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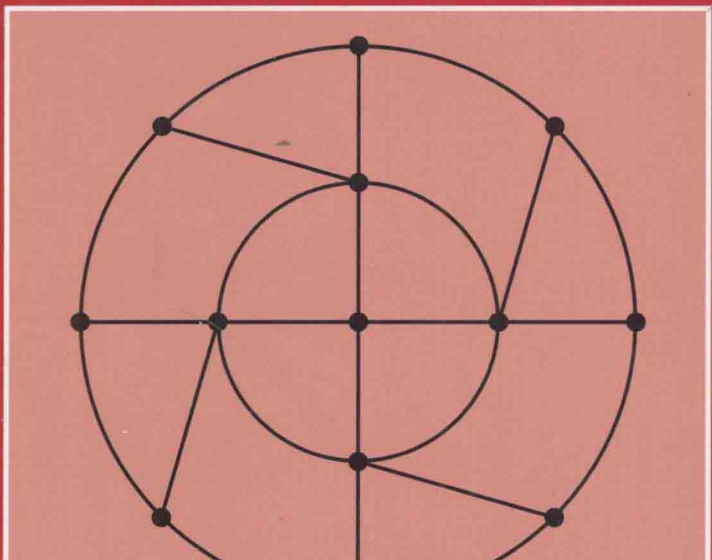
TOPICS IN CHROMATIC GRAPH THEORY

Edited by
Lowell W. Beineke and Robin J. Wilson



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Topics in Chromatic Graph Theory

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Topics in Chromatic Graph Theory

Chromatic graph theory is a thriving area that uses various ideas of 'colouring' (of vertices, edges, etc.) to explore aspects of graph theory. It has links with other areas of mathematics, including topology, algebra and geometry, and is increasingly used in such areas as computer networks, where colouring algorithms form an important feature.

While other books cover portions of the material, no other title has such a wide scope as this one, in which acknowledged international experts in the field provide a broad survey of the subject. All 15 chapters have been carefully edited, with uniform notation and terminology applied throughout. Bjarne Toft (Odense, Denmark), widely recognized for his substantial contributions to the area, acted as academic consultant.

The book serves as a valuable reference for researchers and graduate students in graph theory and combinatorics and as a useful introduction to the topic for mathematicians in related fields.

LOWELL W. BEINEKE is Schrey Professor of Mathematics at Indiana University–Purdue University Fort Wayne (IPFW), where he has worked since receiving his Ph.D. from the University of Michigan under the guidance of Frank Harary. His graph theory interests include topological graph theory, line graphs, tournaments, decompositions and vulnerability. He has published over 100 papers in graph theory and has served as editor of the *College Mathematics Journal*. With Robin Wilson he has co-edited five books in addition to the three earlier volumes in this series. Recent honours include an award instituted in his name by the College of Arts and Sciences at IPFW and a Certificate of Meritorious Service from the Mathematical Association of America.

ROBIN J. WILSON is Emeritus Professor of Pure Mathematics at the Open University, UK, and Emeritus Professor of Geometry at Gresham College, London. After graduating from Oxford, he received his Ph.D. in number theory from the University of Pennsylvania. He has written and co-edited many books on graph theory and the history of mathematics, including *Introduction to Graph Theory, Four Colors Suffice* and *Combinatorics: Ancient & Modern*. His combinatorial research interests formerly included graph colourings and now focus on the history of combinatorics. An enthusiastic popularizer of mathematics, he has won two awards for his expository writing from the Mathematical Association of America.

