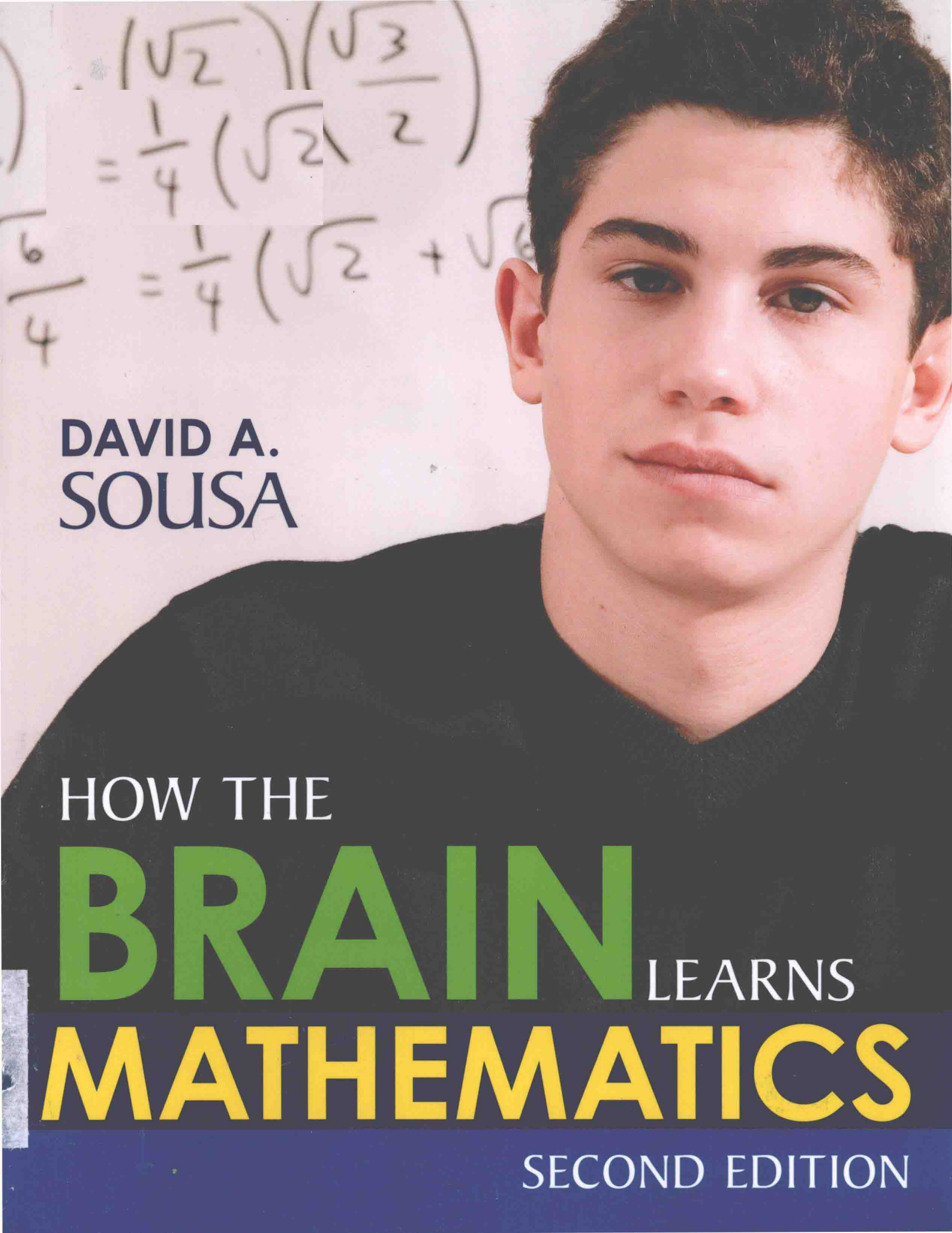




DAVID A.
SOUZA

HOW THE
BRAIN LEARNS
MATHEMATICS

SECOND EDITION

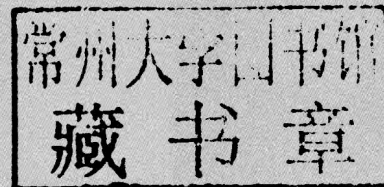
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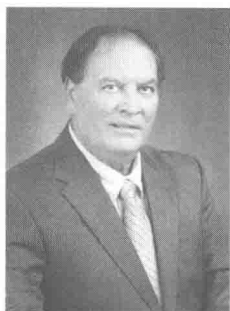
BRAIN LEARNS

