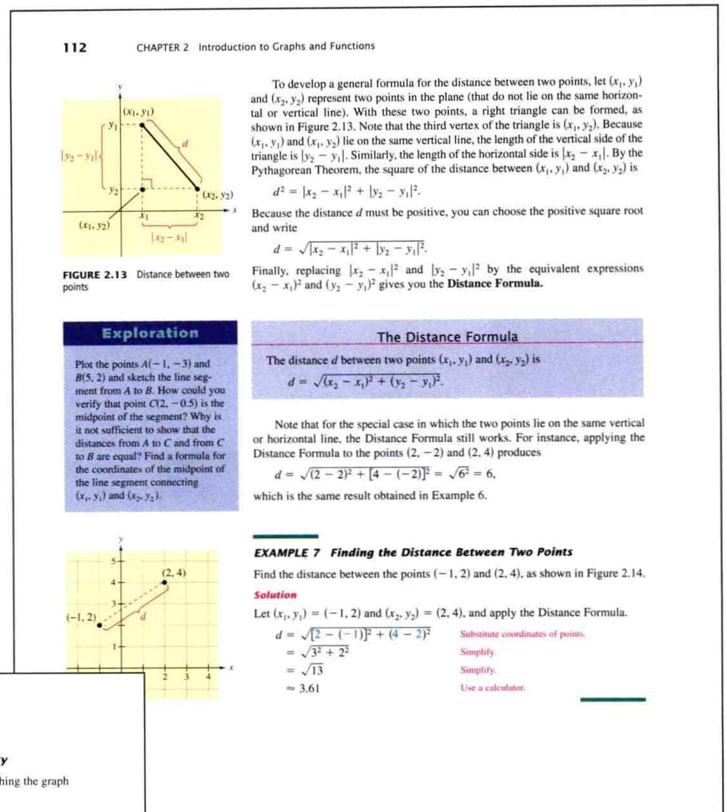
Properties of nth Roots

1. If a is a positive real number and n is *even*, then a has exactly two (real) nth roots, which are denoted by $\sqrt[n]{a}$ and $-\sqrt[n]{a}$.
2. If a is any real number and n is *odd*, then a has only one (real) nth root, which is denoted by $\sqrt[n]{a}$.
3. If a is a negative real number and n is *even*, then a has no (real) nth root.

Integers such as 1, 4, 9, 16, 49, and 81 are called **perfect squares** because they have integer square roots. Similarly, integers such as 1, 8, 27, 64, and 125 are called **perfect cubes** because they have integer cube roots.

Explorations NEW

Before students are exposed to selected topics, Explorations invite them to discover concepts and patterns on their own, often taking advantage of the power of technology. This active participation by students strengthens their intuition and their critical-thinking skills and makes it more likely that they will remember the results. These new, optional boxed features can be omitted if the instructor desires with no loss of continuity in the coverage of material.


112 CHAPTER 2 Introduction to Graphs and Functions

To develop a general formula for the distance between two points, let (x_1, y_1) and (x_2, y_2) represent two points in the plane (that do not lie on the same horizontal or vertical line). With these two points, a right triangle can be formed, as shown in Figure 2.13. Note that the third vertex of the triangle is (x_1, y_2). Because (x_1, y_1) and (x_1, y_2) lie on the same vertical line, the length of the vertical side of the triangle is $|y_2 - y_1|$. Similarly, the length of the horizontal side is $|x_2 - x_1|$. By the Pythagorean Theorem, the square of the distance between (x_1, y_1) and (x_2, y_2) is

$$d^2 = |x_2 - x_1|^2 + |y_2 - y_1|^2.$$

Because the distance d must be positive, you can choose the positive square root and write

$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}.$$

Finally, replacing $|x_2 - x_1|^2$ and $|y_2 - y_1|^2$ by the equivalent expressions $(x_2 - x_1)^2$ and $(y_2 - y_1)^2$ gives you the **Distance Formula.**

FIGURE 2.13 Distance between two points

Exploration

Plot the points $A(-1, -3)$ and $B(5, 2)$ and sketch the line segment from A to B. How could you verify that point $C(2, -0.5)$ is the midpoint of the segment? Why is it not sufficient to show that the distances from A to C and from C to B are equal? Find a formula for the coordinates of the midpoint of the line segment connecting (x_1, y_1) and (x_2, y_2).

The Distance Formula

The distance d between two points (x_1, y_1) and (x_2, y_2) is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Note that for the special case in which the two points lie on the same vertical or horizontal line, the Distance Formula still works. For instance, applying the Distance Formula to the points $(2, -2)$ and $(2, 4)$ produces

$$d = \sqrt{(2 - 2)^2 + [4 - (-2)]^2} = \sqrt{6^2} = 6,$$

which is the same result obtained in Example 6.

EXAMPLE 7 Finding the Distance Between Two Points

Find the distance between the points $(-1, 2)$ and $(2, 4)$, as shown in Figure 2.14.

