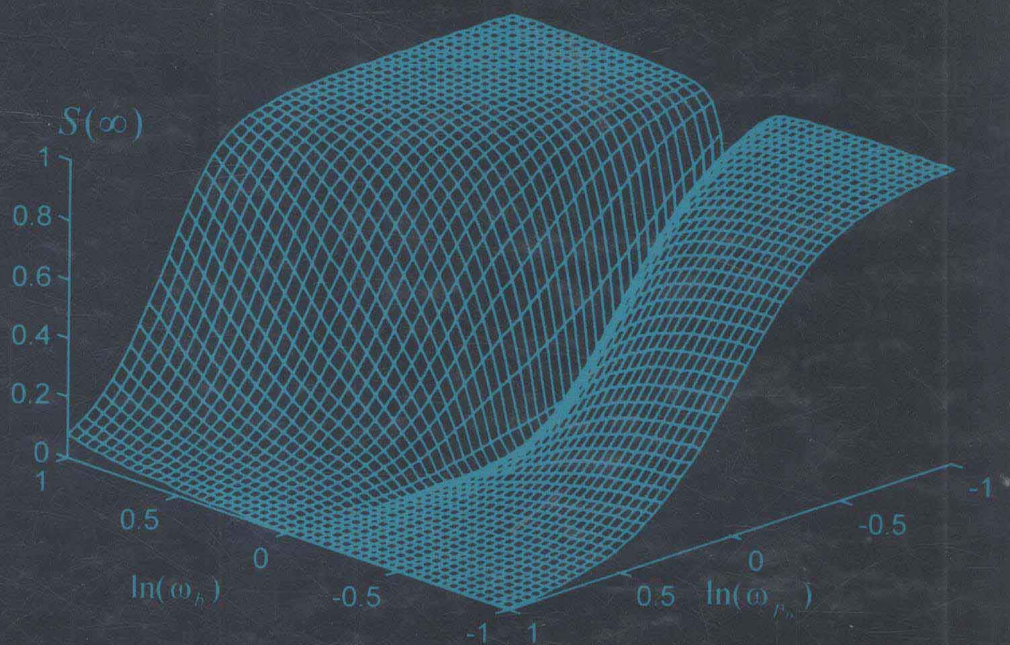


ELECTROMAGNETICS OF COMPLEX MEDIA

*Frequency Shifting by a Transient
Magnetoplasma Medium*



Dikshitulu K. Kalluri

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OF COMPLEX MEDIA

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CRC Press

Boca Raton Boston London New York Washington, D.C.

Acquiring Editor: Robert Stern
Project Editor: Sylvia Wood
Marketing Manager: Jane Stark
Cover design: Dawn Boyd
PrePress: Greg Cuciak

Library of Congress Cataloging-in-Publication Data

Kalluri, Dikshitulu K.

Electromagnetics of complex media : frequency shifting by a
transient magnetoplasma medium / by Dikshitulu K. Kalluri.

p. cm.

Includes bibliographical references and index.

ISBN 0-8493-2522-6 (alk. paper)

1. Electromagnetism. 2. Plasma (Ionized gases) I. Title.

QC760.K36 1998

537--dc21

98-16443

CIP

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International Standard Book Number 0-8493-2522-6

Library of Congress Card Number 98-16443

Printed in the United States of America 1 2 3 4 5 6 7 8 9 0

Printed on acid-free paper

In Loving Memory

*Of my father, Jagannatham Kalluri and
my mother, Venkatalakshmi Kalluri*

Foreword

I am pleased to write a Foreword to this book by Professor D. K. Kalluri, who has been a friend since I first became aware of his work in 1992. In that year, Professor C. J. Joshi of the University of California Los Angeles and co-workers were undertaking a series of experiments, based on ideas developed by a number of theorists, to study the frequency shift of radiation by moving ionization fronts in plasmas. This work was closely related to that which Professor Kalluri had been doing for many years, and so we arranged for him to spend the summer of 1992 in Berkeley. In this way, a fruitful collaboration and a good friendship were initiated.

