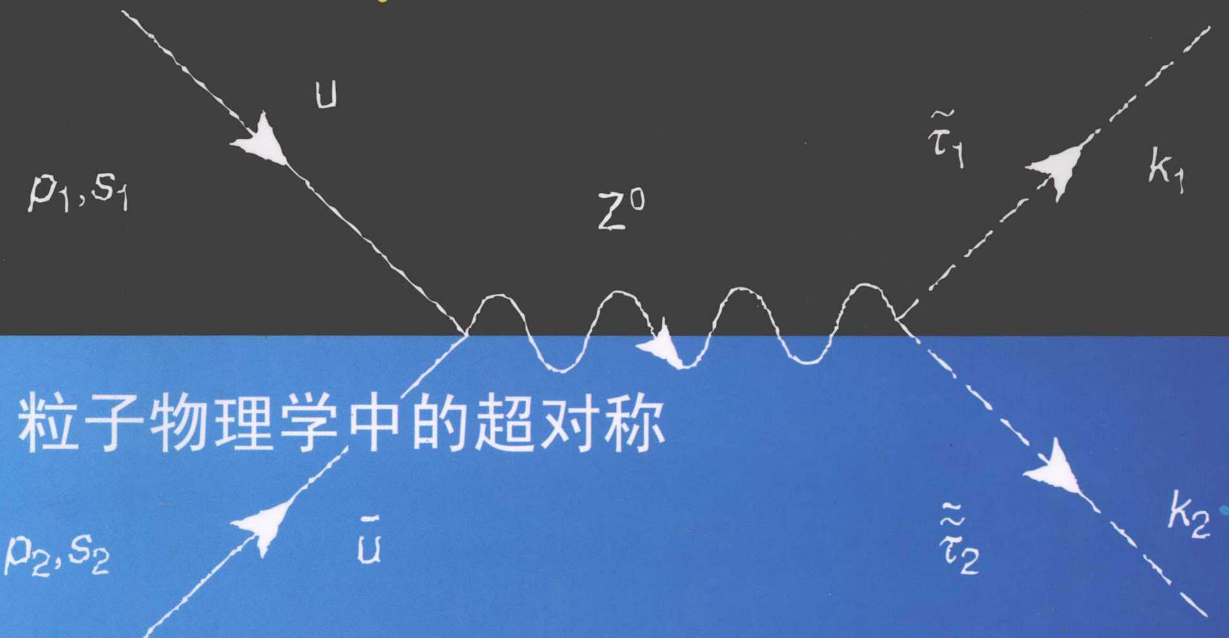
Ian Aitchison

Supersymmetry in Particle Physics

An Elementary Introduction



粒子物理学中的超对称

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SUPERSYMMETRY IN PARTICLE PHYSICS

An Elementary Introduction

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SUPERSYMMETRY IN PARTICLE PHYSICS

Supersymmetry has been a central topic in particle physics since the early 1980s, and represents the culmination of the search for fundamental symmetries that has dominated particle physics for the last 50 years. Traditionally, the constituents of matter (fermions) have been regarded as essentially different from the particles (bosons) that transmit the forces between them. In supersymmetry, however, fermions and bosons are unified.

This is the first textbook to provide a simple pedagogical introduction to what has been a formidably technical field. The elementary and practical treatment brings readers to the frontier of contemporary research, in particular, to the confrontation with experiments at the Large Hadron Collider. Intended primarily for first-year graduate students in particle physics, both experimental and theoretical, this volume will also be of value to researchers in experimental and phenomenological supersymmetry. Supersymmetric theories are constructed through an intuitive 'trial and error' approach, rather than being formal and deductive. The basic elements of spinor formalism and superfields are introduced, allowing readers to access more advanced treatments. Emphasis is placed on physical understanding, and on detailed, explicit derivations of all important steps. Many short exercises are included making for a valuable and accessible self-study tool.

IAN AITCHISON is Emeritus Professor of Physics at the University of Oxford. His research interests include time-dependent effective theories of superconductors, field theories at finite temperature and topological aspects of gauge theories.

