An Introduction to

Mechanics of Materials

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Alpha Science

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Mechanics of Materials

For Krishna, Mishti, Raghu,
Abeer and their Naani

PREFACE

This book has been written in the belief that an introductory course in mechanics of material is a fit place to create a technology and a design focus in an engineering programme even when treating a subject as an engineering science. The engineering science approach is woven around a clear elaboration of the central scheme of dealing with this subject, namely, delinking the geometry aspects of the subject from the materials aspects. This is achieved by using explicitly the three-step scheme of macro (forces) to micro (stresses) conversion, transforming at the micro level (from stresses to strains), and then converting back to the macro level (deformations), or vice versa. The author has found in his experience with teaching this subject over a number of years that this approach provides a student with a clear and firm framework that guides her or him in the development of the subject.

The design focus is achieved by making a student size simple structural elements from the very beginning. The subject becomes quite exciting when a beginning student is able to determine the size of simple things in a straight forward manner. The approach is strengthened through properly designed problems.

The technology focus is brought about by referring to a large number of applications, to standards, and to methods of determining the mechanical properties. Footnotes have been used extensively to extend the discussions and applications.

The level of mathematics has been deliberately kept quite low. A physical approach to elaboration of concepts has been preferred.

The organisation of the material is, accordingly, quite different from those in many textbooks. The chapter on transformation of stress and strain has been shifted back to after calculation of bending stresses so that relevant applications can be handled meaningfully. Energy methods have been introduced very early, but have been so organized such that they can be ignored if not needed. The concepts of indeterminate structures have been introduced from the very beginning to fit into the basic engineering science approach adopted throughout.

The book contains more material than can reasonably be covered in one semester. It has been included in the belief that some readers might wish to over-reach. Sections 1.9, 2.5 – 2.11, 3.8 – 3.11, 5.3, 5.7 – 5.9, 7.5 and 8.5 can perhaps be left out of a one-semester taught course, though much of this is very application oriented.

I must acknowledge here the contribution of innumerable colleagues at IIT Kanpur with whom discussions over many years helped frame my view of the subject. I must thank Kusum, my wife, who kept me at the job of finishing the task.

Vijay Gupta

CONTENTS

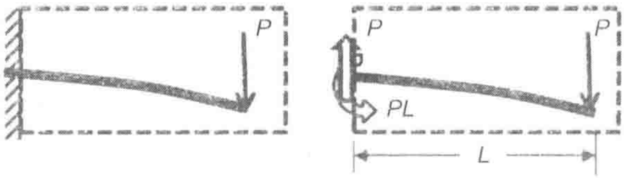
1

STRUCTURES, LOADS AND STRESSES

1.1 MECHANICS OF MATERIAL

The subject matter of a course on mechanics of materials deals with structures. A table or a chair is a structure. A building is a structure. A bridge is a structure. A TV tower is a structure. So is a printed circuit board, the casing of a fax machine, or the body of a car. Among the many purposes of the various structures, one common purpose is to resist and/or transmit forces acting on a structure. By resisting a force we mean that the structure would not break down under the force. The structure of a building is designed to resist the loads which include the weight of the people and things occupying the building, the forces of wind acting on it in a storm, even the load imposed by an earthquake, and the self-load of the building itself. The structure of an aeroplane resists the aerodynamic loads, the weight of its occupants (including the dynamic loads during acceleration and deceleration), the loads due to gusts and turbulence it might fly through, the load imposed by the thrust produced by the engines and, of course, the weight of the structure itself.

How does a structure resist loads? Consider the simple case of a cantilever beam loaded as shown in Fig. 1.1. If the beam is in equilibrium, the net force or moment on the beam or on any part of it must be zero. Let us consider the part of the beam within the dashed-line box shown. It shows the structure without its supports. Instead we show in this all the forces that the *surroundings* apply on the structure. These include, besides the applied load P, the reactions at the support. This is known as a free-body diagram (FBD) or an *isolation* of the structure.



(a) A loaded beam (b) Part of the beam

Fig. 1.1

Clearly, this part of the beam is not in equilibrium with just the external force, P. We need additional external (i.e., external to this part of the beam) forces and/or moments. The open arrows in Fig. 1.1b show the external force and moment required to balance the applied load P. We will, for the time being, refer to these as the *reaction* forces and the moments.

Where do these forces and moment come from?

As we apply the force P to the beam and if these reactions do not kick in, the beam will bend and tend to shear from the stump built into the wall at the left-end. The distortion of the beam so produced results in generation of material forces within the beam that resist this shearing action. When we consider the part of the beam shown in the free body of Fig. 1.1b, these material forces appear as external forces (and moments) on the beam. Of course, there are equal and opposite reaction on the stump of the beam built in the wall.