The transformation of an electromagnetic wave by an inhomogeneous medium is discussed in many books. The Doppler effect, which is a frequency change due to a moving boundary, is also a standard topic in many books. However, the transformation of the frequency of an electromagnetic wave by a general time-varying medium is rarely discussed. The frequency change in a nonmoving media is contrary to the usual experience of wave change. Because of his background in electrical engineering and electromagnetics, Professor Kalluri could weave, in a unique way, this topic of current research interest into a book on *Electromagnetics of Complex Media* varying in space and time. By using simple ideal models, he has focused on the effect of the time-varying parameters in conjunction with one or more additional kinds of complexities in the properties of the medium, and thus made the subject more accessible.

The reader can be assured that Professor Kalluri is highly competent to write this book, for he has contributed original research papers on the frequency shifting of electromagnetic radiation using a transient magnetized plasma. He has expertise, and many publications, dealing with moving media, the use of Laplace transforms to study the effect of boundaries on transient solution, the generation of a downshifted wave whose frequency can be controlled by the strength of a static magnetic field (transformation of a whistler wave), and the turning-off of the external magnetic field (which showed that an original whistler wave is converted into a wiggler magnetic field). In short, the book that follows is written by a highly competent author who treats important subjects. I hope the reader will enjoy it as much as I have.

Andrew M. Sessler
Berkeley, California

Author

Dikshitulu K. Kalluri, Ph.D., is Professor of Electrical and Computer Engineering at the University of Massachusetts Lowell. Born in Chodavaram, India, he received his B.E. degree in electrical engineering from Andhra University, India; a D.I.I Sc. degree in high-voltage engineering from the Indian Institute of Science in Bangalore, India; earned a master's degree in electrical engineering from the University of Wisconsin, Madison, and his doctorate in electrical engineering from the University of Kansas, Lawrence.

Dr. Kalluri began his career at the Birla Institute, Ranchi, India, advancing to the rank of Professor, heading the Electrical Engineering Department, then serving as (Dean) Assistant Director of the institute. He has collaborated with research groups at the Lawrence Berkeley Laboratory, the University of California Los Angeles, the University of Southern California, and the University of Tennessee, and has worked several summers as a faculty research associate at Air Force Laboratories. Since 1984, he has been with the University of Massachusetts Lowell, where he is coordinator of the doctoral program and co-director of the Center for Electromagnetic Materials and Optical Systems (CEMOS). As part of the center, he recently established the Electromagnetics and Complex Media Research Laboratory.

Dr. Kalluri, a fellow of the Institute of Electronic and Telecommunication Engineers and a member of Eta Kappa Nu and Sigma Xi, has published many technical articles and reviews.

Preface

After careful thought I have developed two graduate courses in electromagnetics, each with a different focus. One of them assumes a simple electromagnetic medium, a medium described by scalar permittivity (ϵ), permeability (μ), and conductivity (σ). The geometrical effect, due to the shape and the size of the boundaries, is the main focus. There are many excellent textbooks, for example, *Advanced Electromagnetics* by Balanis, to serve the needs of this course.

Recent advances in material science suggest that materials can be synthesized with any desired electromagnetic properties. The optimum properties for a given application must be understood and sought. To facilitate developing such an appreciation in our graduate students, I have developed another course called the “Electromagnetics of Complex Media.” The focus here is to bring out the major effects due to each kind of complexity in the medium properties. The medium is considered complex if any of the electromagnetic parameters ϵ , μ , and σ are not scalar constants. If the parameters are functions of signal frequency, we have temporal dispersion. If the parameters are tensors, we have anisotropy. If the parameters are functions of position, we have inhomogeneity. A plasma column in the presence of a static magnetic field is at once dispersive, anisotropic, and inhomogeneous. For this reason, I have chosen plasma as the basic medium to illustrate some aspects of the transformation of an electromagnetic wave by a complex medium.

An additional aspect of medium complexity that is of current research interest arises out of the time-varying parameters of the medium. Powerful lasers that produce ultrashort pulses for ionizing gases into plasmas can permit fast changes in the dielectric constant of a medium. An idealization of this process is called the sudden creation of the plasma or sudden switching of the medium. More practical processes require a model of a time-varying plasma with an *arbitrary rise time*.

The early chapters of this book use a mathematical model that usually has one kind of complexity. The medium is often assumed to be unbounded in space or has a simple plane boundary. The field variables and the parameters are often assumed to vary in one spatial coordinate. This eliminates the use of heavy mathematics and permits the focus to be on the effect. The last chapter, however, has a section on the use of the finite-difference time-domain method for the numerical simulation of three-dimensional problems.