Solution

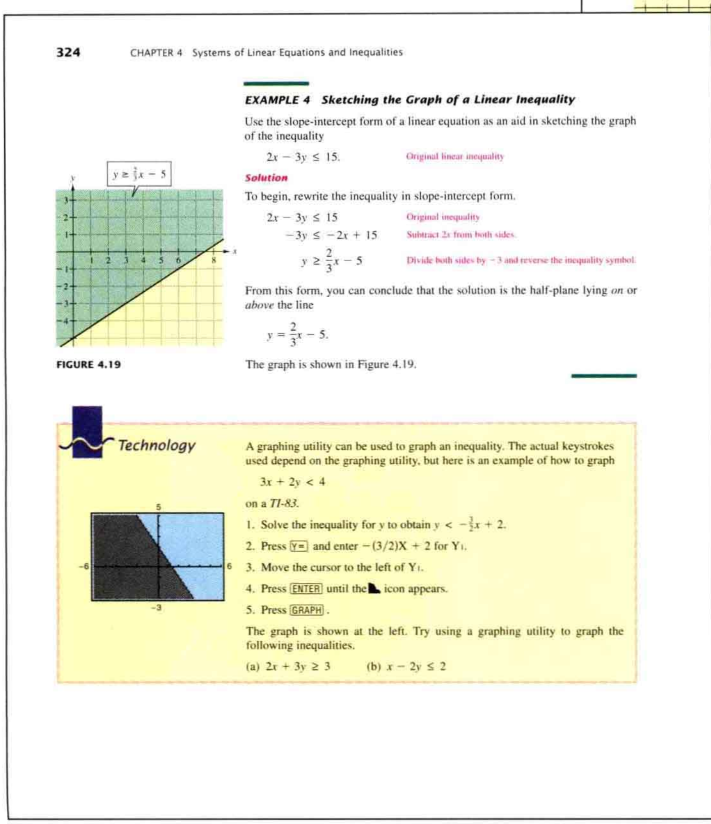
Let $(x_1, y_1) = (-1, 2)$ and $(x_2, y_2) = (2, 4)$, and apply the Distance Formula.

$d = \sqrt{[2 - (-1)]^2 + (4 - 2)^2}$ Substitute coordinates of points.

$= \sqrt{3^2 + 2^2}$ Simplify.

$= \sqrt{13}$ Simplify.

≈ 3.61 Use a calculator.


324 CHAPTER 4 Systems of Linear Equations and Inequalities

EXAMPLE 4 Sketching the Graph of a Linear Inequality

Use the slope-intercept form of a linear equation as an aid in sketching the graph of the inequality

$2x - 3y \le 15.$ Original linear inequality

Solution

To begin, rewrite the inequality in slope-intercept form.

$2x - 3y \le 15$ Original inequality

$-3y \le -2x + 15$ Subtract $2x$ from both sides.

$y \ge \frac{2}{3}x - 5$ Divide both sides by -3 and reverse the inequality symbol.

From this form, you can conclude that the solution is the half-plane lying *on* or *above* the line

$$y = \frac{2}{3}x - 5.$$

The graph is shown in Figure 4.19.

FIGURE 4.19

Technology

A graphing utility can be used to graph an inequality. The actual keystrokes used depend on the graphing utility, but here is an example of how to graph

$3x + 2y < 4$

on a *TI-83*.

1. Solve the inequality for y to obtain $y < -\frac{3}{2}x + 2$.
2. Press Y= and enter $-(3/2)X + 2$ for Y1.
3. Move the cursor to the left of Y1.
4. Press ENTER until the ◣ icon appears.
5. Press GRAPH.

The graph is shown at the left. Try using a graphing utility to graph the following inequalities.

(a) $2x + 3y \ge 3$ (b) $x - 2y \le 2$

Graphics

Visualization is a critical problem-solving skill. Graphing is introduced in Chapter 2, and from that point on, students are encouraged to use graphs to reinforce algebraic or numeric solutions, to interpret data, and to explore concepts. The numerous figures in this text—all computer generated for accuracy—help students develop these skills.

Examples

Each of the nearly 900 examples was carefully chosen to illustrate a particular concept or problem-solving technique and to enhance students' understanding. Students are taught a five-step strategy in the spirit of the AMATYC and NCTM standards, which starts with constructing a verbal model and ends with checking the answer. The examples are titled for easy reference, and comments adjacent to the solutions offer additional explanations.

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EXAMPLE 9 Constructing a Verbal Model

Find two consecutive integers such that the sum of the first integer and three times the second is 87.

Solution

Verbal Model: First integer + 3 · Second integer = 87

Labels: First integer = n
Second integer = $n + 1$

Equation:
$n + 3(n + 1) = 87$ Algebraic model
$n + 3n + 3 = 87$ Distributive Property
$4n + 3 = 87$ Combine like terms.
$4n = 84$ Subtract 3 from both sides.
$n = 21$ Divide both sides by 4.

Thus, the first integer is 21, and the second integer is $21 + 1 = 22$. You can check this by substituting 21 and 22 as the two consecutive integers in the original problem.

Study Tip

It is helpful to break a verbal sentence into parts separated by the word "is." In application problems, "is" often represents an equal sign. To write a verbal model, first identify where the word "is" appears in the sentence.

EXAMPLE 10 A Percent Application

A real estate agency receives a commission of $8092.50 for the sale of a $124,500 house. What percent commission is this?

Solution

Verbal Model: Commission = Percent (decimal form) · Sale price

Labels: Commission = 8092.50 (dollars)
Percent = p (decimal form)
Sale price = 124,500 (dollars)

Equation:
$8092.50 = p(124{,}500)$
$\frac{8092.50}{124{,}500} = p$
$0.065 = p$

The real estate agency receives a commission of 6.5%. Use your ca[...] check this solution in the original statement of the problem.

Applications

A rich and varied selection of real-world applications are integrated throughout the text in examples and in exercises. These applications offer students a constant review of problem-solving skills and emphasize the relevance of the mathematics. Many of the applications use current, real data, and are titled for easy reference.

