Graduate Texts in Physics

Nino Boccara

Modeling Complex Systems

Second Edition

复系统模型 第2版

Springer

世界图书出版公司
www.wpcbj.com.cn

Nino Boccara

Modeling Complex Systems

Second Edition

Springer

Nino Boccara
University of Illinois, Chicago
Dept. Physics
W. Taylor Street 845
60680-5220 Chicago Illinois
USA
boccara@uic.edu

ISSN 1868-4513 e-ISSN 1868-4521
ISBN 978-1-4419-6561-5 e-ISBN 978-1-4419-6562-2
DOI 10.1007/978-1-4419-6562-2
Springer New York Dordrecht Heidelberg London

Library of Congress Control Number: 2010931440

Reprint from English language edition:
Modeling Complex Systems
by Nino Boccara
Copyright © Springer Science+Business Media, LLC 2010
All Rights Reserved

This reprint has been authorized by Springer Science & Business Media for distribution in China Mainland only and not for export therefrom.

Modeling Complex Systems

Graduate Texts in Physics

Graduate Texts in Physics publishes core learning/teaching material for graduate- and advanced-level undergraduate courses on topics of current and emerging fields within physics, both pure and applied. These textbooks serve students at the MS- or PhD-level and their instructors as comprehensive sources of principles, definitions, derivations, experiments and applications (as relevant) for their mastery and teaching, respectively. International in scope and relevance, the textbooks correspond to course syllabi sufficiently to serve as required reading. Their didactic style, comprehensiveness and coverage of fundamental material also make them suitable as introductions or references for scientists entering, or requiring timely knowledge of, a research field.

Pyé-koko di i ka vwè lwen, maché ou ké vwè pli lwen.[1]

[1] Creole proverb from Guadeloupe that can be translated: The coconut palm says it sees far away, walk and you will see far beyond.

Preface of the First Edition

The preface is that part of a book which is written last, placed first, and read least.

Alfred J. Lotka
Elements of Physical Biology
Baltimore: Williams & Wilkins Company 1925

The purpose of this book is to show how models of complex systems are built up and to provide the mathematical tools indispensable for studying their dynamics. This is not, however, a book on the theory of dynamical systems illustrated with some applications; the focus is on modeling, so, in presenting the essential results of dynamical system theory, technical proofs of theorems are omitted, but references for the interested reader are indicated. While mathematical results on dynamical systems such as differential equations or recurrence equations abound, this is far from being the case for spatially extended systems such as automata networks, whose theory is still in its infancy. Many illustrative examples taken from a variety of disciplines, ranging from ecology and epidemiology to sociology and seismology, are given.

This is not only an introductory text directed mainly to advanced undergraduates in most scientific disciplines, but could also serve as a reference book for graduates and young researchers. The material has been taught to junior students at the École de Physique et de Chimie in Paris and the University of Illinois at Chicago. It assumes that the reader has certain fundamental mathematical skills, such as calculus.

Although there is no universally accepted definition of a complex system, most researchers would describe a system of connected agents that exhibits an emergent global behavior not imposed by a central controller, but resulting from the interactions between the agents, as complex. These agents, may be insects, birds, people, or companies, and their number may range from one hundreds to millions.

Finding the emergent global behavior of a large system of interacting agents using analytical methods is usually hopeless, and researchers therefore must rely on computer-based methods. Apart from a few exceptions, most properties of spatially extended systems have been obtained from the analysis of numerical simulations.

Although simulations of interacting multiagent systems are thought experiments, the aim is not to study accurate representations of these systems. The main purpose of a model is to broaden our understanding of general principles valid for the largest variety of systems. Models have to be as simple as possible. What makes the study of complex systems fascinating is not the study of complicated models but the complexity of unsuspected results of numerical simulations.

As a multidisciplinary discipline, the study of complex systems attracts researchers from many different horizons who publish in a great variety of scientific journals. The literature is growing extremely fast, and it would be a hopeless task to try to attain any kind of comprehensive completeness. This book only attempts to supply many diverse illustrative examples to exhibit that common modeling techniques can be used to interpret the behavior of apparently completely different systems.