The main effect of switching a medium is to shift the frequency of the source wave. The frequency change is contrary to the usual experience of wave change we find. The exception is the Doppler effect, which is a frequency change caused by a moving boundary. The moving boundary is a particular case of a time-varying medium.

The primary title indicates that this book will serve the needs of students who study electromagnetics as a basis for a number of disciplines that use “complex

materials." Examples are electro-optics, plasma science and engineering, microwave engineering, and solid-state devices. The aspects of electromagnetic wave transformation by a complex medium that are emphasized in the book are

1. dispersive medium
2. tunneling of power through a plasma slab by evanescent waves
3. characteristic waves in an anisotropic medium
4. transient medium and frequency shifting
5. Green's function for unlike anisotropic media
6. perturbation technique for unlike anisotropic media
7. adiabatic analysis for modified source wave

All the above topics use one-dimensional models.

The following topics are covered briefly in this book: (1) chiral media, (2) surface waves, and (3) periodic media. The topics that are not covered include (1) nonlinear media, (2) parametric instabilities, and (3) random media. I hope to include these topics in the future in an expanded version of the book to serve a two-semester course or to give a choice of topics for a one-semester course.

Problems are added at the end of the book for the benefit of those who would like to use the book as a textbook. The background needed is a one-semester undergraduate electromagnetics course that includes a discussion of plane waves in a simple medium. With this background, a senior undergraduate student or a first-year graduate student can easily follow the book. The solution manual for the problems is available.

The secondary title of the book emphasizes the viewpoint of frequency change and is intended to draw the attention of new researchers who wish to have a quick primer on the theory of using magnetoplasmas for coherent generation of tunable radiation. I hope the book will stimulate experimental and additional theoretical and numerical work on the remarkable effects that can be obtained by the temporal and spatial modification of the magnetoplasma parameters. A large part of the book contains research published by a number of people, including the author of this book, in recent issues of several research journals. Particular attention is drawn to the reprints given in Appendixes B through H. The book also contains a number of unpublished results.

The Rationalized MKS system of units is used throughout the book. The harmonic time and space variations are denoted by $\exp(j\omega t)$ and $\exp(-jkz)$, respectively.

Acknowledgments

I am proud of my present and past doctoral students who have shared with me the trials and tribulations of exploring a new area of research. Among them, I particularly would like to mention Drs. V. R. Goteti and T. T. Huang, and Mr. Joo Hwa Lee. A very special thanks to Joo Hwa, who, in his last semester of doctoral work, undertook the task of critically reviewing the manuscript and used his considerable computer skills to format the text and the figures to the specified standard. I am grateful to the University of Massachusetts Lowell for granting a sabbatical during the spring of 1996 for the purpose of writing this book. The support of my research by the Air Force Office of Scientific Research and the Air Force Laboratories during 1996–1997 is gratefully acknowledged. I am particularly thankful to Dr. K. M. Groves for acting as my focal point at the laboratory and for contributing to the research. The encouragement of my friends and collaborators, Professor Andrew Sessler of the University of California, and Professor Igor Alexeff of the University of Tennessee, helped me a great deal in doing my research and in writing this book.

My peers with whom I had opportunity to discuss research of mutual interest include Professors A. Baños, Jr., S. A. Bowhill, J. M. Dawson, M. A. Fiddy, O. Ishihara, C. J. Joshi, T. C. Katsoules, H. H. Kuehl, S. P. Kuo, M. C. Lee, W. B. Mori, A. G. Nerukh, E. J. Powers, Jr., T. C. K. Rao, B. Reinisch, G. Sales, B. V. Stanic, N. S. Stepanov, D. Wunsch, and B. J. Wurtele and Drs. V. W. Byszewski, S. J. Gitomer, P. Muggli, R. L. Savage, Jr., and S. C. Wilks. The anonymous reviewers of our papers also belong to this group. This group played an important role in providing the motivation to continue our research.

My special thanks go to Dr. Robert Stern at CRC Press for waiting for the manuscript and, when once submitted, processing it with great speed. I appreciate the help and advice given by project editor Sylvia Wood, also at CRC.

Finally, I am most thankful to my wife, Kamala, for assisting me with many aspects of writing the book and to my children Srinath, Sridhar, and Radha for standing by me, encouraging me, and giving up their share of my time for the sake of research.