Group Activities NEW

Group Activities appear at the end of each section. They encourage students to think, talk, and write about mathematics in a peer-assisted learning environment.

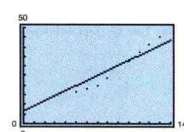
SECTION 6.6 Modeling Data with Quadratic Functions 459

Application

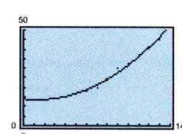
EXAMPLE 4 Finding a Quadratic Model

The total amounts A (in millions of tons) of solid waste materials recycled in the United States in selected years from 1980 through 1993 are shown below. Decide whether a linear model or a quadratic model better fits the data. Then use the model to predict the amount that will be recycled in the year 2000. In the list of data points (t, A), t represents the year, with $t = 0$ corresponding to 1980. *(Source: Franklin Associates, Ltd.)*

(0, 14.5), (5, 16.4), (6, 18.3), (7, 20.1), (8, 23.5), (9, 29.9), (10, 32.9), (11, 37.3), (12, 41.5), (13, 45.0)



Linear



Quadratic

FIGURE 6.27

Solution

Begin by entering the data into a calculator or computer that has least squares regression programs. Then run the regression programs for linear and quadratic models.

Linear: $y = ax + b$ $a = 2.624$ $b = 6.684$
Quadratic: $y = ax^2 + bx + c$ $a = 0.243$ $b = -0.655$ $c = 14.037$

From the graphs in Figure 6.27, you can see that the quadratic model fits better. (The correlation coefficient is $r = 0.994$, which implies that the model is a good fit to the data.) From this model, you can predict the amount that will be recycled in the year 2000 to be

$A = 0.243(20)^2 - 0.655(20) + 14.037 = 98.137$ million tons,

which is more than two times the amount recycled in 1993.

Group Activities

Problem Solving

Modeling Data The amounts y (in gallons) of bottled water consumed in the United States in the years 1980 through 1993 are listed below. The data is given as ordered pairs of the form (t, y), where t is the year, with $t = 0$ representing 1980. Create a scatter plot of the data. With others in your group, decide which type of model best fits the data. Then find the model.

(0, 2.4), (1, 2.7), (2, 3.0), (3, 3.4), (4, 4.0), (5, 4.5), (6, 5.0), (7, 5.7), (8, 6.5), (9, 7.4), (10, 8.0), (11, 8.0), (12, 8.2), (13, 9.2)

Technology NEW

Students are encouraged to use a graphing utility as a tool for exploration, discovery, and problem solving. Many opportunities to visualize concepts, to discover alternative approaches, to execute computations or programs, and to verify the results of other solution methods using technology are integrated throughout the text at point of use. However, students are not required to have access to a graphing utility to use this text effectively. In addition to describing the benefits of using technology, the text also pays special attention to its possible misuse or misinterpretation.

denominators and is called the **least common denominator** (or **LCD**) of the original rational expressions. Once the rational expressions have been written with like denominators, you can simply add or subtract the rational expressions using the rule given at the beginning of this section.

Technology

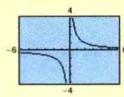
You can use a graphing utility to check your results when adding or subtracting rational expressions. In Example 5, for instance, try graphing the equations

$$y_1 = \frac{7}{6x} + \frac{5}{8x}$$

and

$$y_2 = \frac{43}{24x}$$

in the same viewing rectangle. If the two graphs coincide, as shown below, you can conclude that the solution checks.



EXAMPLE 5 Adding with Unlike Denominators

Add the rational expressions: $\frac{7}{6x} + \frac{5}{8x}$.

Solution

By factoring the denominators, $6x = 2 \cdot 3 \cdot x$ and $8x = 2^3 \cdot x$, you can conclude that the least common denominator is $2^3 \cdot 3 \cdot x = 24x$.

$$\frac{7}{6x} + \frac{5}{8x} = \frac{7(4)}{6x(4)} + \frac{5(3)}{8x(3)} \quad \text{Rewrite fractions using least common denominator.}$$
$$= \frac{28}{24x} + \frac{15}{24x} \quad \text{Like denominators}$$
$$= \frac{28 + 15}{24x} \quad \text{Add fractions.}$$
$$= \frac{43}{24x} \quad \text{Simplified form}$$

EXAMPLE 6 Subtracting with Unlike Denominators

Subtract the rational expressions: $\frac{3}{x-3} - \frac{5}{x+2}$.

Solution

The only factors of the denominators are $(x - 3)$ and $(x + 2)$. Therefore, the least common denominator is $(x - 3)(x + 2)$.

$$\frac{3}{x-3} - \frac{5}{x+2} = \frac{3(x+2)}{(x-3)(x+2)} - \frac{5(x-3)}{(x-3)(x+2)}$$
$$= \frac{3x+6}{(x-3)(x+2)} - \frac{5x-15}{(x-3)(x+2)}$$
$$= \frac{(3x+6) - (5x-15)}{(x-3)(x+2)}$$
$$= \frac{3x+6-5x+15}{(x-3)(x+2)}$$
$$= \frac{-2x+21}{(x-3)(x+2)}$$

NOTE In Example 1, the solutions are rational numbers, which means that the equation could have been solved by factoring. Try solving the equation by factoring.

Solving Equations by the Quadratic Formula

When using the Quadratic Formula, remember that *before* the formula can be applied, you must first write the quadratic equation in standard form.