For Danny

Preface

This book is intended to be an elementary and practical introduction to supersymmetry in particle physics. More precisely, I aim to provide an accessible, self-contained account of the basic theory required for a working understanding of the 'Minimal Supersymmetric Standard Model' (MSSM), including 'soft' symmetry breaking. Some simple phenomenological applications of the model are also developed in the later chapters.

The study of supersymmetry (SUSY) began in the early 1970s, and there is now a very large, and still growing, research literature on the subject, as well as many books and review articles. However, in my experience the existing sources are generally suitable only for professional (or intending) theorists. Yet searches for SUSY have been pursued in experimental programmes for some time, and are prominent in experiments planned for the Large Hadron Collider at CERN. No direct evidence for SUSY has yet been found. Nevertheless, for the reasons outlined in Chapter 1, supersymmetry at the TeV scale has become the most highly developed framework for guiding and informing the exploration of physics beyond the Standard Model. This dominant role of supersymmetry, both conceptual and phenomenological, suggests a need for an entry-level introduction to supersymmetry, which is accessible to the wider community of particle physicists.

The first difficulty presented by conventional texts on supersymmetry – and it deters many students – is one of notation. Right from the start, discussions tend to be couched in terms of a spinor notation that is generally not familiar from standard courses on the Dirac equation – namely, that of either 'dotted and undotted 2-component Weyl spinors', or '4-component Majorana spinors'. This creates something of a conceptual discontinuity between what most students already know, and what they are trying to learn; it becomes a pedagogical barrier. By contrast, my approach builds directly on knowledge of Dirac spinors in a conventional representation, using 2-component ('half-Dirac') spinors, without necessarily requiring the more sophisticated dotted and undotted formalism. The latter is, however, introduced early on (in Section 2.3), but it can be treated as an optional extra; the essential elements of SUSY and the MSSM (contained in Sections 3.1, 3.2, 4.2, 4.4, 5.1 and Chapters 7 and 8) can be understood quite reasonably without it.

Apart from its simple connection to standard Dirac theory, a second advantage of the 2-component formalism is, I think, that it is simpler to use than the Majorana one for motivating and establishing the forms of simple SUSY-invariant Lagrangians. Again, a more powerful route is available via the superfield formalism, to which I provide access in Chapter 6, but the essentials do not depend on it.

On the other hand, I don't think it is wise to eschew the Majorana formalism altogether. For one thing, there are some important sources which adopt it exclusively, and which students might profitably consult. Furthermore, the Majorana formalism appears to be the one generally used in SUSY calculations, since, with some modifications, it allows the use of short-cuts familiar from the Dirac case. So I provide an early introduction to Majorana spinors as well, in Section 2.5; and at various places subsequently I point out the Majorana equivalents for what is going on. I make use of Majorana forms in Section 8.2, where I recover the Standard Model interactions in the MSSM, and also in the calculations of Section 5.2 and of Chapter 12. I believe that the indicated arguments justify the added burden, to the interested reader, of having to acquire some familiarity with a second language.

Moving on from notation, my approach is generally intuitive and constructive, rather than formal and deductive. It is very much a do-it-yourself treatment. Thus in Sections 2.1 and 2.2 I provide a gentle and detailed introduction to the use of Weyl spinors in the 'half-Dirac' notation. Care is taken to introduce a simple (free) SUSY theory very slowly and intuitively in Section 3.1, and this is followed by an appetite-whetting preview of the MSSM, as a relief from the diet of formalism. The simple SUSY transformations learned in Section 3.1 are used to motivate the SUSY algebra in Section 4.2 (rather than just postulating it), and simple consequences for supermultiplet structure are explained in Section 4.4. The more technical matter of the necessity for auxiliary fields (even in such a simple case) is discussed at the end of Chapter 4.

The introduction of interactions in a chiral multiplet follows reasonably straightforwardly in Section 5.1 (the Wess–Zumino model). The more technical – but theoretically crucial – property of cancellation of quadratic divergences is illustrated for some simple cases in Section 5.2.

After the optional detour into chiral superfields, the main thread is taken up again in Chapter 7, where supersymmetric gauge theories are introduced via vector supermultiplets, which are then combined with chiral supermultiplets. Here the superfield formalism has been avoided in favour of a more direct try-it-and-see approach similar to that of Section 3.1.