After a general introduction followed by an overview of various modeling techniques used to explain a specific phenomenon, namely, the observed coupled oscillations of predator and prey population densities, the book is divided into two parts. The first part describes models formulated in differential equations or recurrence equations in which local interactions between the agents are replaced by uniform long-range ones and whose solutions can only give the time evolution of spatial averages. Despite the fact that such models offer rudimentary representations of multiagent systems, they are often able to give a useful qualitative picture of the system's behavior. The second part is devoted to models formulated in terms of automata networks in which the local character of the interactions between the individual agents is explicitly taken into account. Chapters of both parts include a few exercises that, as well as challenging the reader, are meant to complement the material in the text. Detailed solutions to all exercises are provided.

Nino Boccara

Preface of the Second Edition

In this second edition, I essentially made some additions. I first added a few extra footnotes to give some details on the main contributors cited in the text, some more recent references, and a few new exercises also accompanied by their solutions. Some exercises have been adapted from original publications, whose reference has always been given. Moreover, I added a framed text listing all the important points discussed in the chapter just after the chapter title, and a chapter's summary in which are listed, with their definitions, all the most important notions and essential results obtained in the chapter, at the end of the chapter. Finally, at the end of the book before the list of references, I added a glossary listing the meanings of all the specialized terms used in the text. Chapter 6, which is dedicated to spatial models, has been slightly expanded to include a somewhat more realistic agent-based model than cellular automata models, in which the agents are not constrained to occupy periodic locations. I supplemented the already rather extensive list of references with a few more recent ones. The list of references includes either articles or books to which I am referring to in the text or articles and books which, I think, could be useful to the reader wishing to go beyond the material I have presented.

Nino Boccara

Notations

The essential mathematical notations used in the text are grouped below.

$\square$	Indicates the end of a proof
$\mathbb{R}$	Set of real numbers
$\mathbb{R}_+$	Set of positive numbers
$\mathbb{C}$	Set of complex numbers
$\mathbb{Z}$	Set of all integers
$\mathbb{Z}_L$	Set of integers modulo L
$\mathbb{N}$	Set of positive integers
$\mathbb{N}_0$	Set of nonnegative integers
$\mathbb{Q}$	Set of rational numbers
$\emptyset$	Empty set
$x \in \mathcal{X}$	x is an element of the set $\mathcal{X}$
$\mathcal{S} \subset \mathcal{X}$	$\mathcal{S}$ is a subset of $\mathcal{X}$
$A \cup B$	Set of elements either in A or in B
$A \cap B$	Set of elements that are in both A and B
A_c	Complement of set A
$A \setminus B$	Set of all elements in A that are not in B
$A \triangle B$	Set of elements either in A or in B but not in A and B
$\|A\|$	Number of elements of set A
$\{x \in \mathcal{M} \mid P(x)\}$	All elements of $\mathcal{M}$ that have the property $P(x)$
$a \sim b$	a is equivalent to b
$a \Rightarrow b$	a implies b
$a \Leftrightarrow b$	a implies b and conversely
$\lfloor x \rfloor$	Largest integer less than or equal to x
$\lceil x \rceil$	Smallest integer greater than or equal to x
$\overline{z}$	Complex conjugate of z
$m(A)$	Lebesgue measure of the set A
$f : X \to Y$	f is a mapping from the set X into the set Y

