



Topics on Chaotic Systems

Selected Papers from CHAOS 2008
International Conference

Christos H Skiadas
Ioannis Dimotikalis
Charilaos Skiadas
editors



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International Conference

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editors

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TOPICS ON CHAOTIC SYSTEMS

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PREFACE

This collection includes the main part of the best papers presented in the International Conference (CHAOS2008) on Chaotic Modeling, Simulation and Applications, Chania, Crete, Greece, June 3–6, 2008. The task was to bring together various groups working in the area of Nonlinear Systems and Dynamics, Chaotic Theory and Application for exchanging views and reporting research findings. The topics included are relevant to the study of nonlinear systems and dynamics in an interdisciplinary research, and include many very interesting applications. This book provides a valuable collection of new ideas, methods, and techniques in the field of Nonlinear Dynamics, Chaos, Fractals and their applications in General Science and Engineering Sciences.

The book focuses on many fields such as Chaos and Dynamical Systems, Nonlinear Systems, Fractals, Chaotic Attractors, Mechanics, Hydro-Fluid Dynamics, Chaotic Advection, Chaos in Meteorology and Cosmology, Bifurcation, Hamiltonian and Quantum Chaos, Plasma Physics, Chaos in Biology and Genetics, Chaos in Medicine and Physiology, Chaotic Control, Time Series Analysis and Forecasting Chaotic Systems, Chaos in Economy and Markets, Traffic Flow, and Chaotic Simulations. These contributions present new ideas and methods for solving problems by analyzing the relevant data. Also, the use of recent advances in different fields are emphasized, especially on chaotic simulation methods and techniques.

We would like to acknowledge the valuable support and hospitality provided by the Mediterranean Agronomic Institute, Chania, Greece. Sincere thanks must also go to those whose contributions have been essential in organizing the Conference and creating these Proceedings. Finally, We would like to thank Anthi Katsirikou, Mary Karadima, Aggeliki Oikonomou and George Matalliotakis for their valuable support.

February 10, 2009

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Alexander G. Ramm

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The Effects of Machine Components on Bifurcation and Chaos as Applied to Multimachine System

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The second system of the IEEE second benchmark model of Subsynchronous Resonance (SSR) is considered. The system can be mathematically modeled as a set of first order nonlinear ordinary differential equations with the compensation factor ($\mu = X_c/X_L$) as a bifurcation (control) parameter. So, bifurcation theory can be applied to nonlinear dynamical systems, which can be written as $dx/dt = F(x; \mu)$. The effects of machine components, i.e. damper winding, automatic voltage regulator (AVR), and power system stabilizer (PSS) on SSR in power system are studied. The results show that these components affect the locations, number and type of the Hopf bifurcations.

Keywords: Hopf bifurcation; chaos; subsynchronous resonance; damper windings; PSS.

1. Introduction

In power systems series compensation is considered as a powerful technique based on economic and technical considerations for increasing effectively the power transfer capability as well as improving the stability of these systems. However, this introduces problems as well as with the benefits, namely the electromechanical interaction between electrical resonant circuits of the transmission system and the torsional natural frequencies of the turbine-generator rotor. This phenomenon is called subsynchronous resonance (SSR), and it can cause shaft fatigue and possible damage or failure.

The phenomenon of subsynchronous resonance occurs mainly in series capacitor-compensated transmission systems. SSR has been studied extensively since 1970, when a major transmission network in southern California experienced shaft failure to its turbine-generator unit with series compensation. The subsynchronous torques on the rotor is a matter of concern because the turbine-generator shaft itself has natural modes of oscillation

that are typical of any spring mass system. It happens that the shaft oscillatory modes are at subsynchronous frequencies. Should the induced subsynchronous torques coincide with one of the shaft natural modes of oscillation, the shaft will oscillate at this natural frequency, sometimes with high amplitude.

Three types of SSR can identify the interaction of the system and the generator under the subsynchronous resonance. They have been called torsional interaction effect, induction generator effect, and transient torque effect. In this research, we focus on the torsional interaction effect, which results from the interaction of the electrical subsynchronous mode with the torsional mode. Several methods have been used in SSR study. The most common of these methods are eigenvalue analysis, frequency scanning, and time-domain analysis. The eigenvalue analysis is used in this research. It is a very valuable technique because it provides both the frequencies of oscillation and the damping at each frequency.