At roughly the half-way stage in the book, all the elements necessary for understanding the construction of the MSSM (or variants thereof) are now in place. The model is defined in Chapter 8, and immediately applied to exhibit gauge-coupling unification. Elementary ideas of SUSY breaking are introduced in Chapter 9, together with the phenomenolgically important notion of 'soft' supersymmetry-breaking parameters. The remainder of the book is devoted to simple applications: Higgs physics (Chapter 10), sparticle masses (Chapter 11) and sparticle production processes (Chapter 12).

Throughout, emphasis is placed on providing elementary, explicit and detailed derivations of important formal steps wherever possible. Many short exercises are included, which are designed to help the reader to engage actively with the text, and to keep abreast of the formal development through practice at every stage.

In keeping with the stated aim, the scope of this book is strictly limited. A list of omitted topics would be long indeed. It includes, for example: the superfield formalism for vector supermultiplets; Feynman rules in super-space; wider phenomenological implications of the MSSM; local supersymmetry (supergravity); more detail on SUSY searches; SUSY and cosmology; non-perturbative aspects of SUSY; SUSY in dimensions other than 4, and for values of N other than $N = 1$. Fortunately, a number of excellent and comprehensive monographs are now available; readers interested in pursuing matters beyond where I leave them, or in learning about topics I omit, can confidently turn to these professional treatments.

I am very conscious that the list of references is neither definitive nor comprehensive. In a few instances (for example, in reviewing the beginnings of SUSY and the MSSM) I have tried to identify the relevant original contributions, although I have probably missed some. Usually, I have not attempted to trace priorities carefully, but have referred to more comprehensive reviews, or have simply quoted such references as came to hand as I worked my own way into the subject. I apologize to the many researchers whose work, as a consequence, has not been referenced here.

The book has grown out of lectures to graduate students at Oxford working in both experimental and theoretical particle physics. In this, its genesis is very similar to my book with Tony Hey, *Gauge Theories in Particle Physics*, first published in 1982 and now in its third (two-volume) edition. The present book aims to reach a similar readership: in particular, I have tried to design the level so that it follows smoothly on from the earlier one. Indeed, as the title suggests, this book may be seen as 'volume 3' in the series.

However, I would expect theorists and experimentalists to use the book differently. For theorists, it should be a relatively easy read, setting them up for immediate access to the professional literature and more advanced monographs. On the other hand, many experimentalists are likely to find some of the formal parts indigestible, even with the support provided. They should be able to find a reasonably friendly route to the physics they want to learn via the 'essential elements' mentioned earlier (that is, Sections 2.1, 2.2, 3.1, 3.2, 4.2, 4.4, 5.1 and Chapters 7 and 8), to be followed by whatever applications they are most interested in. Much of this material should not be beyond final year maths or physics undergraduates who have taken courses in relativistic quantum mechanics, introductory quantum field theory, and gauge theories. By the same token, the book may also be useful to a wide range of physicists in other areas, who wish to gain a first-hand appreciation of the excitement and anticipation which surround the possible discovery of supersymmetry at the TeV scale.

Ian J. R. Aitchison
February 2007

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Above all, and once again, I owe a special debt to my good friend George Emmons, who has been an essential part of this project from the beginning: his comments and queries revealed misconceptions on my part, as well as obscurities in the presentation, and led to many improvements in the developing text; he read carefully through several successive LaTex drafts, spotting many errors, and he corrected the final proofs. His encouragement and support have been invaluable.

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Contents

1
Introduction and motivation

Supersymmetry (SUSY) – a symmetry relating bosonic and fermionic degrees of freedom – is a remarkable and exciting idea, but its implementation is technically rather complicated. It can be discouraging to find that after standard courses on, say, the Dirac equation and quantum field theory, one has almost to start afresh and master a new formalism, and moreover one that is not fully standardized. On the other hand, 30 years have passed since the first explorations of SUSY in the early 1970s, without any direct evidence of its relevance to physics having been discovered. The Standard Model (SM) of particle physics (suitably extended to include an adequate neutrino phenomenology) works extremely well. So the hard-nosed seeker after truth may well wonder: why spend the time learning all this intricate SUSY formalism? Indeed, why speculate at all about how to go 'beyond' the SM, unless or until experiment forces us to? If it's not broken, why try and fix it?