$f : x \mapsto f(x)$	Mapping f takes the point x to the point $f(x)$
$f^{-1}(x)$	Set of all preimages of x
$f \circ g$	Composite of mappings f and g, g being applied first
$\mathbf{x} \in \mathbb{R}^n$	$\mathbf{x}$ is an element of the n-dimensional vector space $\mathbb{R}^n$
$\mathbf{x} = (x_1, x_2, \ldots, x_n)$	$x_1, x_2, \ldots, x_n$ are the components of vector $\mathbf{x}$
$\|\mathbf{x}\|$	Norm of vector $\mathbf{x}$
$\dim(\mathcal{S})$	Dimension of (manifold) $\mathcal{S}$
$[a, b]$	Closed interval; i.e., $\{x \in \mathbb{R} \mid a \leq x \leq b\}$
$]a, b[$	Open interval; i.e., $\{x \in \mathbb{R} \mid a < x < b\}$
$[a, b[$	$\{x \in \mathbb{R} \mid a \leq x < b\}$
$]a, b]$	$\{x \in \mathbb{R} \mid a < x \leq b\}$
$d(x, y)$	Distance between points x and y in a metric space
$N(\mathbf{x})$	Neighborhood of $\mathbf{x}$
$\dot{\mathbf{x}}$	Time derivative of vector $\mathbf{x}$
$D\mathbf{X}(\mathbf{x})$	Derivative of vector field $\mathbf{X}$ at $\mathbf{x}$
$D_{\mathbf{x}}\mathbf{X}(\mathbf{x}, \boldsymbol{\mu})$	Derivative with respect to $\mathbf{x}$ of vector field $\mathbf{X}$ at $(\mathbf{x}, \boldsymbol{\mu})$
$\operatorname{tr} \mathbf{A}$	Trace of square matrix $\mathbf{A}$
$\det \mathbf{A}$	Determinant of square matrix $\mathbf{A}$
$\operatorname{diag}[\lambda_1, \ldots, \lambda_n]$	Diagonal matrix whose diagonal elements are $\lambda_1, \ldots, \lambda_n$
$\operatorname{diag}[B_1, \ldots, B_n]$	Block-diagonal matrix whose diagonal blocks are $B_1, \ldots, B_n$
$[\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_n]$	$n \times n$ matrix whose element a_{ij} is the j-component of the n-dimensional vector $\mathbf{a}_i$
$\operatorname{spec}(\mathbf{A})$	Spectrum of linear operator $\mathbf{A}$
$f(x) \overset{x \to 0}{=} O(g(x))$	There exist two positive constants A and a such that $\|f(x)\| \leq A\|g(x)\|$ for $\|x\| < a$
$f(x) \overset{x \to 0}{=} o(g(x))$	For any $\varepsilon > 0$, there exists $\delta > 0$ such that $\|f(x)\| \leq \varepsilon\|g(x)\|$ for $\|x\| < \delta$
$f(x) \sim g(x)$	$f(x)$ and $g(x)$ have the same asymptotic behavior
$f(x + 0)$	$\lim_{\varepsilon \to 0} f(x + \varepsilon)$, where $\varepsilon > 0$
$f(x - 0)$	$\lim_{\varepsilon \to 0} f(x + \varepsilon)$, where $\varepsilon < 0$
$a \approx b$	a is approximately equal to b
$a \lessapprox b$	a is less than or approximately equal to b
$a \gtrapprox b$	a is greater than or approximately equal to b
$G(N, M)$	Graph of order N and size M
$A(G)$	Adjacency matrix of graph G
$V(G)$	Set of vertices of graph G
$E(G)$	Set of edges of graph G
$N(x)$	Neighborhood of vertex x of a graph
$d(x)$	Degree of the vertex x of a graph
$d_{\text{in}}(x)$	In-degree of the vertex x of a digraph
$d_{\text{out}}(x)$	Out-degree of the vertex x of a digraph

L_{N}	Characteristic path length of network N.
C_{N}	Clustering coefficient of network N.
D_{N}	Diameter of network N.
$\binom{n}{k}$	Binomial number.
$P(X = x)$	Probability that the random variable X is equal to x.
F_X	Cumulative distribution function of the random variable X defined by $F_X(x) = P(X \leq x)$.
f_X	Probability density function of the absolutely continuous random variable X.
$X_n \xrightarrow{\mathrm{d}} X$	Sequence of random variables (X_n) converges in distribution to X.
$\widetilde{m}$	Median of a distribution defined by $F(\widetilde{m}) = \frac{1}{2}$.
$\langle X \rangle$	Average value of random variable X.
$m_r(X)$	Moment of order r of a random variable X; i.e., $m_r(X) = \langle X^r \rangle$.
$\sigma^2(X)$	Variance of a random variable X; i.e., $\sigma^2(X) = m_2(X) - m_1^2(X)$.
φ_X	Characteristic function of the random variable X.
$N(m, \sigma^2)$	Normal random variable on mean m and variance σ^2.
$\widehat{f}$	Fourier transform of function f.
$L_{\alpha,\beta}$	Probability density function of stable Lévy distribution.
f_{t}	Probability density of a Student's t-distribution.
$W = \{W_t \mid t \geq 0\}$	Stochastic process.

Contents