Recently, power system dynamics has been studied using the nonlinear dynamics point of view, which utilizes the bifurcation theory. Actually, power systems have rich bifurcation phenomena. Bifurcation is used to indicate a qualitative change in the features of a system, such as the number and types of solution upon a small variation in the parameters of a system. It has been revealed that there are different types of bifurcation in power system models. In general, the power system model can be represented by a system of nonlinear algebraic and ordinary differential equations.

The bifurcation theorem was used by Zhu *et al.* [1] to demonstrate the existence of a Hopf bifurcation in a single machine infinite busbar (SMIB) power system, in which the dynamics of the damper windings and the AVR are neglected. Nayfeh *et al.* [2] applied the bifurcation theory to a practical series capacitor compensated single machine power system, the BOARD-MAN turbine-generator system. Harb *et al.* [3] applied a bifurcation analysis together with the method of multiple scales and Floquet theory to the CHOLLA #4 turbine-generator system. Tomim *et al.* [4] proposed an index that identifies Hopf bifurcation points in power systems susceptible to subsynchronous resonance. Dobson and Barocio [5] analyzed general perturbations of a weak resonance and found two distinct behaviors, including interactions near strong resonances in which the eigenvalues quickly change direction.

In this paper, we focus on the torsional interaction effect, which results from the interaction of the electrical subsynchronous mode with the torsional mode. The second system of the IEEE second benchmark model is

considered. We use bifurcation theory and chaos to investigate the complex dynamics of the considered system. The type of the Hopf bifurcation is determined by numerical integration of the system, with specific amount of initial disturbances, slightly before and after the bifurcation value. On further increase of the compensation factor, the system experiences chaos via torus attractor. Chaos is a bounded steady-state behavior that is not an equilibrium solution or a periodic solution or a quasiperiodic solution.⁶

2. System Description

The system considered is the two different machine infinite bus system, shown in Figure 1(a). The two machines have a common torsional mode connected to a single series compensated transmission line. The model and the parameters are provided in the second system of the IEEE second benchmark model.

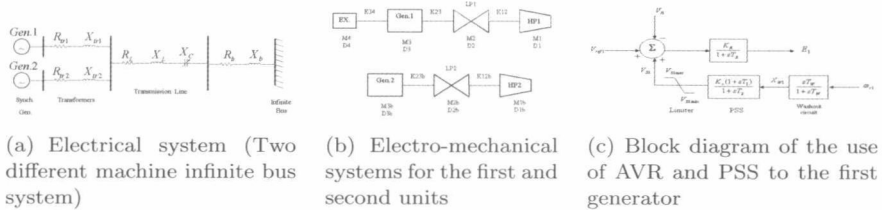


Fig. 1.

The electro-mechanical systems for the first and second units are shown in Figure 1(b). The first unit consists of exciter (EX), generator (Gen.1), low-pressure (LP1) and high-pressure (HP1) turbine sections. And the second unit consists of generator (Gen.2), low-pressure (LP2) and high-pressure (HP2) turbine sections. Every section has its own angular momentum constant M and damping coefficient D , and every pair of successive masses have their own shaft stiffness constant K , as shown in Figure 1(b). The data for electrical and mechanical system are provided in [7]. Replacement of these generators with a single equivalent generator will change the resonance characteristics and therefore is not justified. Consequently, each generator is represented in its own rotor frame of reference and suitable a transformation is made.

3. Mathematical Model

The mathematical model of the electrical and mechanical system will be presented in this section. Actually, the electrical system includes the dynamic nonlinear mathematical model of a synchronous generator and that of the transmission line. The generator model considered in this study includes five equations, d -axis stator winding, q -axis stator winding, d -axis rotor field winding, q -axis rotor damper winding and d -axis rotor damper winding equations. Each mass of the mechanical system can be modeled by a second order ordinary differential equation (swing equation), which is presented in state space model as two first order ordinary differential equations.