MATHEMATICS

*A mathematician, like a painter or a poet, is a maker of patterns.
If his patterns are more permanent than theirs, it is because they are made with ideas.*

—Godfrey Harold Hardy
A Mathematician's Apology

About the Author



David A. Sousa, EdD, is an international consultant in educational neuroscience and author of 16 books that suggest ways educators and parents can translate current brain research into strategies for improving learning. A member of the Cognitive Neuroscience Society, he has conducted workshops in hundreds of school districts on brain research, instructional skills, and science education at the preK–12 and university levels. He has made presentations to more than 200,000 educators at national conventions of educa-

tional organizations and to regional and local school districts across the United States, Canada, Europe, Australia, New Zealand, and Asia.

Dr. Sousa has a bachelor's degree in chemistry from Bridgewater State University in Massachusetts, a Master of Arts in Teaching degree in science from Harvard University, and a doctorate from Rutgers University. His teaching experience covers all levels. He has taught senior high school science and served as a K–12 director of science, a supervisor of instruction, and a district superintendent in New Jersey schools. He was an adjunct professor of education at Seton Hall University for 10 years and a visiting lecturer at Rutgers University.

Prior to his career in New Jersey, Dr. Sousa taught at the American School of Paris (France) and served for 5 years as a Foreign Service Officer and science advisor at the USA diplomatic missions in Geneva (Switzerland) and Vienna (Austria).

Dr. Sousa has edited science books and published dozens of articles in leading journals on staff development, science education, and educational research. His most popular books for educators include *How the Brain Learns*, now in its fourth edition; *How the Special Needs Brain Learns*, second edition; *How the Gifted Brain Learns*; *How the Brain Learns to Read*, second edition; *How the Brain Influences Behavior*; *How the ELL Brain Learns*; *Differentiation and the Brain* (with Carol Tomlinson); and *How the Brain Learns Mathematics*, which was selected by the Independent Publishers' Association as one of the best professional development books of 2008. *The Leadership Brain* suggests ways for educators to lead today's schools more effectively. *Brainwork: The Neuroscience of How We Lead Others* was written for business and organizational leaders. His books have been published in French, Spanish, Chinese, Arabic, Korean, Russian, and several other languages.

Dr. Sousa is past president of the National Staff Development Council (now called Learning Forward). He has received numerous awards from

professional associations, school districts, and educational foundations for his commitment to research, staff development, and science education. He received the Distinguished Alumni Award and an honorary doctorate from Bridgewater (Mass.) State University, and an honorary doctorate in humane letters from Gratz College in Philadelphia.

Dr. Sousa has been interviewed by Matt Lauer on NBC's *Today* show, on other television programs, and on National Public Radio about his work with schools using brain research. He makes his home in South Florida.

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Introduction

Numbers rule the universe.

—Pythagoras

EVERYONE CAN DO MATHEMATICS ■

Human beings are born with some remarkable capabilities. One is language. In just a few years after birth, toddlers are carrying on running conversations without the benefit of direct instruction. Over the next few years, their sentences become more complex and their vocabulary grows exponentially. By the age of 10, they understand about 10,000 words and speak their native language with 95 percent accuracy.

Another innate talent is number sense—the ability to determine the number of objects in a small collection, to count, and to perform simple addition and subtraction, also without direct instruction. Yet by the age of 10, some of these children are already saying, “I can’t do math!” But you never hear them saying, “I can’t do language!” Why this difference?

One reason is that spoken language and number sense are survival skills; abstract mathematics is not. In elementary schools we present complicated notions and procedures to a brain that was first designed for survival in the African savanna. Human culture and society have changed a lot in the past 5,000 years, but the human brain has not. So how does the brain cope when faced with a task, such as multiplying a pair of two-digit numbers, for which it was not prepared? Thanks to modern imaging devices that can look inside the living brain, we can see which cerebral circuits are called into play when the brain tackles a task for which it has limited innate capabilities. That the human brain *can* rise to this challenge is testimony to its remarkable ability to assess its environment and make calculations that can safely land humans on the moon and send a space probe into orbit around a planet hundreds of millions of miles away.