EXAMPLE 1 The Quadratic Formula: Two Distinct Solutions

$$x^2 + 6x = 16 \quad \text{Original equation}$$
$$x^2 + 6x - 16 = 0 \quad \text{Write in standard form.}$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{Quadratic Formula}$$
$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(-16)}}{2(1)} \quad \text{Substitute: } a = 1, b = 6, c = -16.$$
$$x = \frac{-6 \pm \sqrt{100}}{2} \quad \text{Simplify.}$$
$$x = \frac{-6 \pm 10}{2} \quad \text{Simplify.}$$
$$x = 2 \text{ or } x = -8 \quad \text{Solutions}$$

The solutions are 2 and -8. Check these in the original equation. Or, try using a graphic check, as shown in Figure 6.6.

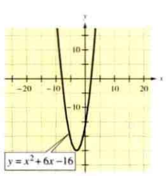


FIGURE 6.6

Study Tip

If the leading coefficient of a quadratic equation is negative, we suggest that you begin by multiplying both sides of the equation by -1, as shown in Example 2. This will produce a positive leading coefficient, which is less cumbersome to work with.

EXAMPLE 2 The Quadratic Formula: Two Distinct Solutions

$$-x^2 - 4x + 8 = 0 \quad \text{Leading coefficient is negative.}$$
$$x^2 + 4x - 8 = 0 \quad \text{Multiply both sides by } -1.$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{Quadratic Formula}$$
$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(-8)}}{2(1)} \quad \text{Substitute: } a = 1, b = 4, c = -8.$$
$$x = \frac{-4 \pm \sqrt{48}}{2} \quad \text{Simplify.}$$
$$x = \frac{-4 \pm 4\sqrt{3}}{2} \quad \text{Simplify.}$$
$$x = \frac{2(-2 \pm 2\sqrt{3})}{2} \quad \text{Divide out common factor.}$$
$$x = -2 \pm 2\sqrt{3} \quad \text{Solutions}$$

The solutions are $-2 + 2\sqrt{3}$ and $-2 - 2\sqrt{3}$. Check these in the original equation.

Notes

Many instructional notes accompany definitions, rules, and examples to give additional insight or describe generalizations.

Study Tips NEW

Study Tips help students avoid common errors, address special cases, and expand upon concepts. They appear in the margin at point of use.

Exercises

The nearly 7000 section exercises contain numerous computational and applied problems dealing with a wide range of topics. The exercise sets are designed to build competence, skill, and understanding; each exercise set is graded in difficulty to allow students to gain confidence as they progress. Each pair of consecutive problems is similar, with the answers to the odd-numbered problems given at the end of the text. Detailed solutions to all odd-numbered exercises are given in the *Student Study and Solutions Guide.*

SECTION 6.4 Applications of Quadratic Equations **441**

6.4 Exercises

See Warm-Up Exercises, p. A45

1. *Unit Analysis* Describe the units of the product.

$\frac{9 \text{ dollars}}{\text{hour}} \cdot (20 \text{ hours})$

2. *Unit Analysis* Describe the units of the product.

$\frac{20 \text{ feet}}{\text{minute}} \cdot \frac{1 \text{ minute}}{60 \text{ seconds}} \cdot (45 \text{ seconds})$

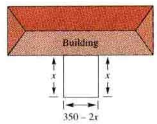
Number Problems In Exercises 3–6, find two positive integers that satisfy the requirement.

3. The product of two consecutive integers is 8 less than 10 times the smaller integer.
4. The product of two consecutive integers is 80 more than 15 times the larger integer.
5. The product of two consecutive even integers is 50 more than 3 times the larger integer.
6. The product of two consecutive odd integers is 22 less than 15 times the smaller integer.

Dimensions of a Rectangle In Exercises 7–16, complete the table of widths, lengths, perimeters, and areas of rectangles.

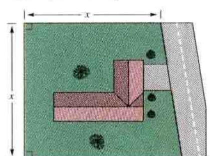
	Width	*Length*	*Perimeter*	*Area*
7.	$0.75l$	l	42 in.	
8.	w	$1.5w$	40 m	
9.	w	$2.5w$		250 ft^2
10.	w	$1.5w$		216 cm^2
11.	$\frac{1}{3}l$	l		192 in.^2
12.	$\frac{3}{4}l$	l		2700 in.^2
13.	w	$w + 3$	54 km	
14.	$l - 6$	l	108 ft	
15.	$l - 20$	l		$12{,}000 \text{ m}^2$
16.	w	$w + 5$		500 ft^2

17. *Lumber Storage Area* A retail lumberyard plans to store lumber in a rectangular region adjoining the sales office (see figure). The region will be fenced on three sides and the fourth side will be bounded by the wall of the office building. Find the dimensions of the region if 350 feet of fencing is available and the area of the region is to be 12,500 square feet.



18. *Fencing the Yard* You have 100 feet of fencing. Do you have enough to enclose a rectangular region whose area is 630 square feet? Is there enough to enclose a circular region of area 630 square feet? Explain.

19. *Fencing the Yard* A family has built a fence around three sides of their property. In total, they used 550 feet of fencing. By their calculations, the lot is one acre (43,560 square feet). Is this correct? Explain your reasoning.



Warm-Ups

For each text section (except for Section 1.1), there is a corresponding set of ten Warm-Up exercises in the Appendix, as indicated by an icon. The Warm-Ups enable students to review and practice the previously learned skills necessary to master the new skills presented in the section. Answers to Warm-Ups appear in the Appendix as well.