Using the direct and quadrature d - q axes and Park's transformation, we can write the complete mathematical model that describes the dynamics of the system as follows. For the first generator:

$$\begin{aligned}
 & - (X_{tr1} + X_L + X_b + X_{d1}) \frac{di_{d1}}{dt} - (X_L + X_b) \cos \delta_{r12} \frac{di_{d2}}{dt} \\
 & - (X_L + X_b) \sin \delta_{r12} \frac{di_{q2}}{dt} + X_{md1} \frac{di_{f1}}{dt} + X_{md1} \frac{di_{D1}}{dt} = \\
 & \omega_b \left\{ (R_{tr1} + R_L + R_b + R_{a1}) i_{d1} \right. \\
 & + \left[(R_L + R_b) \cos \delta_{r12} + \left(\frac{\omega_{r2}}{\omega_b} + 1 \right) (X_L + X_b) \sin \delta_{r12} \right] i_{d2} \\
 & - \left[\frac{\omega_{r1}}{\omega_b} (X_{tr1} + X_L + X_b) + (X_L + X_b) + \omega_{r1} X_{q1} \right] i_{q1} \\
 & + \left[(R_L + R_b) \sin \delta_{r12} - \left(\frac{\omega_{r2}}{\omega_b} + 1 \right) (X_L + X_b) \cos \delta_{r12} \right] i_{q2} + X_{mq1} \omega_{r1} i_{Q1} \\
 & \left. + e_{cd} \sin \delta_{r1} - e_{cq} \cos \delta_{r1} + V_{0D} \sin \delta_{r1} - V_{0Q} \cos \delta_{r1} \right\}
 \end{aligned} \tag{1}$$

$$\begin{aligned}
 & - (X_{tr1} + X_L + X_b + X_{q1}) \frac{di_{q1}}{dt} - (X_L + X_b) \cos \delta_{r12} \frac{di_{q2}}{dt} \\
 & + (X_L + X_b) \sin \delta_{r12} \frac{di_{d2}}{dt} + X_{mq1} \frac{di_{Q1}}{dt} = \\
 & \omega_b \left\{ (R_{tr1} + R_L + R_b + R_{a1}) i_{q1} \right. \\
 & + \left[(R_L + R_b) \cos \delta_{r12} + \left(\frac{\omega_{r2}}{\omega_b} + 1 \right) (X_L + X_b) \sin \delta_{r12} \right] i_{q2} \\
 & + \left[\frac{\omega_{r1}}{\omega_b} (X_{tr1} + X_L + X_b) + (X_L + X_b) + \omega_{r1} X_{d1} \right] i_{d1} \\
 & + \left[-(R_L + R_b) \sin \delta_{r12} + \left(\frac{\omega_{r2}}{\omega_b} + 1 \right) (X_L + X_b) \cos \delta_{r12} \right] i_{d2} - X_{md1} \omega_{r1} i_{f1} \\
 & \left. - X_{md1} \omega_{r1} i_{D1} + e_{cd} \cos \delta_{r1} + e_{cq} \sin \delta_{r1} + V_{0D} \cos \delta_{r1} + V_{0Q} \sin \delta_{r1} \right\}
 \end{aligned} \tag{2}$$

$$-X_{md1} \frac{di_{d1}}{dt} + X_{f1} \frac{di_{f1}}{dt} + X_{md1} \frac{di_{D1}}{dt} = \omega_b \left[-R_{f1} i_{f1} + \frac{R_{f1} E_{fd1}}{X_{md1}} \right] \tag{3}$$

$$-X_{mq1} \frac{di_{q1}}{dt} + X_{Q1} \frac{di_{Q1}}{dt} = -\omega_b R_{Q1} i_{Q1} \tag{4}$$

$$-X_{md1} \frac{di_{d1}}{dt} + X_{md1} \frac{di_{f1}}{dt} + X_{D1} \frac{di_{D1}}{dt} = -\omega_b R_{D1} i_{D1} \tag{5}$$

where: $v_{f1} = \frac{R_{f1}E_{fd1}}{X_{md1}}$, and for $i, j = 1, 2$ and $i \neq j$, $\sin \delta_{rij} = \sin(\delta_{ri} - \delta_{rj})$, $\cos \delta_{rij} = \cos(\delta_{ri} - \delta_{rj})$.