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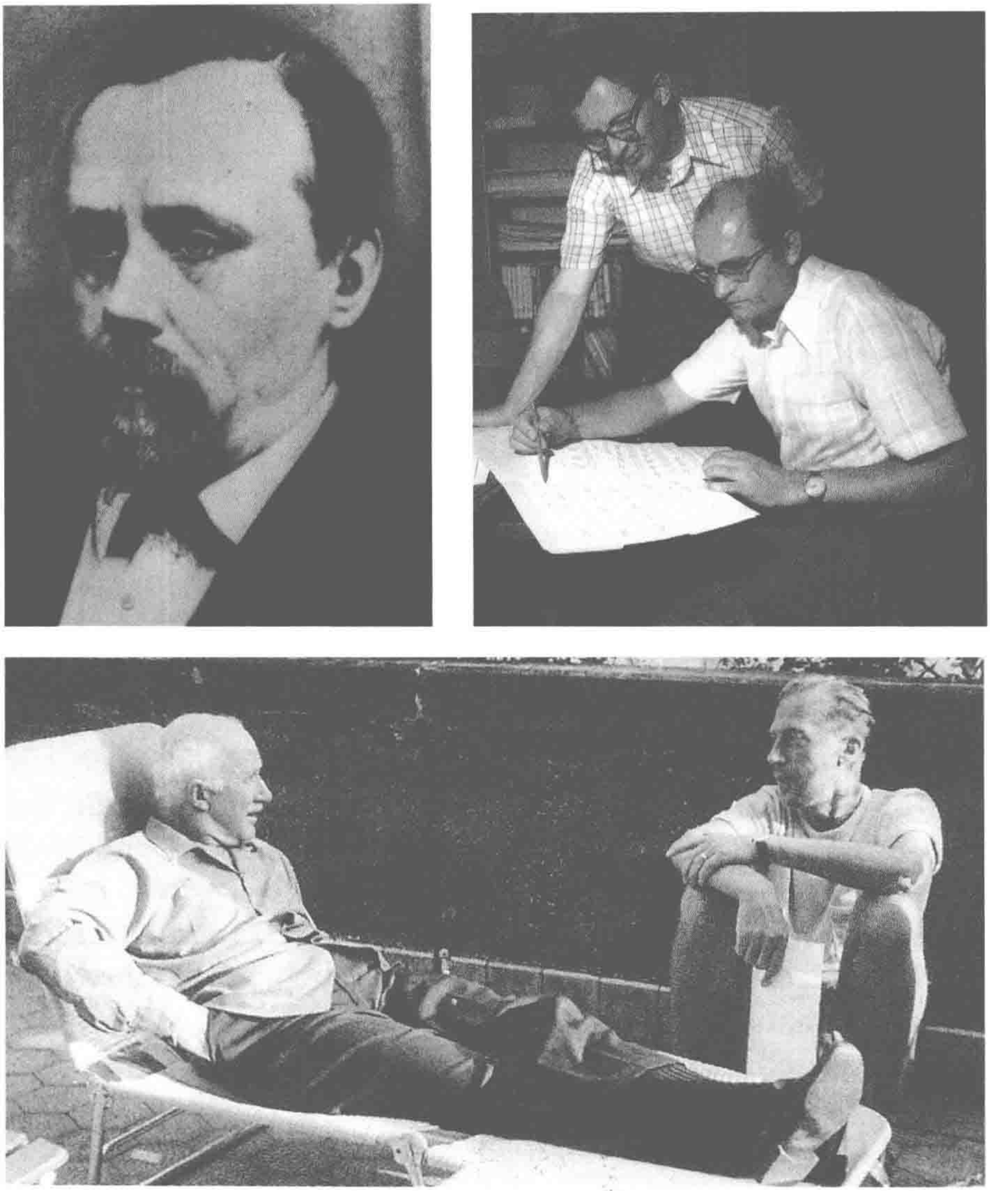

Above: Francis Guthrie, who proposed the four-colour problem, and Kenneth Appel and Wolfgang Haken, who solved it. *Below*: Gerhard Ringel (right) and Ted Youngs, who solved the Heawood conjecture. (Courtesy of Robin Wilson.)

Foreword

Bjarne Toft

The four colour problem is
the tip of the iceberg,
the thin end of the wedge
and the first cuckoo of spring.
W. T. Tutte, 1978

A fundamental process in mathematics is that of partitioning a set of objects into classes according to certain rules. Chromatic graph theory deals with a situation where the rules are almost as simple as one can imagine: for each pair of objects we are told whether they may be put in the same class or not. However, the simplicity of the rules does not mean that the problems encountered are simple – on the contrary. Starting from the four-colour problem around 1850, the theory has developed into a many-sided body of problems, theories, results and applications, and even though many problems have been solved, sometimes in surprising ways, the number of simply stated but challenging problems remains large and growing. This explains the popularity of the area and why it attracts so many active researchers.