Children often say, “I can’t do math!” But you never hear them say, “I can’t do language!” Why this difference?

WHAT IS MATHEMATICS? ■

To most people, mathematics is about calculating numbers. Some may even expand the definition to include the study of quantity (arithmetic),

space (plane and solid geometry), and change (calculus). But even this definition does not encompass the many areas where mathematics and mathematicians are found. A broader definition of mathematics comes from W. W. Sawyer (1982). In the 1950s, he described mathematics as the “classification and study of all possible patterns” (p. 12). He explained that *pattern* was meant “to cover almost any kind of regularity that can be recognized by the mind” (p. 12).

Other mathematicians who share Sawyer’s view have shortened the definition even further: Mathematics is the science of patterns. Devlin (2000) not only agrees with this definition but has used it as the title of one of his books. He explains that patterns include order, structure, and logical relationships, and go beyond the visual patterns found in tiles and wallpaper to those that occur everywhere in nature. For example, patterns can be found in the orbits of the planets, the symmetry of flowers, how people vote, the spots on a leopard, the outcomes of games of chance, the relationship between the words that make up a sentence, and the sequence of sounds we recognize as music. Some patterns are numerical and can be described with numbers, such as voting patterns of a nation or the odds of winning the lottery. Other patterns, such as a leopard’s spots, are visual designs not connected to numbers at all.

Mathematics can be defined simply as the science of patterns.

Devlin (2000) further points out that mathematics can help make the invisible visible. Two-thousand years ago, the Greek mathematician Eratosthenes was able to calculate the diameter of Earth with considerable accuracy and without ever stepping foot off the planet. The equations developed by 18th-century mathematician Daniel Bernoulli explain how a jet plane flying overhead stays aloft. Thanks to Isaac Newton, we can calculate the effects of the unseen force of gravity. More recently, linguist Noam Chomsky has used mathematics to explain the invisible and abstract patterns of words that we recognize as a grammatical sentence.

If mathematics is the science of patterns and if visible and invisible patterns exist all around us, then mathematics is not just about numbers but about the world we live in. If that is the case, then why are so many students turned off by mathematics before they leave high school? What happens in those classrooms that gives students the impression that mathematics is a sterile subject filled with meaningless abstract symbols? Clearly, educators have to work harder at planning a mathematics curriculum that is exciting and relevant and at designing lessons that carry this excitement into every day’s instruction.

I will leave the discussion of what content to include in a preK–12 mathematics curriculum to experts in that area, especially now that the Common Core State Standards for Mathematics (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) have been released and adopted by many states. My purpose here is to suggest how the research in cognitive neuroscience can be used to plan lessons in mathematics that are more likely to result in learning and retention.

Why Is Learning Mathematics So Hard?

Succeeding in high school mathematics is still no easy feat. Take a look at Table I.1. The results of the 2013 National Assessment of Educational

Table I.1 Proficiency Levels for Grades 4, 8, and 12 in Mathematics on NAEP, 2005–2013

Year	Grade 4				Grade 8				Grade 12			
	<i>Below Basic</i>	<i>At Basic</i>	<i>Proficient</i>	<i>Advanced</i>	<i>Below Basic</i>	<i>At Basic</i>	<i>Proficient</i>	<i>Advanced</i>	<i>Below Basic</i>	<i>At Basic</i>	<i>Proficient</i>	<i>Advanced</i>
2013	17	41	34	8	26	38	27	9	35	39	23	3
2011	18	42	34	7	27	39	26	8	—	—	—	—
2009	18	43	33	6	27	39	26	8	36	38	23	3
2007	18	43	34	6	29	39	25	7	—	—	—	—
2005	20	44	31	5	31	39	24	6	39	38	21	2

SOURCE: NAEP (2013).

