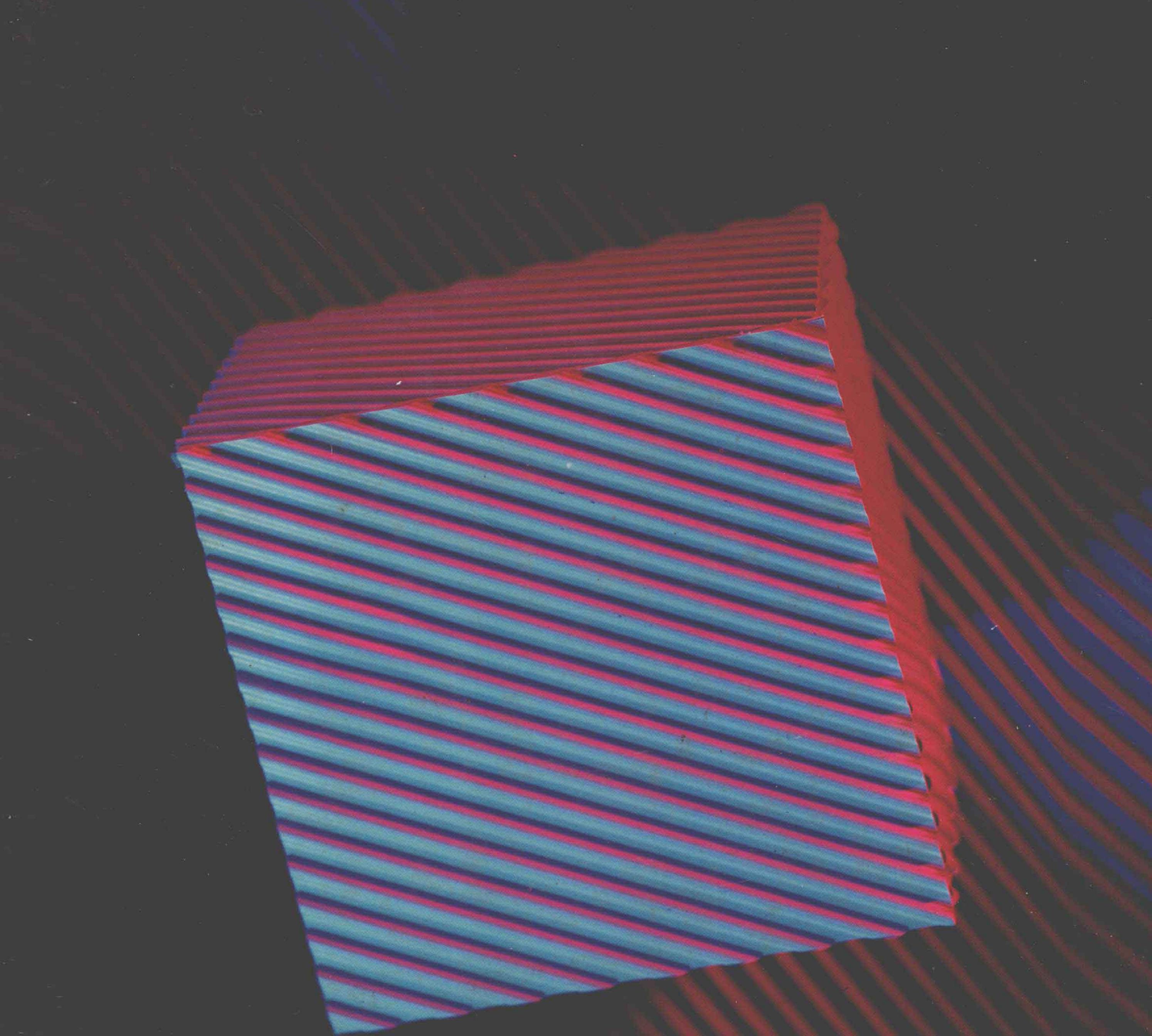
COLLEGE ALGEBRA

A GRAPHING APPROACH

DEMANA / WAITS



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A GRAPHING APPROACH

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ADDISON-WESLEY PUBLISHING COMPANY

Reading, Massachusetts • Menlo Park, California • New York
Don Mills, Ontario • Wokingham, England • Amsterdam • Bonn
Sydney • Singapore • Tokyo • Madrid • San Juan

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This book was produced with TEXtures.