We can summarize the above as:

- The external forces acting on a structure result in deformation of the structural members.
- The deformations so caused result in resisting forces *within* the material of the members.
- When we consider the equilibrium of a part of the member, these internal forces come into play as external forces and balance the applied forces or moments.

1.2 DEFORMATION AND RESISTING FORCES

Consider a vertical rod anchored as shown in Fig. 1.2. It is common knowledge that when you apply a longitudinal force P to this rod, it elongates a definite amount (depending on its dimensions and its material). Consider a portion of the rod enclosed by the broken-line rectangle. The free-body diagram (FBD) of this part is shown in Fig. 1.2b. Since the rod is in equilibrium after the elongation, there must be a force that balances the applied force P. Where does that force come from? Clearly, there are internal forces which are holding this part of the rod from running away from the rest of the rod. These *internal forces*, as seen in the previous section, are the consequence of the distortion produced in the bar. Now more the force we apply, more is the elongation, suggesting that the resisting force that develops in the rod depends on the elongation. Robert Hooke, a British scientist is credited to be the first to explore the relationship between this resisting force and the elongation. He found out in the year 1678 that for a *given material*, the resisting force does not depend on the elongation but on the relative elongation produced. He introduced the term *strain* to denote the elongation relative to the original length of the bar. If l denotes the original length, and δ the elongation, the strain[1] is defined by

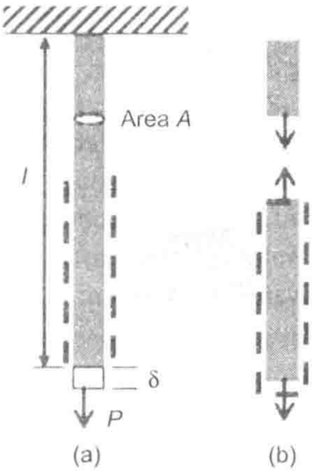


Fig. 1.2 A bar loaded longitudinally

Strain, $\varepsilon = \delta / l$ (1.1)

Strain is dimensionless and has no units.

Hooke also found out that it is not the force, but the *intensity* of force measured as the *force per unit area* that is related to strain. He called it stress. If A is the area of cross-section of the bar, the stress[2] is given by the resisting force P divided by A.

Stress, $\sigma = P/A$ (1.2)

[1]Since the strain in structural members is generally quite small, of the order of 10^{-3} or 10^{-4}, it is common to express strain using the symbol μ to represent a strain equal to 10^{-6}. Thus, a strain of 120 μ equals 120×10^{-6}.

[2]Note here that the stress is the intensity of the *resisting force P*, and not of the loading P. The two are equal only in this simple case of uni-axial loading. It further assumes that the load *intensity* is uniform across the section. A more general formulation of stress is introduced in Section 1.4.

The stress and strain of this example are termed as tensile, and the member is said to be in tension.

Stress has the dimensions of force per unit area (hydraulic pressure, too, has the same dimensions) and has SI units of Newton per meter squared (N/m^2), which is termed as *Pascal* and is abbreviated as Pa.

He further found that the stress and strain have, *in a large part*, a simple *linear* relationship for bars made of the same material:

Stress α Strain, or, $\sigma = E\varepsilon$ (1.3)

The constant of proportionately, E, is termed as the *elastic modulus*, and depends on the material of the bar. Strain being dimensionless, the dimensions (and units) of E are the same as those of the stress.

Combining Eqs. 1.1–1.3, we get:

$$\delta = \frac{PL}{AE} \tag{1.4}$$

The value of the elastic modulus E for most construction materials is quite high, denoting that it takes fairly large forces to produce small elongations. Table A.1 in Appendix A gives the values of the elastic modulus for some common materials. Steel has about the largest value of the elastic modulus of about 200 GPa[3]. Cast iron has about half this value. Aluminium is still lower at 70 GPa.

The summary statements of the previous section can now be recast as:

- The external forces acting on a structure result in strains in the structural members.
- The strains so produced result in stresses within the material of the members.
- The stresses, for the most part, are proportional to the strains produced.
- The constant of proportionality is termed as the modulus of elasticity.

Another point to note is that the *strength* of a structure depends on the stresses that develop within the structure. A structure fails either when it ruptures or when the deformation exceeds acceptable limits. Both conditions are characterised by the level of stresses within the structure. It will be seen in the next chapter that there are two quantities (which are properties of material, rather than of the structure) that are used to denote the strength: the *yield stress*, which is the stress level at which a structure (or parts of it) develops significant deformations, and the *ultimate stress*, at which (a member of) the structure has catastrophic failure. A structure is designed such that the stress in no part of it exceeds yield stress if limiting deformation is the goal, or the ultimate stress if we can tolerate deformations but not breakage.

[3] A GPa is 10^9 Pa, or 10^9 N/m^2