Dikshitulu K. Kalluri

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1 Isotropic Plasma: Dispersive Medium

1.1 INTRODUCTION

Plasma is a *quasineutral* mixture of charged particles and neutral particles. It is characterized by two independent parameters for each of the particle species: the particle density N and the temperature T . Plasma physics deals with such mixtures, and there is a vast amount of literature on this topic. A few references of direct interest to the reader of this book are given at the end of this chapter. References 1–3 deal with modeling of a magnetized plasma as an electromagnetic medium. The models are adequate in exploring some of the applications in which the medium can be considered to have time-invariant electromagnetic parameters.

There are some applications for which the thermal effects are unimportant; such a plasma is called a *cold plasma*. A *Lorentz plasma*¹ is a further simplification of the medium. In this model it is assumed that the electrons interact with each other only through collective space charge forces and that the heavy positive ions and neutral particles are at rest. The positive ions serve as a background that ensures the overall charge neutrality of the mixture. In this book the Lorentz plasma will be the dominant model used to explore the major effects of a nonperiodically time-varying electron density profile $N(t)$. Departure from the model will be made only when necessary to bring in other relevant effects. References 1 and 2 show the approximations made to arrive at the Lorentz plasma model.

1.2 BASIC FIELD EQUATIONS FOR A COLD ISOTROPIC PLASMA

The electric field $\mathbf{E}(\mathbf{r}, t)$, the magnetic field $\mathbf{H}(\mathbf{r}, t)$, and the velocity field $\mathbf{v}(\mathbf{r}, t)$ of the electrons in the isotropic Lorentz plasma satisfy the following equations:

$$\nabla \times \mathbf{E} = -\mu_0 \frac{\partial \mathbf{H}}{\partial t}, \quad (1.1)$$

$$\nabla \times \mathbf{H} = \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mathbf{J}, \quad (1.2)$$

$$m \frac{d\mathbf{v}}{dt} = -q\mathbf{E}, \quad (1.3)$$

where \mathbf{J} is the free-electron current density in the plasma, q is the absolute value of the charge of an electron, and m is the mass of an electron. The relation between

the current density and the electric field in the plasma will depend on the ionization process⁴⁻⁷ that creates the plasma. Assume that the electron density profile $N(t)$ in the plasma is known and that the created electrons have zero velocity at the instant of their birth, then from Equation 1.3 the velocity at t of the electrons born at t_i is given by

$$\mathbf{v}_i(\mathbf{r}, t) = -\frac{q}{m} \int_{t_i}^t \mathbf{E}(\mathbf{r}, \tau) d\tau. \quad (1.4)$$

The change in current density at t due to the electrons born at t_i can be computed as follows:

$$\Delta \mathbf{J}(\mathbf{r}, t) = -q \Delta N_i \mathbf{v}_i(\mathbf{r}, t). \quad (1.5)$$

Here ΔN_i is the electron density added at t_i and is given by

$$\Delta N_i = \left[\frac{\partial N}{\partial t} \right]_{t=t_i} \Delta t_i. \quad (1.6)$$

Therefore, the current density is given by (see Appendix A)

$$\mathbf{J}(\mathbf{r}, t) = \frac{q^2}{m} \int_0^t \frac{\partial N(\mathbf{r}, \tau)}{\partial \tau} d\tau \int_{\tau}^t \mathbf{E}(\mathbf{r}, \alpha) d\alpha + \mathbf{J}(\mathbf{r}, 0). \quad (1.7)$$

The expression for \mathbf{J} may be simplified (see Appendix A):

$$\mathbf{J}(\mathbf{r}, t) = \epsilon_0 \int_0^t \omega_p^2(\mathbf{r}, \tau) \mathbf{E}(\mathbf{r}, \tau) d\tau + \mathbf{J}(\mathbf{r}, 0). \quad (1.8)$$

Here, ω_p^2 is the square of the plasma frequency proportional to the electron density N and is given by

$$\omega_p^2(\mathbf{r}, t) = \frac{q^2 N(\mathbf{r}, t)}{m \epsilon_0}. \quad (1.9)$$

See Reference 1 for a physical explanation of the term *plasma frequency*. By differentiating Equation 1.8 a differential equation for \mathbf{J} is obtained:

$$\frac{d\mathbf{J}}{dt} = \epsilon_0 \omega_p^2(\mathbf{r}, t) \mathbf{E}(\mathbf{r}, t). \quad (1.10)$$