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3.2 Exercises

See Warm-Up Exercises, p. A41

1. *Falling Object* In an experiment, students measured the speed s (in meters per second) of a falling object t seconds after it was released. The results are given in the table.

t	0	1	2	3	4
s	0	11.0	19.4	29.2	39.4

A model for the data is $s = 9.7t + 0.4$.

(a) Plot the data and graph the model on the same set of coordinate axes.

(b) Create a table showing the given data and the approximations given by the model.

(c) Use the model to predict the speed of the object after falling 5 seconds.

(d) Interpret the slope in the context of the problem.

2. *Cable TV* The average monthly basic rate R (in dollars) for cable TV for the years 1989 through 1994 in the United States is given in the table. *(Source: Paul Kagan Associates, Inc.)*

Year	1989	1990	1991
R	15.21	16.78	18.10

Year	1992	1993	1994
R	19.08	19.39	21.62

A model for the data is $R = 1.17t + 16.61$, where t is the time in years, with $t = 0$ corresponding to 1990.

(a) Plot the data and graph the model on the same set of coordinate axes.

(b) Create a table showing the given data and the approximations given by the model.

(c) Use the model to predict the average monthly basic rate for cable TV for the year 2000.

(d) Interpret the slope in the context of the problem.

3. *Property Tax* The property tax in a township is directly proportional to the assessed value of the property. The tax on property with an assessed value of $17,072 is $1067.

(a) Find a mathematical model that gives the tax T in terms of the assessed value v.

(b) Use the model to find the tax on property with an assessed value of $11,500.

(c) Determine the tax rate.

4. *Revenue* The total revenue R is directly proportional to the number of units sold x. When 25 units are sold, the revenue is $6225.

(a) Find a mathematical model that gives the revenue R in terms of the number of units sold x.

(b) Use the model to find the revenue when 32 units are sold.

(c) Determine the price per unit.

5. *The English and Metric Systems* The label on a roll of tape gives the amount of tape in inches and centimeters. These amounts are 500 and 1270.

(a) Use the information on the label to find a mathematical model that relates inches to centimeters.

(b) Use part (a) to convert 15 inches to centimeters.

(c) Use part (a) to convert 650 centimeters to inches.

(d) Use a graphing utility to graph the model in part (a). Use the graph to confirm the results in parts (b) and (c).

6. *The English and Metric Systems* The label on a bottle of soft drink gives the amount in liters and fluid ounces. These amounts are 2 and 67.63.

(a) Use the information on the label to find a mathematical model that relates liters to fluid ounces.

(b) Use part (a) to convert 27 liters to fluid ounces.

(c) Use part (a) to convert 32 fluid ounces to liters.

(d) Use a graphing utility to graph the model in part (a). Use the graph to confirm the results in parts (b) and (c).

1.2 Exercises

See Warm-Up Exercises, p. A39

In Exercises 1–26, name the property of real numbers that justifies the statement.

1. $3 + (-5) = -5 + 3$
2. $-5(7) = 7(-5)$
3. $5(2a) = (5 \cdot 2)a$
4. $5 + 0 = 5$
5. $(10 + 8) + 3 = 10 + (8 + 3)$
6. $7(9 + 15) = 7 \cdot 9 + 7 \cdot 15$
7. $(5 + 10)(8) = 8(5 + 10)$
8. $3 + (12 - 9) = (3 + 12) - 9$
9. $25 + 35 = 35 + 25$
10. $7 \cdot 1 = 7$
11. $(-4 \cdot 10) \cdot 8 = -4(10 \cdot 8)$
12. $3(6 + b) = 3 \cdot 6 + 3 \cdot b$
13. $3x + 0 = 3x$
14. $8y \cdot 1 = 8y$
15. $25 - 25 = 0$
16. $10x \cdot \frac{1}{10x} = 1$
17. $6(-10) = -10(6)$
18. $2(6 \cdot 3) = (2 \cdot 6)3$
19. $10(2x) = (10 \cdot 2)x$
20. $0 + 8w = 8w$
21. $\frac{1}{y} \cdot y = 1$
22. $4 \cdot \frac{1}{4} = 1$
23. $1 \cdot (5t) = 5t$
24. $(x + 1) - (x + 1) = 0$
25. $3(2 + x) = 3 \cdot 2 + 3x$
26. $(6 + x) - m = 6 + (x - m)$

In Exercises 27–36, use the property of real numbers to fill in the missing part of the statement.

27. Associative Property of Multiplication
$3(6y) =$ ▒
28. Commutative Property of Addition
$10 + (-6) =$ ▒
29. Commutative Property of Multiplication
$15(-3) =$ ▒
30. Associative Property of Addition
$6 + (5 - y) =$ ▒
31. Distributive Property
$5(6 + z) =$ ▒
32. Distributive Property
$-3(4 + x) =$ ▒
33. Commutative Property of Addition
$25 + (-x) =$ ▒
34. Additive Inverse
$13x - 13x =$ ▒
35. Multiplicative Identity
$(x + 8) \cdot 1 =$ ▒
36. Additive Identity
$(8x) + 0 =$ ▒

True or False? In Exercises 37–40, decide whether the statement is true or false. Explain.

37. $-6x + 6x = 0$
38. $-9 + 5 = -5 + 9$
39. $6(7 + 2) = 6(7) + 2$
40. $-4(8 + 1) = -4(8) - 4(1)$

41. *Think About It* Does every real number have a multiplicative inverse? Explain.
42. *Think About It* What is the additive inverse of a real number? Give an example of the Additive Inverse Property?