This book presents a picture of this many-sided body as it has evolved so far. Experts from various parts of the area present main ideas, methods and results, and describe what is important. Map-colouring dominated the field for many years, but with authors like K. Wagner, H. Hadwiger, R. L. Brooks, W. T. Tutte, G. A. Dirac, G. Hajós, T. Gallai and P. Erdős, among others, the theory became more general, abstract and applicable.

The chapters cover much ground. The first one outlines the general theory of colouring graphs on surfaces. Other types of graphs, such as perfect graphs, geometric graphs, random graphs and hypergraphs are then treated in chapters of their own, as are special types of colourings, such as edge-colourings, list-colourings and integer flows. Classical topics, such as Brooks's theorem, Hadwiger's conjecture and chromatic polynomials, are described and updated to current knowledge. Applications and relations to other fields, such as scheduling, games and algorithms, are also included. The final chapter presents some 20 unsolved problems: solutions to most of these are probably beyond what can be achieved with current knowledge.

The area continues to surprise, and the achievements of the past few years in particular have witnessed a treasure trove of results, methods, ideas and problems. We now know more of W. T. Tutte's iceberg, even if much still lies hidden below the surface, waiting for discovery!

Preface

The field of graph theory has undergone tremendous growth during the past century. As recently as the 1950s, the graph theory community had few members and most were in Europe and North America; today there are hundreds of graph theorists and they span the globe. By the mid 1970s, the subject had reached the point where we perceived a need for a collection of surveys of various areas of graph theory: the result was our three-volume series *Selected Topics in Graph Theory*, comprising articles written by distinguished experts and then edited into a common style. Since then, the transformation of the subject has continued, with individual branches (such as chromatic graph theory) expanding to the point of having important subdivisions themselves. This inspired us to conceive of a new series of books, each a collection of articles within a particular area of graph theory written by experts within that area. The first three of these books were the companion volumes to the present one, on algebraic graph theory, topological graph theory and structural graph theory. This is thus the fourth volume in the series.

A special feature of these books is the engagement of academic consultants (here, Bjarne Toft) to advise us on topics to be included and authors to be invited. We believe that this has been successful, with the result that the chapters of each book cover the full range of area within the given area. In the present case, the area is chromatic graph theory, with chapters written by authors from around the world. Another important feature is that, where possible, we have imposed uniform terminology and notation throughout, in the belief that this will aid readers in going from one chapter to another. For a similar reason, we have not tried to remove a small amount of material common to some of the chapters.

We hope that these features will facilitate usage of the book in advanced courses and seminars. We sincerely thank the authors for cooperating in these efforts, even though it sometimes required their abandoning some of their favourite conventions – for example, computer scientists commonly use the term *node*, whereas graph theorists use *vertex*; not surprisingly, the graph theorists prevailed on this one. We also asked our contributors to endure the ordeal of having their early versions subjected to detailed critical reading. We believe that as a result the final product

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is significantly better than it would otherwise have been (as a collection of individual chapters with differing styles and terminology). We want to express our heartfelt appreciation to all of our contributors for their cooperation in these endeavours.

We extend special thanks to Bjarne Toft for his service as Academic Consultant – his advice has been invaluable. We are also grateful to Cambridge University Press for publishing these volumes; in particular, we thank Roger Astley, Charlotte Thomas and Clare Dennison for their advice, support, patience and cooperation. Finally we extend our appreciation to several universities for the ways in which they have assisted with our project: the first editor (LWB) is grateful to his home institution of Indiana University–Purdue University Fort Wayne, while the second editor (RJW) has had the cooperation of the Open University as well as the Mathematical Institute and Pembroke College in Oxford University.

LOWELL W. BEINEKE
ROBIN J. WILSON

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