Library of Congress Cataloging-in-Publication Data

Demana, Franklin D., 1938–
College algebra, a graphing approach/ by Franklin Demana and Bert K. Waits; with the assistance of Alan Osborne and Gregory D. Foley.

p. cm.
Includes index.
ISBN 0-201-52811-8
1. Algrebra–Graphic methods. I. Waits, Bert K. II. Title.
QA155.5.D45 1989
512.0285–dc20 89-6790

ABCDEFGHIJ-MU-954321089

The authors dedicate this book to their wives, Christine Demana and Barbara Waits, without whose patience, love, and understanding this book would not have been possible.

Preface

This text is designed for a two-quarter or one-semester college algebra course. It was written because the authors believe that conventional texts that do not incorporate technology do not prepare students well for further study in mathematics and science in the 1990s. For example, only about 17% of the entering freshmen at The Ohio State University with four or more years of high school college-preparatory mathematics, including a precalculus course, are ready to begin their collegiate study of mathematics with calculus. This is consistent with national evidence that performance in college calculus courses is dismal at best.

Calculator- and computer-based graphing technology is incorporated in this text to enhance the teaching and learning of precalculus mathematics. Students are expected to have *regular and frequent* access to graphing calculators or computers with appropriate software for homework outside of class and perhaps for occasional classroom laboratory activities as well. Use of computer graphing or graphing calculator technology in this text is *not* optional.

Modern technology has evolved to the stage that it should be routinely used by mathematics students at all levels. Computer- or calculator-based graphing removes the need for contrived problems and opens the door for students to explore and solve realistic and interesting applications. The teaching and learning of traditional topics can be improved with the full use of these new tools. Computer- and calculator-based technology can turn the mathematics classroom into a mathematics laboratory. Technology gives rise to interactive instructional approaches that permit a focus on problem solving and encourage generalizations based on strong geometric evidence. The new instructional approaches that are possible with today's technology make you and your students active partners in an exciting, rewarding, enjoyable, and intensive educational experience. It is in this spirit of exploration and experimentation that this text is written.

Content Features

- **Applications** Problems are used to introduce and develop much of the mathematics in this text. Students using this approach become flexible problem solvers. Using problems as a basis for discussion makes the mathematics understandable to students, and students come to value mathematics because they appreciate its power. More realistic applications are possible because of the speed and power of technology (see Example 4 of Section 3.5 and Example 5 of Section 9.6). You can select exercises with a business and economics theme, a science theme, or other themes.

- **Graphs** In the past careful, time consuming numerical and analytical techniques were used to produce accurate graphs that were rarely used. Now accurate graphs are obtained quickly and used to study the numerical and analytical properties of functions (see Example 5 of Section 6.3). Important questions are often generated by students viewing graphs. Sometimes these questions are answered using algebraic manipulation, which is then often well received by students. Other times they are answered by student exploration, taking advantage of the speed and power of graphing technology (see Example 1 of Section 5.5).

- **Scale** Scale is a crucial issue. The shape of a graph depends on the viewing window or rectangle in which the graph is viewed (see Example 6 of Section 1.2). Care in selecting viewing rectangles must be exercised because familiar graphs may not be recognized when they are distorted.

- **Multiple Representations** Algebraic representations (equations, inequalities, etc.) and geometric representations (graphs of algebraic representations) are established for a given problem situation (see Figure 1.3.1 of Section 1.3). Then connections among algebraic representations, geometric representations, and the problem situation are exploited to provide understanding about mathematical concepts and to give a geometric base for the algebraic ideas. Modeling receives special attention in this text (see Example 1 of Section 1.3).