In Exercises 43–50, give (a) the additive inverse and (b) the multiplicative inverse of the quantity.

43. 10
44. 18
45. -16
46. -52
47. $6z,\ z \neq 0$
48. $2y,\ y \neq 0$
49. $x + 1,\ x \neq -1$
50. $y - 4,\ y \neq 4$

In Exercises 51–58, rewrite the expression using the Associative Property of Addition or the Associative Property of Multiplication.

51. $(x + 5) - 3$
52. (z −
53. $32 + (-4 + y)$
54. 15 +
55. $3(4 \cdot 5)$
56. (10
57. $6(2y)$
58. 8(3x

True or False NEW

To help students understand the logical structure of algebra, a set of True or False questions is included toward the end of selected exercise sets. These questions help students focus on concepts, common errors, and the correct statements of definitions and rules.

Think About It NEW

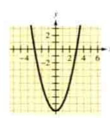
These exercises are thought-provoking, conceptual problems that help students grasp underlying theories.

Graphing Utilities

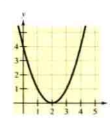
Many exercises in the text can be solved using technology; however, the symbol ▦ identifies all exercises in which students are specifically instructed to use a graphing utility. Students are encouraged to use scientific and graphing calculators to discover patterns, to experiment, to calculate, and to create graphic models.

In Exercises 63–66, explain how the x-intercepts of the graph correspond to the solutions of the polynomial equation when $y = 0$.

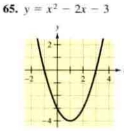
63. $y = x^2 - 9$



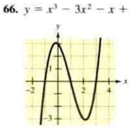
64. $y = x^2 - 4x + 4$



65. $y = x^2 - 2x - 3$



66. $y = x^3 - 3x^2 - x + 3$



▦ In Exercises 67–70, use a graphing utility to graph the equation and find any x-intercepts of the graph. Verify algebraically that any x-intercepts are solutions of the polynomial equation when $y = 0$.

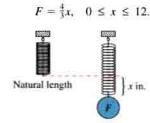
67. $y = \frac{1}{2}x - 2$
68. $y = -3x + 6$
69. $y = x^2 - 6x$
70. $y = x^2 - 11x + 28$

▦ In Exercises 71–80, use a graphing utility to solve the equation graphically.

71. $7 - 2(x - 1) = 0$
72. $2x - 1 = 3(x + 1)$
73. $4 - x^2 = 0$
74. $x^2 + 2x = 0$
75. $x^2 - 2x + 1 = 0$
76. $1 - (x - 2)^2 = 0$
77. $2x^2 + 5x - 12 = 0$
78. $(x - 2)^2 - 9 = 0$
79. $x^3 - 4x = 0$
80. $2 + x - 2x^2 - x^3 = 0$

81. *Hooke's Law* The force F (in pounds) to stretch a spring x inches from its natural length is given by

$$F = \tfrac{4}{3}x,\quad 0 \le x \le 12.$$



(a) Use the model to complete the table.

x	0	3	6	9	12
F					

(b) Sketch a graph of the model.

(c) Use the graph in part (b) to determine how the length of the spring changes each time the force is doubled.

▦ 82. *Dairy Farms* The number of farms in the United States with milk cows has been decreasing. The numbers of farms N (in thousands) for the years 1988 through 1994 are given in the table.

Year	1988	1989	1990	1991	1992	1993	1994
N	216	203	193	181	171	159	150

A model for this data is

$$N = -11.0t + 192.9,$$

where t is the time in years, with $t = 0$ corresponding to 1990. *(Source: U.S. Department of Agriculture)*

(a) Use a graphing utility to plot the data and graph the model.

(b) How well does the model represent the data? Explain your reasoning.

(c) Use the model to predict the number of farms with milk cows in 1997.

(d) Explain why the model may not be accurate in the future.

Section Projects NEW

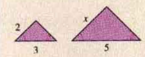
Section Projects appear at the end of every exercise set. These extended applications are often multi-part exercises that make use of real data to develop critical-thinking and problem-solving skills. Section Projects are designed for individual or group assignments.

SECTION 3.3 Applications of Linear Equations 219

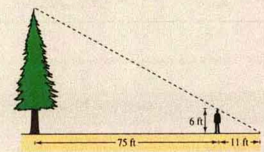
Section Project

Proportions You can answer each of the following questions by writing and solving a proportion using the fact that corresponding sides of similar triangles are proportional.

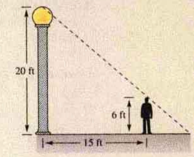
Geometry Solve for the length x of the side of the triangle (see figure).



Tree Height A man who is 6 feet tall walks directly toward the tip of a shadow of a tree. When the man is 75 feet from the tree, he notices his own shadow beyond the shadow of the tree. Find the height of the tree, if the length of the shadow of the tree beyond this point is 11 feet (see figure).



Shadow Length Find the length of the shadow of a man who is 6 feet tall and is standing 15 feet from a streetlight that is 20 feet high (see figure).



98 CHAPTER 1 Concepts of Elementary Algebra

Chapter Summary

After studying this chapter, you should have acquired the following skills. These skills are keyed to the Review Exercises that begin on page 99. Answers to odd-numbered Review Exercises are given in the back of the book.