Progress (NAEP) mathematics tests of twelfth-grade students revealed that 23 percent were considered proficient in mathematics skills. This was the same percentage found in the 2009 assessment. No educator or parent can feel reassured by results showing such a low percentage of high school seniors performing at this proficiency level in mathematics. For fourth graders, 34 percent were proficient, compared with 33 percent for 2009 and 34 percent for 2011. As for the eighth graders, 27 percent scored proficient in 2013, compared with 26 percent for both the 2009 and 2011 assessments. The improvement was not significant (NAEP, 2013). Despite all the attention and high-stakes testing focused on mathematics instruction in recent years, achievement results have barely moved.

Explanations for this lackluster performance abound. Some say that learning mathematics is difficult because it is so abstract and requires more logical and ordered thinking. Others say that the various symbols used in mathematics make it similar to tackling a foreign language. Education critics maintain that only a few students are really developmentally incapable of handling mathematics and that the poor performance stems mainly from inadequate instruction. They cite the so-called “math wars” as hindering major progress in mathematics curriculum development, similar to what the “reading wars” did to reading instruction during the 1990s.

Impact of Teacher Preparation

Another potential factor affecting students’ success in mathematics is the content knowledge of their teachers. Numerous studies have shown that middle and high school students learn more when their teachers have certifications or degrees in mathematics (e.g., Wayne & Youngs, 2003). Although states have been increasing the course requirements for individuals to be licensed to teach mathematics in secondary schools, problems persist. A recent study of 115 prospective middle school mathematics teachers at a large U.S. public university revealed that a substantial number of them had a limited knowledge of algebra for teaching (Huang & Kulm, 2012). The students made numerous mistakes when solving quadratic equations

and in algebraic reasoning and manipulation. They also had difficulties with connecting algebraic and graphical representations.

A survey by the U.S. Department of Education showed that only 63 percent of the nation's nearly 144,000 high school mathematics teachers have both a college major and certification in mathematics (Hill, 2011). Nearly 26 percent have only a major or only certification in mathematics, and about 11 percent have neither a major nor certification in mathematics. This last group is referred to as out-of-field teachers. Other surveys find that out-of-field teachers are more likely to be found in high-poverty schools or in schools where they are assigned to the most challenging students (e.g., Kalogrides, Loeb, & Betelle, 2011).

Responses From Mathematics Educators

The National Council of Teachers of Mathematics (NCTM) published the *Principles and Standards for School Mathematics* in 2000, proposing five process standards and five content standards for preK–12 mathematics instruction. Since then, interpretation of the standards in the elementary and middle school grades became so broad that NCTM decided to refocus the curriculum at those grade levels.

In 2006, NCTM released *Curriculum Focal Points for Mathematics in Prekindergarten Through Grade 8*, which identified three important mathematical topics at each level, described as “cohesive clusters of related knowledge, skills, and concepts” that form the necessary foundation for understanding concepts in higher-level mathematics. The publication was intended to bring more coherence to the very diverse mathematics curricula currently in use. It provided a framework for states and districts to design more focused curricular expectations and assessments for preK–8 mathematics curriculum development. Shortly thereafter, the National Mathematics Advisory Panel (2008) published its final report, making recommendations for curriculum changes, teacher education, instructional practices and materials, and assessment. In the meantime, the National Governors Association and the Council of Chief State School Officers launched an effort to develop standards in mathematics and English/language arts. They were finally published in 2010 as the Common Core State Standards for Mathematics, and have been adopted by most of the states. We will discuss these standards further in Chapters 4 and 5.

Whether this new effort will succeed in improving student achievement in mathematics remains to be seen. In the meantime, teachers enter classrooms every day prepared to help their students feel confident enough to master mathematics principles and operations. One thing seems certain: Students who are poor in mathematics in their early years remain poor in mathematics in their later years.

■ ABOUT THIS BOOK

I am often asked to give specific examples of how the fruits of scientific research have made an impact on educational practice. That question is a lot easier to answer now than it was 20 years ago because recent discoveries in cognitive neuroscience have given us a deeper understanding of the

brain. Thanks to brain-scanning technology we now have more knowledge about our short-term and long-term memory systems, the impact of emotions on learning, how we acquire language and motor skills, and how the brain learns to read. But only more recently have researchers begun to examine closely the neural mechanisms involved in processing arithmetic and mathematical operations.