- **Foreshadowing Calculus** Maxima and minima of functions are found in this text by using graphs. Intervals where functions are increasing or decreasing (see Example 5 of Section 3.6) and limiting behavior of functions are determined graphically. We do *not* borrow the techniques of calculus – rather we lay the foundation for the later study of calculus by providing students with rich intuitions and understandings about functions and graphs (see Example 5 of Section 4.4).

- **Answers** Emphasis on exact answers is reduced, and approximate answers are emphasized. Technology provides a proper balance between exact answers that are rarely needed in the real world and approximate answers. What is usually needed is an answer with prescribed accuracy (see Example 14 of Section 2.2). Graphing techniques, such as zoom-in, provide an excellent geometric vehicle for discussing error in answers. Students can read answers from graphs with accuracy up to the limits of machine precision.

- **Algebraic Manipulation** You will find a good deal of ordinary algebraic manipulation in this text. However, the algebraic techniques often arise from problem situations or are used to answer questions generated by graphs (see Example 2 of Section 8.4). Students are more willing to perform algebraic manipulations when they are developed in context but are *not* the focus of a lesson.

- **Geometric Transformations** The exploratory nature of graphing technology helps students learn how a given graph may be obtained from a basic graph using the following geometric transformations: horizontal or vertical shifting, horizontal stretching or shrinking, vertical stretching or shrinking, reflection with respect to a coordinate axis. This technology-enhanced approach develops students' abilities so that they can sketch graphs of complicated functions (see Example 1 of Section 3.4, Example 6 of Section 4.4, Example 4 of Section 5.3, Example 5 of Section 5.7, and Example 6 of Section 6.6).

- **Trigonometric Functions** Students using a technology-enhanced approach quickly understand the common features of the graphs of a complete class of functions. We are able to use this understanding to reduce the time spent on trigonometric functions because their graphs can be determined quickly. Some instructors in the field test of the preliminary version of this text were uncomfortable with reducing the time spent on these topics because of their past experiences. Others had no trouble when they remained faithful to the new approach. We deliberately spend less time on identities and believe this to be appropriate.

Pedagogical Features

Each chapter begins with an introductory overview of the material to be covered. The first paragraph of a section gives an overview of the section. The text requires considerably more reading than conventional texts. Students are expected to read the text and work through the examples.

- **Exercises** Directions for some exercises state *not* to use graphing technology. Whenever possible, it is good practice to have students verify responses to these exercises with graphing technology. Many exercises will require the use of technology. We have *not* written an example for each type of exercise given. Some exercises that are more difficult or extend the ideas of the section are marked with an asterisk. There are more exercises than can be reasonably assigned to any one student. The exercises were designed to provide you with flexibility to accommodate students with different backgrounds and interests.

- **Intensity** The extensive examples found within a section, coupled with the interrelated explanations, provide for an intensity of topic coverage. Thus, it is usually not possible to cover a section in one class meeting. Some examples should be left as reading exercises for students. You may find it helpful sometimes to work one problem through completely even if it takes the entire class, especially early in the course. However, you may find it necessary to omit some topics. For those who need a one-quarter or one-semester course, *Precalculus: Functions and Graphs* by Franklin Demana and Bert K. Waits gives a briefer version of the material contained in this text.

- **The Role of the Instructor Using Technology** The role of an instructor changes from a lecturer to a facilitator of learning. Students become active partners in the learning process as a consequence of technology. They learn to explore and experiment with mathematical concepts because of the speed and power of technology. You can use a single computer or an overhead graphing calculator in the classroom to provide an interactive lecture-demonstration, or use a guided-discovery approach in a computer lab or in a classroom where each student has a graphing calculator. You can use realistic problems to motivate and teach mathematical concepts because technology makes such problems accessible. Technology provides a much richer classroom environment that fosters student involvement in the educational experience. You should encourage students to explore and experiment with the technology.