As regards the formalism, most standard sources on SUSY use either the 'dotted and undotted' 2-component (Weyl) spinor notation found in the theory of representations of the Lorentz group, or 4-component Majorana spinors. Neither of these is commonly included in introductory courses on the Dirac equation (although perhaps they should be), but it is perfectly possible to present simple aspects of SUSY using a notation which joins smoothly on to standard 4-component Dirac equation courses, and a brute force, 'try-it-and-see' approach to constructing SUSY-invariant theories. That is the approach to be followed in this book, at least to start with. However, as we go along the more compact Weyl spinor formalism will be introduced, and also (more briefly) the Majorana formalism. Later, we shall include an introduction to the powerful superfield formalism. All this formal concentration is partly because the simple-minded approach becomes too cumbersome after a while, but mainly because discussions of the phenomenology of the Minimal Supersymmetric Standard Model (MSSM) generally make use of one or other of these more sophisticated notations.

What of the need to go beyond the Standard Model? Within the SM itself, there is a plausible historical answer to that question. The V–A current–current (four-fermion) theory of weak interactions worked very well for many years, when used at lowest order in perturbation theory. Yet Heisenberg [1] had noted as early as 1939 that problems arose if one tried to compute higher-order effects, perturbation theory apparently breaking down completely at the then unimaginably high energy of some 300 GeV (the scale of $G_F^{-1/2}$). Later, this became linked to the non-renormalizability of the four-fermion theory, a purely theoretical problem in the years before experiments attained the precision required for sensitivity to electroweak radiative corrections. This perceived disease was alleviated but not cured in the 'Intermediate Vector Boson' model, which envisaged the weak force between two fermions as being mediated by massive vector bosons. The non-renormalizability of such a theory was recognized, but not addressed, by Glashow [2] in his 1961 paper proposing the SU(2) × U(1) structure. Weinberg [3] and Salam [4], in their gauge-theory models, employed the hypothesis of spontaneous symmetry breaking to generate masses for the gauge bosons and the fermions, conjecturing that this form of symmetry breaking would not spoil the renormalizability possessed by the massless (unbroken) theory. When 't Hooft [5] demonstrated this in 1971, the Glashow–Salam–Weinberg theory achieved a theoretical status comparable to that of quantum electrodynamics (QED). In due course the precision electroweak experiments spectacularly confirmed the calculated radiative corrections, even yielding a remarkably accurate prediction of the top quark mass, based on its effect as a virtual particle . . . but note that even this part of the story is not yet over, since we have still not obtained experimental access to the proposed symmetry-breaking (Higgs [6]) sector. If and when we do, it will surely be a remarkable vindication of theoretical preoccupations dating back to the early 1960s.

It seems fair to conclude that worrying about perceived imperfections of a theory, even a phenomenologically very successful one, can pay off. In the case of the SM, a quite serious imperfection (for many theorists) is the 'SM fine-tuning problem', which we shall discuss in a moment. SUSY can suggest a solution to this perceived problem, provided that supersymmetric partners to known particles have masses no larger than a few TeV (roughly).

In addition to the 'fine-tuning' motivation for SUSY – to which, as we shall see, there are other possible responses – there are some quantitative results (Section 1.2), and theoretical considerations (Section 1.3) , which have inclined many physicists to take SUSY and the MSSM (or something like it) very seriously. As always, experiment will decide whether these intuitions were correct or not. A lot of work has been done on the phenomenology of such theories, which has influenced the Large Hadron Collider (LHC) detector design. Once again, it will surely be extraordinary if, in fact, the world turns out to be this way.