Questions This Book Will Answer

This book will answer questions such as these:

- What innate number capabilities are we born with?
- How much number manipulation and basic arithmetic can young children learn without direct instruction?
- Why are native speakers of Asian languages able to learn counting sooner and faster than are English-speaking children?
- What kind of number word system could help English-speaking children learn to count easier and faster?
- Why is learning mathematics so difficult for so many students?
- For everyday classroom practice, what are the implications of the current research on how we learn to calculate?
- How do the mind-sets of teachers and students affect mathematics instruction and learning?
- How does the brain manage to deal with abstract mathematics concepts?
- How is the omnipresent technology affecting students' attention and memory systems?
- What strategies are effective in teaching students with reading difficulties to learn mathematics?
- How can we tell if a student's difficulties in mathematics are the result of environmental factors or developmental deficits?
- What strategies should teachers of mathematics consider when planning lessons?
- What have brain-imaging studies revealed about the nature of dyscalculia?
- How can elementary and secondary school teachers successfully detect mathematics difficulties?
- What instructional strategies work best with students who have difficulties in mathematics?
- What instructional strategies are successful with English language learners who are having difficulties in mathematics?
- How can teachers use research on how the brain learns mathematics to design an instructional model for teaching preK–12 mathematics?
- How can integrating the arts into mathematics lessons improve instruction and learning?

Chapter Contents

Chapter 1—Developing Number Sense. Children's ability to determine quantities begins soon after birth. This chapter examines the components of this innate number sense and how it leads to counting and basic

arithmetic operations. It looks at the regions of the brain that work together and manipulate numbers and the ways language affects how quickly and easily children learn to count.

Chapter 2—Learning to Calculate. Because counting large numbers is not a survival skill, the brain must learn mathematical concepts and procedures. This chapter explores the various developmental stages the brain must go through to understand number relationships and manipulations, such as in multiplication. It discusses why the brain views learning to multiply as an unnatural act, and it suggests some other ways to look at teaching multiplication that may be easier for students to learn.

Chapter 3—Reviewing the Elements of Learning. This chapter presents some of the recent findings from cognitive neuroscience, including discoveries about the power of feedback, memory systems and how technology affects them, the nature and value of practice and rehearsal, lesson timing, formative assessments, and the benefits of writing in mathematics classes. Gender differences and fixed and growth mindsets in mathematics, as well as learning and teaching styles, are also discussed. The chapter concludes with a section on strategies to motivate students in mathematics.

Chapter 4—Teaching Mathematics to the Preschool and Kindergarten Brain. Although young children have an innate number sense, certain instructional strategies can enhance those capabilities and prepare children to be more successful in learning arithmetic operations. This chapter suggests some of those strategies.

Chapter 5—Teaching Mathematics to the Preadolescent Brain. Here we look at the development and characteristics of the preadolescent brain and how they affect the individual's emotional and rational behavior. The chapter offers suggestions on how lesson plans can be modified, from the primary grades up through middle school, to take into account the nature of this developing brain so more of these students will be successful in learning mathematics. Also included is a discussion of how the Common Core State Standards for Mathematics were developed and what they mean for mathematics instruction.

Chapter 6—Teaching Mathematics to the Adolescent Brain. Similar to the previous chapter, we review the nature of the adolescent brain and suggest what considerations need to be made to adapt lessons to meet these students' needs. Included here are discussions of mathematical reasoning and instructional choices—such as layering the curriculum, the flipped classroom, and graphic organizers—that can be very effective strategies for making mathematics relevant and interesting to today's students.

Chapter 7—Recognizing and Addressing Mathematics Difficulties. Numerous suggestions are offered in this chapter to enable teachers to identify and help students experiencing difficulties in learning mathematics, including math anxiety. This chapter discusses the major differences between the environmental and developmental factors that contribute to mathematics difficulties. It presents some tested strategies that teachers of all grade levels can use with students who are poor in mathematics to help them understand number operations and gain a more accurate and deeper understanding of mathematical concepts. A new section discusses strategies for English language learners who are having difficulties in mathematics.