



**KENNETH C. WILLIAMS**

**INTRODUCTION TO  
GAME THEORY**  
**A BEHAVIORAL APPROACH**

**INTERNATIONAL  
EDITION**

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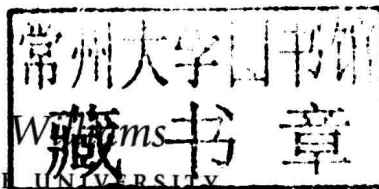
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A BEHAVIORAL APPROACH

International Edition

*Kenneth C. Williams*  
MICHIGAN STATE UNIVERSITY



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# PREFACE

**B**ehavioral game theory uses psychological reasoning to explain behavior within a game theory model and tests this behavior using laboratory experiments. As will become evident from this book, I truly believe that many (although not all) social science research problems lend themselves to an axiomatic treatment and empirical testing. I understand that research methods are an acquired taste and many people do not share my view, so this book is an attempt to expose behavioral game theory to a wider audience.

I believe (as do others) that the reason that game theory and laboratory experiments are not commonplace in the social sciences is because they have a reputation of being difficult because of the math they involve. The point of this book is to show that math should not be an obstacle, because the basic game theory material is fairly easy to learn and the math is often not the key component of the concepts. Rather, game theory is a way of thinking logically about a problem and formulating research questions based on theory. This method forces a researcher to strip down a problem to its core elements to expose its internal machinery. This allows us to carefully inspect how the parts of the problem fit and operate together to produce some outcome. Experiments can be used to observe whether the parts work in the way that they were predicted to work. In my opinion, this approach is important, especially for young scholars, because it forces them to dissect and systematically study problems.

Behavioral game theory requires a knowledge of basic game theory and basic experimental methods. Outside of psychology, few students in the social sciences learn about experimental methods, and outside of economics, few students learn about game theory. The blending of the two areas provides a unique way to study human decision-making. This book attempts to teach the basic material in a nontechnical way that is accessible to a lay audience. I find that repetitive learning is useful with this material, since people who are being exposed to its ideas for the first time can easily become confused. Consequently, there are some concepts that I attempt to “hammer home” by repeating them several times in slightly different ways in the hope that the various explanations might make the concept make more sense.

I consider the problem sets at the end of each chapter to be an integral part of the overall book. The problem sets are designed so that the reader can get a hands-on feel of how game theory models and experiments actually work, since they require the reader to construct models and think about designs to test them. While some of the problems are more challenging than others, I hope that if a solution is not apparent, readers will take this approach: think about how the problem should have been asked, and then solve that problem. This book is a result of courses that I have taught on research methods, experimental methods, game theory, and behavioral game theory over the years. As all instructors know, some concepts work perfectly in the classroom and others are a disaster. There is a learning curve in developing a course, in which the more the course is taught, the more disastrous material can be removed and replaced with more appropriate material. I have taken courses from Peter Diamond and Drew Fudenberg, so I have seen minor flubs in the classroom from the best. My course, as well as this book, has undergone this transformation, and I must apologize to students who took my early courses on this topic and those students and reviewers who read an early draft of this manuscript.

Since this book is mostly concerned with displaying the merits of laboratory experiments in the study of human behavior, one of the biggest problems I encountered was selecting experiments to be included in the various sections. My goal was to select a diverse set of experiments so that the reader could be exposed to a wide range of different experimental designs. Consequently, some of the experiments I have included may seem odd or peripheral to a particular area. My criteria for selecting the experiments were that the experiment had to be easy to understand, and that the experiments should be different from each other in terms of design as much as possible. Although I know that I have excluded many experiments that should have been included, I hope that my selection provides the readers with exposure to a diverse range of experiments.

Some people criticize game theory for its simplicity, especially regarding the rationality assumption. My view of rationality—and a view that I have emphasized in this book—is that the assumptions of rationality are a modeling technique that allows us to model fundamental human behavior using math functions. Game theory models are concerned with institutions, so we have to have a way to model decision-making behavior in order to study the impact of institutions on behavior. Rationality is not a theory about how real people behave when they make decisions in a consistent manner, but a modeling technique that allows us to mimic a fundamental aspect of human decision choice using a mathematical function.

I like to think of this book as a collaborative effort with my students and the many anonymous reviewers who took the time to write very detailed comments. I am very thankful to the publisher for selecting a very diverse set of reviewers so that I received a diverse range of comments. I attempted to be meticulous about responding to every comment, since I knew that if someone felt compelled to write a comment on an issue, then I needed to respond to that issue. Even those comments that I initially felt were misguided ultimately became very useful in helping me clarify my discussion. There were two reviewers in particular who spent a considerable amount of time on early drafts of this manuscript, giving me very detailed constructive and critical comments and pointing out the errors (of which there were many) that I had made. You know who you are, and I thank you.

I would also like to take the chance here to say a few goodbyes, the first being to one of my advisors, Mel Hinich. I have met a few true geniuses in my lifetime, and I would have to put Mel near the top of that list. The reason that he surpassed others was his command of knowledge in an amazingly large number of diverse fields. Looking at the breadth of his research, it is almost impossible to classify him. He understood that true knowledge came from the integration of different fields of study. He once told me that almost all researchers hide in their own research holes and fail to look up and see what other researchers are doing in their holes. He felt that some of these holes could be connected, and that this connection was the only way that we could discover truth. I can take solace in the fact that his influence on me was common knowledge between us. In a similar vein, I would like to pay homage to Richard McKelvey, with whom I had the honor of working on an experimental project as a graduate student. I recall that I was supposed to conduct the experiment, and he was rather perturbed and felt that I should not be doing it, since I knew the theory behind the experiment. This was a valuable lesson about the seriousness of experimental controls. Lastly, I would like to pay my respects to Haywood Alker, whose narratives still mystify me. I would also be remiss if I did not thank my other advisors, especially my head advisor Peter Ordeshook, and the also important Mat McCubbins (my Austin home), Benjamin Page, and Brian Roberts. Also around helping out during my grad school days were Gary Cox (my other Austin home), Mike Munger, Terry Sullivan, Tom Schwartz, Jim Sidanius, and Gavin Duffy (who introduced me to artificial intelligence).

I also want to thank my editor Jennifer Carpenter and her assistant Maegan Sherlock for helping to develop this project, and Keith Faivre for editing assistance. It's a luxury to write without having to worry about page limits! Of course, I need to thank Becky Morton, who over the years has forced me to think "a lot" about "the proper way" to conduct experiments. Thanks also to Sugato Dasgupta for fixing some mistakes for me, and to Rick Wilson for advice about in-class experiments from which I "borrowed" some designs. I would also like to thank the following reviewers for their feedback: Philip Arena, University at Buffalo; Anna Bassi, University of North Carolina at Chapel Hill; John Morgan, University of California, Berkeley; and Alex Weisiger, University of Pennsylvania. Finally, I would like to thank my family, especially my parents Elijah and Julia and brothers Reggie and Greg, for all of the support they have given me over the years. I would like to extend a special thanks to my wife Marcie, the Soviet historian and lawyer who really doesn't "get" the social science thing but always provides useful comments and even suggested the most common names she knew, Olga and Igor, for the featured players in this book. Finally, I wanted to thank my daughter Katie for her patience in understanding that Dad's games are not really fun games to play.



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