• Plot real numbers on a real number line and compare them by using inequality symbols. *(Section 1.1)*	**Review Exercises 1–4**
• Evaluate expressions containing operations with real numbers. *(Section 1.1)*	**Review Exercises 5–24, 47**
• Identify the rule of algebra that is illustrated by an equation. *(Sections 1.2, 1.3)*	**Review Exercises 25–30**
• Expand expressions using the Distributive Property. *(Sections 1.2, 1.3)*	**Review Exercises 31–34**
• Simplify expressions by removing symbols of grouping. *(Section 1.3)*	**Review Exercises 35–38**
• Simplify expressions by applying the properties of exponents. *(Section 1.3)*	**Review Exercises 39–44**
• Solve problems involving geometry. *(Sections 1.3, 1.4)*	**Review Exercises 45, 46, 129, 130**
• Use expressions or equations to solve real-life problems. *(Sections 1.1, 1.8)*	**Review Exercises 48, 127, 128**
• Interpret graphs representing real-life data. *(Sections 1.1–1.4, 1.7)*	**Review Exercises 49, 50**
• Simplify expressions by performing arithmetic operations. *(Section 1.4)*	**Review Exercises 51–66**
• Multiply polynomials using the special product formulas. *(Section 1.4)*	**Review Exercises 67–74**
• Factor expressions completely. *(Sections 1.5, 1.6)*	**Review Exercises 75–96**
• Solve linear equations. *(Section 1.7)*	**Review Exercises 97–104, 115, 116, 120, 121, 123, 124**
• Solve literal equations. *(Section 1.7)*	**Review Exercises 105, 106**
• Solve polynomial equations. *(Section 1.8)*	**Review Exercises 107–114, 117–119, 122, 125, 126**
• Use a calculator to evaluate expressions containing operations with real numbers. *(Section 1.1)*	**Review Exercise 131**

Chapter Summary NEW

The Chapter Summary reviews the skills covered in the chapter. Section references make this an effective study tool, and correlation to the Review Exercises offers guided practice.

Mid-Chapter Quiz NEW

Each chapter contains a Mid-Chapter Quiz. This feature allows students to perform a self-assessment midway through the chapter. Answers to Mid-Chapter Quizzes appear at the end of the text.

520 CHAPTER 7 Rational Expressions and Rational Functions

Mid-Chapter Quiz

Take this quiz as you would take a quiz in class. After you are done, check your work against the answers given in the back of the book.

1. Determine the domain of the rational expression $\frac{y+2}{y(y-4)}$.
2. Evaluate the rational function $h(x) = (x^2 - 9)/(x^2 - x - 2)$ as indicated. If not possible, state the reason.
(a) $h(-3)$ (b) $h(0)$ (c) $h(-1)$ (d) $h(5)$

In Exercises 3–8, write the expression in reduced form.

3. $\frac{9y^2}{6y}$ 4. $\frac{8u^3v^2}{36uv^3}$ 5. $\frac{4x^2-1}{x-2x^2}$
6. $\frac{(z+3)^2}{2z^2+5z-3}$ 7. $\frac{7ab+3a^2b^2}{a^2b}$ 8. $\frac{2mn^2-n^3}{2m^2+mn-n^2}$

Review Exercises

The Review Exercises at the end of each chapter offer students an opportunity for additional practice. Answers to all the odd-numbered Review Exercises appear at the end of the text.

550 CHAPTER 7 Rational Expressions and Rational Functions

Review Exercises

In Exercises 1–4, find the domain of the rational expression.

1. $\frac{3y}{y-8}$ 2. $\frac{t+4}{t+12}$
3. $\frac{u}{u^2-7u+6}$ 4. $\frac{x-12}{x(x^2-16)}$

In Exercises 5–12, simplify the rational expression.

5. $\frac{6x^4y^2}{15xy^2}$ 6. $\frac{2(y^3z)^2}{28(yz^2)^2}$
7. $\frac{5b-15}{30b-120}$ 8. $\frac{4a}{10a^2+26a}$
9. $\frac{9x-9y}{y-x}$ 10. $\frac{x+3}{x^2-x-12}$
11. $\frac{x^2-5x}{2x^2-50}$ 12

23. $\frac{x^2-7x}{x+1} \div \frac{x^2-14x+49}{x^2-1}$
24. $\left(\frac{6x}{y^2}\right)^2 \div \left(\frac{3x}{y}\right)^3$
25. $\frac{4}{9} - \frac{11}{9}$ 26. $\frac{2(3y+4)}{2y+1} + \frac{3-y}{2y+1}$
27. $\frac{15}{16} - \frac{5}{24} - 1$ 28. $-\frac{3}{8} + \frac{7}{6} - \frac{1}{12}$
29. $\frac{1}{x+5} + \frac{3}{x-12}$ 30. $\frac{2}{x-10} + \frac{3}{4-x}$
31. $5x + \frac{2}{x-3} - \frac{3}{x+2}$
32. $4 - \frac{4x}{x+6} + \frac{7}{x-5}$

Chapter Test

Each chapter contains an end-of-chapter test for students to assess their progress. Answers appear at the end of the text.

CHAPTER TEST 553

Chapter Test

Take this test as you would take a test in class. After you are done, check your work against the answers given in the back of the book.

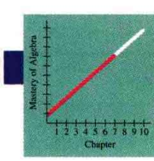
1. Find the domain of the rational expression $\frac{3y}{y^2-25}$.
2. Simplify the rational expression $\frac{2-x}{3x-6}$.

In Exercises 3–11, perform the operation(s) and simplify.