- **Visualization** Graphing helps students learn how to see and describe graphs and their characteristics. You use the power of visualization by carefully selecting a sequence of visual experiences to help students understand or discover mathematical concepts. Graphing technology makes the addition of geometric representations to the usual numeric and algebraic representations very natural. Exploring the connections between representations and problems deepens student understanding about mathematics and helps students value mathematics.

- **Review** Each chapter ends with a list of key terms introduced in that chapter and an extensive set of exercises. These exercises can be used to prepare students for exams or used for quizzes.

Supplements for the Instructor

- **Instructor's Manual** The Instructor's Manual contains an introductory chapter that gives an extensive overview and advice about using a technology-enhanced approach. There is a chapter-by-chapter commentary including remarks about selected exercises. Two versions are given for each chapter test. The Instructor's Manual contains answers to all exercises. The text contains the answers to approximately half of the exercises – usually the odd ones.

- **Tests** The Instructor's Manual that accompanies this text contains two versions of each chapter test.

- **Software** Instructors of class-size adoptions of this text receive *Master Grapher*, a powerful, interactive computer software package designed by the authors that graphs functions, conic equations, parametric equations, polar equations, and functions of two variables. *Master Grapher* comes in versions that work on Apple II, Macintosh, IBM, and most IBM-compatible computers.

- **Graphing Calculator and Computer Graphing Laboratory Manual** This laboratory manual gives details about the use of graphing calculators and computers to support the text. Students also receive a copy of the manual. It contains a user's guide to the *Master Grapher* graphing software as well as guides to the use of a Casio or Sharp graphing calculator.

A Request

For many of you using this text, incorporating technology into the teaching of mathematics is a new venture. You are a pathfinder in the true sense of the word and, consequently, will enjoy some highs when instruction goes well but some lows when your experiences do not match your expectations.

We would like to hear from you concerning your successes and frustrations. This is partly to guide the design of a revision of this text and partly to gain understanding of what it means for instructors and institutions to shift to using technology for mathematics teaching.

Acknowledgements

The authors are indebted to a great many people who participated in the development of this text. We very much appreciate the important and constructive suggestions of our colleagues Alan Osborne and Greg Foley. Special thanks are due to Janice McDonald and Barbara Waldron, our skillful TEX typists, and to Amy Edwards, who prepared the artwork for the manuscript and worked tirelessly to help the authors meet important deadlines. We also appreciate the efforts of David New, our resident TEXpert, who facilitated production of the book; and Jill Baumer, Cindy Bernlohr, Christine Browning, Sandy Davey, Tony DeGennaro, Penny Dunham, Ann Farrell, Bishnu Naraine, Jeri Nichols, Anne Sadeghipour, Linda Taylor, and Laurie Wern, who typed drafts, prepared answers, proofread, and made numerous helpful suggestions. The staff at Addison-Wesley, especially Stephanie Botvin and David Pallai, have provided enthusiastic support and expert guidance throughout the production process, making us proud to be part of the Addison-Wesley family.

We sincerely thank our colleagues who participated in the 1988–89 field test of a preliminary version of the textbook and who made valuable contributions that helped shape this book.

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Contents

1 Relations, Functions, and Graphs

• Introduction

In this chapter we introduce models, algebraic representations, geometric representations, and complete graphs—important concepts that will be used throughout the textbook. The definitions of these terms will not be precise, but their meanings will become clear as examples are given throughout the textbook. Roughly speaking, algebraic representations are equations, inequalities, or systems of equations or inequalities, and geometric representations are graphs. We use algebraic representations and geometric representations as models of problem situations. We introduce functions and relations and draw graphs in the Cartesian coordinate system in this chapter. We will explain what is meant by the domain, range, and graph of a relation, function, equation, algebraic representation, and problem situation. Functions, relations, equations, inequalities, systems of equations and inequalities, and graphs are examples of models. Models can be very complex, and generally there is no single, correct model for a given problem situation. The person analyzing the problem situation usually chooses a model to represent the problem situation. Ideally a model should contain all the pertinent information about the problem situation and not be unduly complicated.

