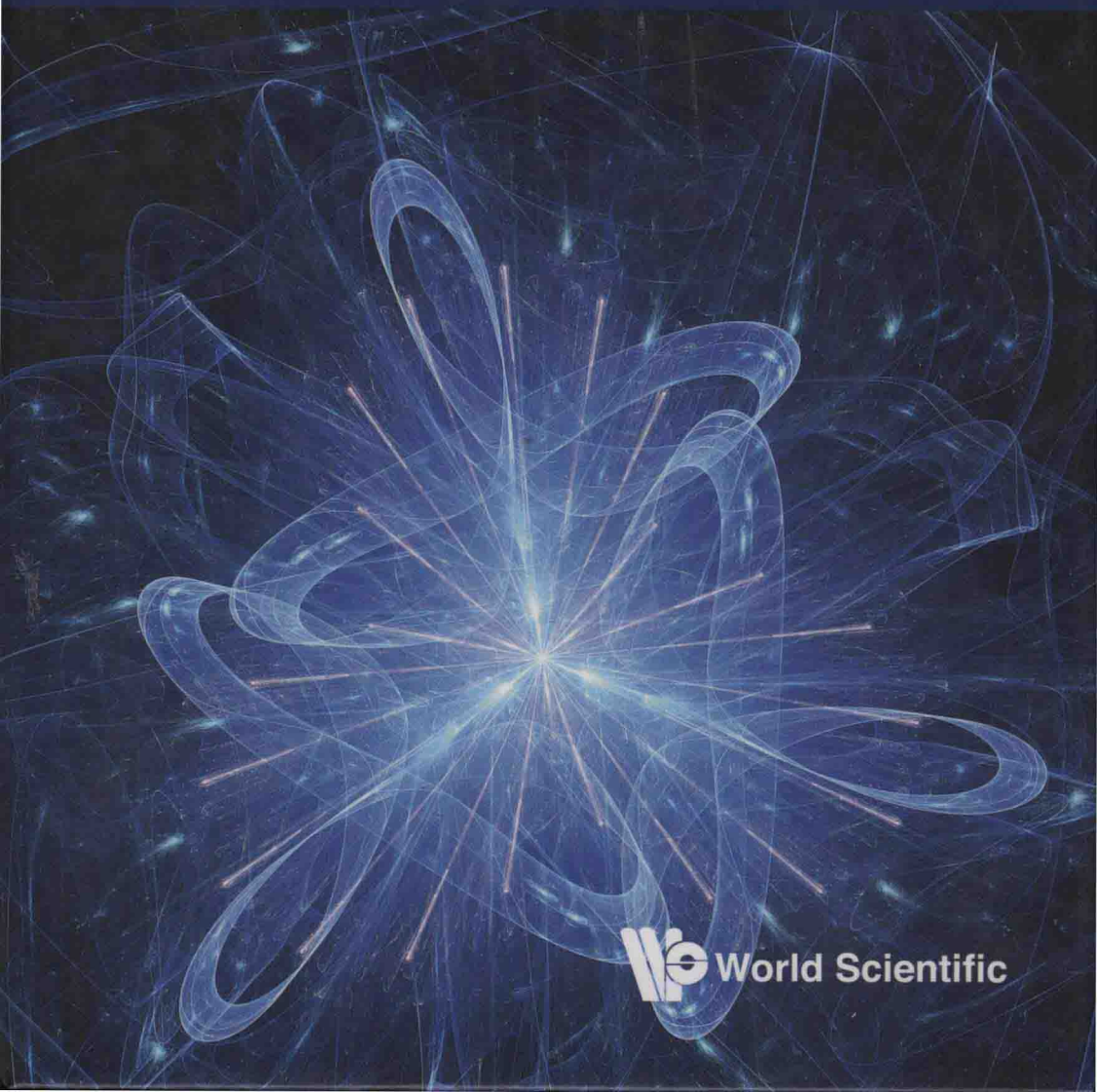
DIFFERENTIABILITY AND FRACTALITY IN DYNAMICS OF PHYSICAL SYSTEMS

Ioan Merches • Maricel Agop



World Scientific

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The true sign of intelligence
is not knowledge but imagination.

Albert Einstein

Preface

This monograph is a collection of ideas developed by several authors on differentiability and fractality, taken as basic frames of approaching the study of physical systems dynamics. Here are some results we want to put into evidence, together with the specific "arsenal" of methods and procedures.

Using the 1-forms in Cartan sense theory, we are offering an elegant procedure of analyzing the motion, in order to obtain the main results of the theory of relativity, and some new results as well. This way, the form of the inertia transformations — *i.e.* the geometric structure of the space-time "aggregate" — is not at all prejudiced. In other words, the irreducible attributes of motion of a body, two bodies, or a solid rigid are dependent on a set of Euclidean invariants. Within such an approach, without abandoning the space-time standard frame, gravitation is re-found as an inertial mass interaction. In addition, a repulsive interaction — responsible for Hubble's dilatation — is found; this interaction is very sensitive at intergalactic distances.

The wave-corpuscle duality, reducible to the equivalence of two 1-forms, inertial and ondulator, imply motions on curves of constant informational energy. This way not only quantization is statuated, but also dynamics implying velocity limits — not necessarily the speed of light in vacuum.

Motion of a charged particle in a combined field (composed by an electromagnetic and a magnetic constant fields) mimes by numeric

simulation not only various scenarios of evolution towards chaos (period doubling, bifurcations on subharmonics, etc.), but also pattern generation (motion on stationary orbits, etc.). Under these circumstances, trajectories are continuous and non-differentiable curves (fractal curves), and this fact imposes a new space-time geometry, *i.e.* the fractal geometry.

There is also developed a dynamics of the physical object in a fractal space-time, together with some instructive and expressive applications: the fractal model of the atom, the potential gap, the linear harmonic oscillator, the free particle, Heisenberg's uncertainty relations, etc.

The physical systems that display chaotic behavior are recognized to acquire self-similarity and manifest strong fluctuations at all possible scales. Since fractality appears as an universal property of these systems, it is necessary to construct a new physics, either through non-differentiability (Nottale's scale relativity model), or through fractional derivative (fractional physics model). In the first variant, an extended model of Nottale's theory (Scale relativity theory with an arbitrary constant fractal dimension) is established and some applications (fractal model of the atom, particle in a box, harmonic oscillator and free particle) are given. In the second variant, the theory of fractional scale relativity and some applications (fractal covariant mechanics induced by modified scale laws, fractional Schrödinger equation, and emergence of complex gravity) are obtained and discussed.

At the end of this introductory presentation we want to emphasize the fact that our endeavor is addressed to researches, scientists and all those working in the field. Even if this book is not a textbook, it can successfully be used by MSc and PhD students interested in this unlimited and fascinating subject.

The authors express their gratitude to Professor Ieronim Mihaila, from the Faculty of Mathematics, University of Bucharest, for his competent advice during elaboration of the book.

* * *

We dedicate this monograph to the memory of Professor Octav Onicescu, prominent researcher and scientist, member of the Romanian Academy.

The Authors
Iasi, November 2014

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