Similarly, for the second generator the generator model includes five equations as follows:

$$\begin{aligned} & - (X_{tr2} + X_L + X_b + X_{d2}) \frac{di_{d2}}{dt} - (X_L + X_b) \cos \delta_{r21} \frac{di_{d1}}{dt} \\ & - (X_L + X_b) \sin \delta_{r21} \frac{di_{q1}}{dt} + X_{md2} \frac{di_{f2}}{dt} + X_{md2} \frac{di_{D2}}{dt} = \\ & \omega_b \left\{ (R_{tr2} + R_L + R_b + R_{a2}) i_{d2} \right. \end{aligned} \quad (6)$$

$$\begin{aligned} & + \left[(R_L + R_b) \cos \delta_{r21} + \left(\frac{\omega_{r1}}{\omega_b} + 1 \right) (X_L + X_b) \sin \delta_{r21} \right] i_{d1} \\ & - \left[\frac{\omega_{r2}}{\omega_b} (X_{tr2} + X_L + X_b) + (X_L + X_b) + \omega_{r2} X_{q2} \right] i_{q2} \\ & + \left[(R_L + R_b) \sin \delta_{r21} - \left(\frac{\omega_{r1}}{\omega_b} + 1 \right) (X_L + X_b) \cos \delta_{r21} \right] i_{q1} + X_{mq2} \omega_{r2} i_{Q2} \\ & + e_{cd} \sin \delta_{r2} - e_{cq} \cos \delta_{r2} + V_{0D} \sin \delta_{r2} - V_{0Q} \cos \delta_{r2} \left. \right\} \\ & - (X_{tr2} + X_L + X_b + X_{q2}) \frac{di_{q2}}{dt} - (X_L + X_b) \cos \delta_{r21} \frac{di_{q1}}{dt} \\ & + (X_L + X_b) \sin \delta_{r21} \frac{di_{d1}}{dt} + X_{mq2} \frac{di_{Q2}}{dt} = \\ & \omega_b \left\{ (R_{tr2} + R_L + R_b + R_{a2}) i_{q2} \right. \end{aligned} \quad (7)$$

$$\begin{aligned} & + \left[(R_L + R_b) \cos \delta_{r21} + \left(\frac{\omega_{r1}}{\omega_b} + 1 \right) (X_L + X_b) \sin \delta_{r21} \right] i_{q1} \\ & + \left[\frac{\omega_{r2}}{\omega_b} (X_{tr2} + X_L + X_b) + (X_L + X_b) + \omega_{r2} X_{d2} \right] i_{d2} \\ & + \left[-(R_L + R_b) \sin \delta_{r21} + \left(\frac{\omega_{r1}}{\omega_b} + 1 \right) (X_L + X_b) \cos \delta_{r21} \right] i_{d1} - X_{md2} \omega_{r2} i_{f2} \\ & - X_{md2} \omega_{r2} i_{D2} + e_{cd} \cos \delta_{r2} + e_{cq} \sin \delta_{r2} + V_{0D} \cos \delta_{r2} + V_{0Q} \sin \delta_{r2} \left. \right\} \end{aligned} \quad (8)$$

$$- X_{md2} \frac{di_{d2}}{dt} + X_{f2} \frac{di_{f2}}{dt} + X_{md2} \frac{di_{D2}}{dt} = \omega_b \left[-R_{f2} i_{f2} + \frac{R_{f2} E_2}{X_{md2}} \right] \quad (9)$$

$$- X_{mq2} \frac{di_{q2}}{dt} + X_{Q2} \frac{di_{Q2}}{dt} = -\omega_b R_{Q2} i_{Q2} \quad (10)$$

$$- X_{md2} \frac{di_{d2}}{dt} + X_{md2} \frac{di_{f2}}{dt} + X_{D2} \frac{di_{D2}}{dt} = -\omega_b R_{D2} i_{D2} \quad (10)$$

where $v_{f2} = \frac{R_{f2}E_2}{X_{md2}}$.

Voltage drop across X_c :

$$\frac{de_{cd}}{dt} = \omega_b [X_c I_{LD} + e_{cq}] \quad (11)$$

$$\frac{de_{cq}}{dt} = \omega_b [X_c I_{LQ} - e_{cd}] \quad (12)$$

where:

$$I_{LD} = I_{q1} \cos \delta_1 + I_{q2} \cos \delta_2 + I_{d1} \sin \delta_1 + I_{d2} \sin \delta_2$$

$$I_{LQ} = I_{q1} \sin \delta_1 + I_{q2} \sin \delta_2 - I_{d1} \cos \delta_1 - I_{d2} \cos \delta_2$$