In this textbook we pay a good bit of attention to the way the following topics are interrelated: problem situations, algebraic representations of problem situations, and their graphs. Textbooks often focus most of their attention on working with various algebraic representations that are common in mathematics and give too little attention to geometric representations and their connections with algebraic representations and problem situations. Applications and graphical problem solving are important themes in this textbook.

Functions are important models in mathematics. We use graphing calculators or computer graphing software to obtain graphs of functions and certain relations. Knowledge of the symmetry properties are used to help determine their complete graphs. The vertical line test is used to decide if the relation determined by a graph is a function. We will see that graphing is an extremely important tool in mathematics. The absolute value function, the greatest integer function, and piecewise-defined functions will be introduced in this chapter. The distance formula will be given and used to obtain an equation for a circle.

1.1 Cartesian Coordinate System and Complete Graphs

In this section we introduce the Cartesian coordinate system and explain what is meant by domain, range, and the graph of a relation, equation, and problem situation. The meaning of *complete graph* will be introduced. We also explain what is meant by an algebraic representation of a problem situation and a geometric representation of a problem situation. A geometric representation will be given for a problem situation and compared with the graph of an algebraic representation of the problem situation.

Data that arise in problem situations can be graphed. A branch of mathematics called *data analysis* is devoted to analyzing data and determining appropriate algebraic representations of data. In this textbook, data will generally be obtained from well defined algebraic relationships. In the first example we use a familiar problem situation to give an example of an algebraic representation and a geometric representation of a problem situation. In this case the algebraic representation will be an equation and the geometric representation a portion of a straight line.

• EXAMPLE 1: Quality Rent-a-Car charges \$15 plus \$0.20 a mile to rent a car. Give an algebraic representation and a geometric representation that shows the relationship between the number of miles driven and the charges.

SOLUTION: If 50 miles are driven, the charges are $0.20(50) + 15$ or \$25. If x represents the number of miles driven and y the corresponding charges in dollars, then the equation $y = 0.2x + 15$ can be used to compute additional possibilities.

x (miles driven)	50	75	100	200
y (rental charges)	25	30	35	55

This equation is an algebraic representation of the problem situation. We can represent each of the possible miles driven and corresponding charges as a point on a graph. For example, the point representing 50 miles driven ($x = 50$) and charges of \$25 ($y = 25$) is located above 50 on the horizontal axis and across from 25 on the vertical axis (Figure 1.1.1). Can any real number greater than or equal to zero be the number of miles driven? Many would say that the answer is no. They would claim that miles driven should be given as

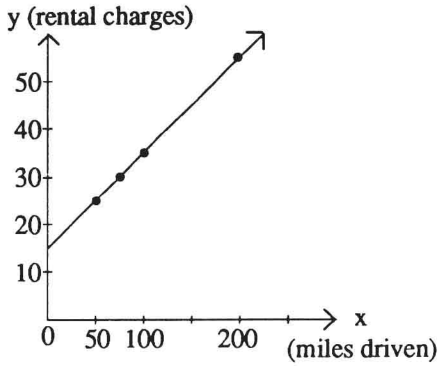

Figure 1.1.1

positive integers, or maybe as positive numbers in tenths. Regardless of your point of view, we cannot graph all the possible points because there are infinitely many. However, if you continue to plot additional points, you will be convinced that the graph of the relationship between the number of miles driven and the charges appears to be the ray indicated in Figure 1.1.1. The ray is a geometric representation of the problem situation. The arrow on the graph indicates that the graph continues in the direction suggested by the arrow, even though we can only draw a part of it on the page. •

The collection of points on the ray indicated in Figure 1.1.1 that corresponds only to possible miles driven is a *graph of the Quality Rent-a-Car problem situation* because the graph geometrically describes the complete relationship between the number of miles driven and the charges. Because of the scale in Figure 1.1.1 the graph will appear to be the same whether you use all positive real numbers, only positive numbers in tenths, or only positive integers as possible number of miles driven.