3. $\frac{4z^3}{5} \cdot \frac{25}{12z^2}$ 4. $\frac{y^2+8y+16}{2(y-2)} \cdot \frac{8y-16}{(y+4)^3}$
5. $(4x^2-9) \cdot \frac{2x+3}{2x^2-x-3}$ 6. $\frac{(2xy^2)^3}{15} \div \frac{12x^3}{21}$
7. $\left(\frac{3x}{x+2}\right)$ 8. $\left(9x - \frac{1}{x}\right)$ 9. $2x + \frac{1-4x^2}{}$

Cumulative Test

In this edition, Cumulative Tests have been placed at the end of each chapter (except Chapter 1). These tests reinforce the message that is presented throughout the text—that mathematics is a continuing story and requires constant synthesis and review. Answers appear at the end of the text.



Cumulative Test: Chapters 1–7

Take this test as you would take a test in class. After you are done, check your work against the answers given in the back of the book.

In Exercises 1 and 2, simplify the expression.

1. $5(x+2) - 4(2x-3)$ 2. $0.12x + 0.05(2000 - 2x)$

In Exercises 3 and 4, use the function to find and simplify the expression for $f(a+2)$.

3. $f(x) = x^2 - 3$ 4. $f(x) = \frac{3}{x+5}$

In Exercises 5 and 6, simplify the rational expression.

5. $\frac{-16x^2}{12x}$ 6. $\frac{6u^5v^{-3}}{27uv^3}$

In Exercises 7–9, perform the operation and simplify. (Assume that all variables

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If you have suggestions for improving this text, please feel free to write to us. Over the past two decades we have received many useful comments from both instructors and students, and we value these very much.

Roland E. Larson
Robert P. Hostetler
Carolyn F. Neptune

Supplements

Intermediate Algebra: Graphs and Functions, Second Edition by Larson, Hostetler, and Neptune is accompanied by a comprehensive supplements package. Most items are keyed directly to the text.

Printed Resources for the Instructor

Instructor's Annotated Edition by Larson, Hostetler, and Neptune
- Includes the entire student edition of the text
- Answers to all exercises and tests
- Teaching tips at point of use
- Additional examples and exercises with answers at point of use

Instructor's Guide by Carolyn F. Neptune, Johnson County Community College
- Detailed solutions to all even-numbered Section Exercises
- Transparency Masters

Test Item File by David C. Falvo, The Pennsylvania State University, The Behrend College
- Over 4,000 test items keyed to the text by section and organized by objective
- Six Chapter Tests per chapter
- Questions given in both multiple-choice and fill-in formats
- Answers to all test items and to chapter tests
- Also available as a computerized test bank

Printed Resources for the Student

Student Study and Solutions Guide by Carolyn F. Neptune, Johnson County Community College
- Step-by-step solutions to all odd-numbered Section Exercises, all Review Exercises, and all Mid-Chapter Quiz, Chapter Test, and Cumulative Test problems

Graphing Technology Guide: Algebra by Benjamin N. Levy and Laurel Technical Services
- Keystroke instructions for a wide variety of Texas Instruments, Casio, Sharp, and Hewlett-Packard graphing calculators, including the most current models
- Examples with step-by-step solutions
- Extensive graphics screen output
- Technology tips

Media Resources for the Instructor

Computerized Testing

- Test-generating software for Windows and Macintosh
- Over 4,000 test items
- Also available as a printed test bank

Media Resources for the Student

Videotape Series by Dana Mosely

- Comprehensive section-by-section coverage
- Detailed explanation of important concepts
- Numerous examples and applications, often illustrated by computer-generated graphics

Tutorial Software

- Interactive tutorial software with comprehensive section-by-section coverage
- Diagnostic feedback
- Additional examples
- Chapter self-tests
- Glossary

How to Study Algebra

Studying Mathematics Studying mathematics is a linear process: The material you learn each day builds upon material you learned previously. There are no shortcuts—you must keep up with the coursework every day.

Making a Plan Make your own course plan right now! A good rule of thumb is to study two to four hours for every hour in class. After your first major test, you will know if your efforts were sufficient. If you did not make the grade you wanted, then you should increase your study time, improve your study efficiency, or both.

Preparing for Class Before class, review your notes from the previous class. Then, read the portion of the text that is to be covered, paying special attention to the definitions and rules that are highlighted. This takes self-discipline, but it pays off because you will benefit much more from your instructor's presentation.

Attending Class Attend every class. Arrive on time with your text, a pen or pencil and paper for notes, and your calculator. If you must miss a class, get the notes from another student, go to your tutor for help, or view the appropriate mathematics videotape. You *must* learn the material that was covered in the missed class before attending the next class.

Participating in Class As you read the text before class, write down any questions you may have about the material. Ask your instructor these questions during class. This way, you will understand the material better, and you will be prepared to do your homework.

Taking Notes During class, take notes on definitions, examples, concepts, and rules. Focus on the instructor's cues to identify important material. Then, as soon after class as possible, review your notes and add any explanations that are necessary to make your notes understandable *to you.*

Doing the Homework Learning algebra is like learning to play the piano or basketball. You cannot develop skills just by watching someone do it; you must do it yourself. The best time to do your homework is right after class, when the concepts are still fresh in your mind. This increases your chances of retaining the information in long-term memory.

Finding a Study Partner When you get stuck on a problem, it may help to work with a partner. Even if you feel you are giving more help than you are getting, you will find that teaching others is an excellent way to learn.

Building a Math Library Build a library of books that can help you with your math courses. Consider using the *Student Study and Solutions Guide* for this text. As you will probably take other math courses after this one, we suggest that you keep the text. It will be a valuable reference book. Adding computer software and math videotapes is another way to build your math library.