1.1 The SM fine-tuning problem

The electroweak sector of the SM contains within it a parameter with the dimensions of energy (i.e. a 'weak scale'), namely

$$v \approx 246\,\text{GeV}, \tag{1.1}$$

where $v/\sqrt{2}$ is the vacuum expectation value (or 'vev') of the neutral Higgs field, $\langle 0|\phi^0|0\rangle = v/\sqrt{2}$. The occurrence of the vev signals the 'spontaneous' breaking of electroweak gauge symmetry (see, for example [7], Chapter 19), and the associated parameter v sets the scale, in principle, of all masses in the theory. For example, the mass of the $\mathrm{W}^{\pm}$ (neglecting radiative corrections) is given by

$$M_{\mathrm{W}} = gv/2 \sim 80\,\text{GeV}, \tag{1.2}$$

and the mass of the Higgs boson is

$$M_{\mathrm{H}} = v\sqrt{\frac{\lambda}{2}}, \tag{1.3}$$

where g is the SU(2) gauge coupling constant, and λ is the strength of the Higgs self-interaction in the Higgs potential

$$V = -\mu^2\phi^{\dagger}\phi + \frac{\lambda}{4}(\phi^{\dagger}\phi)^2, \tag{1.4}$$

where $\lambda > 0$ and $\mu^2 > 0$. Here ϕ is the SU(2) doublet field

$$\phi = \begin{pmatrix} \phi^{+} \\ \phi^{0} \end{pmatrix}, \tag{1.5}$$

and all fields are understood to be quantum, no 'hat' being used.

Recall now that the *negative* sign of the 'mass^2' term $-\mu^2$ in (1.4) is essential for the spontaneous symmetry-breaking mechanism to work. With the sign as in (1.4), the minimum of V interpreted as a classical potential is at the non-zero value

$$|\phi| = \sqrt{2}\mu/\sqrt{\lambda} \equiv v/\sqrt{2}, \tag{1.6}$$

where $\mu \equiv \sqrt{\mu^2}$. This classical minimum (equilibrium value) is conventionally interpreted as the expectation value of the quantum field in the quantum vacuum (i.e. the vev), at least at tree level. If '$-\mu^2$' in (1.4) is replaced by the positive quantity 'μ^2', the classical equilibrium value is at the origin in field space, which would imply $v = 0$, in which case all particles would be massless. Hence it is vital to preserve the sign, and indeed magnitude, of the coefficient of $\phi^{\dagger}\phi$ in (1.4).

The discussion so far has been at tree level (no loops). What happens when we include loops? The SM is renormalizable, which means that finite results are obtained for all higher-order (loop) corrections even if we extend the virtual momenta

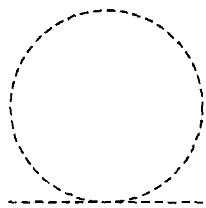

Figure 1.1 One-loop self-energy graph in ϕ^4 theory.

in the loop integrals all the way to infinity; but although this certainly implies that the theory is well defined and calculable up to infinite energies, in practice no one seriously believes that the SM is really all there is, however high we go in energy. That is to say, in loop integrals of the form

$$\int^{\Lambda} \mathrm{d}^4 k \; f(k, \text{external momenta}) \tag{1.7}$$

we do not think that the cut-off Λ *should* go to infinity, physically, even though the reormalizability of the theory assures us that no inconsistency will arise if it does. More reasonably, we regard the SM as part of a larger theory which includes as yet unknown 'new physics' at high energy, Λ representing the scale at which this new physics appears, and where the SM must be modified. At the very least, for instance, there surely must be some kind of new physics at the scale when quantum gravity becomes important, which is believed to be indicated by the Planck mass

$$M_{\mathrm{P}} = (G_{\mathrm{N}})^{-1/2} \simeq 1.2 \times 10^{19}\,\mathrm{GeV}. \tag{1.8}$$

If this is indeed the scale of the new physics beyond the SM or, in fact, if there is *any* scale of 'new physics' even several orders of magnitude different from the scale set by v, then we shall see that we meet a problem with the SM, once we go beyond tree level.

The 4-boson self-interaction in (1.4) generates, at one-loop order, a contribution to the $\phi^\dagger\phi$ term, corresponding to the self-energy diagram of Figure 1.1, which is proportional to

$$\lambda \int^{\Lambda} \mathrm{d}^4 k \, \frac{1}{k^2 - M_{\mathrm{H}}^2}. \tag{1.9}$$

This integral clearly diverges quadratically (there are four powers of k in the numerator, and two in the denominator), and it turns out to be *positive*, producing a correction

$$\sim \lambda \Lambda^2 \phi^\dagger \phi \tag{1.10}$$