Scale

In Figure 1.1.1 each scale mark on the horizontal axis represents 50 (miles) and each scale mark on the vertical axis represents 5 (dollars). If we had chosen the scale marks on the y-axis and x-axis to represent one dollar and one mile, respectively, then we would need a very large piece of paper to display the same information that is shown in Figure 1.1.1. The scale will often have to be different on the two axes, and should be based on the size of the numbers involved in the problem situation.

We will see that we do *not* always need a graph of a problem situation to answer questions about the problem. Often a graph that contains the graph of the problem situation will be used to answer questions about the problem situation. For example, we can use the graph in Figure 1.1.1 to answer questions about renting cars from Quality Rent-a-Car.

• EXAMPLE 2: Sarah is charged $50 for renting a car at Quality Rent-a-Car. How many miles did she drive the car?

SOLUTION: To solve this problem graphically, we draw a horizontal line from the point marked 50 (\$50) on the vertical axis (rental charges) to the graph in Figure 1.1.1. Then, we draw a vertical line down to the horizontal axis (miles driven) to read the answer to this question from the graph. The result is shown in Figure 1.1.2. We estimate from the graph that Sarah drove 175 miles. In this case, 175 miles is the exact solution. This can be confirmed by direct substitution, that is, $0.2(175) + 15 = 50$, or algebraically by solving the equation $0.2x + 15 = 50$ for x. •

Next we give some of the standard terminology used in graphing.

The Real Number Line

We call $1, 2, 3, \ldots$ the **positive integers**, and $-1, -2, -3, \ldots$ the **negative integers**. The positive and negative integers together with zero form the **integers**. The positive integers together with 0 are called the **whole numbers.** A **rational number** is any number of the form $\frac{a}{b}$, where a and b are integers with $b \neq 0$. The decimal form of a rational number either terminates or is nonterminating *and* repeating. For example, $\frac{3}{4} = 0.75$ terminates and $\frac{13}{22} = 0.5909090\ldots$ is nonterminating (the digits 9 and 0 repeat forever as suggested by the pattern). A real number that is not rational is called an **irrational number**. Some examples of irrational numbers are $\sqrt{2}$, $-\sqrt{3}$, π, and $\frac{1}{\pi+1}$. Irrational numbers have infinite decimal representations that do *not* repeat; for example, $0.4404004000400004\ldots$. Notice that the number of 0's between a pair of consecutive 4's increases by one as we move from left to right through the decimal representation.

The **real number line** provides a geometric representation of the real numbers. Each real number corresponds to one and only one point on the number line, and each point on the number line corresponds to one and only one real number. We first choose a point on the number line to be labeled zero (0). Then we mark equally spaced points on each side of 0. The points to the right of 0 are labeled $1, 2, 3, \ldots$, and the points to the left of 0 are labeled $-1, -2, -3, \ldots$ (Figure 1.1.3). Every real number, including irrational numbers,

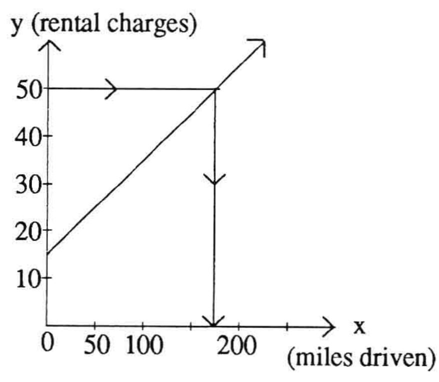

Figure 1.1.2

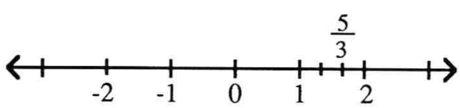

Figure 1.1.3