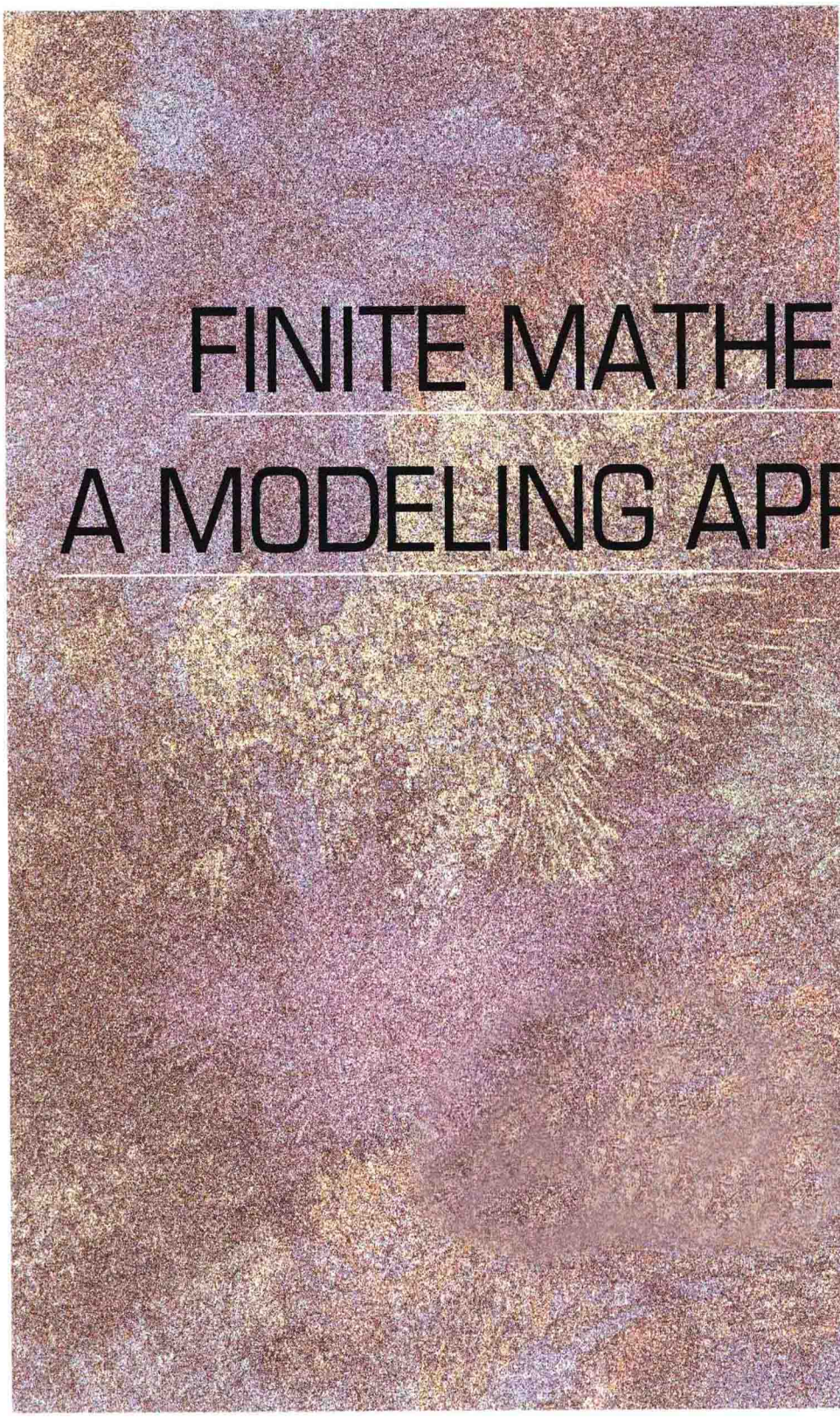
FINITE MATHEMATICS

A MODELING APPROACH

Richard Bronson *Gary Bronson*



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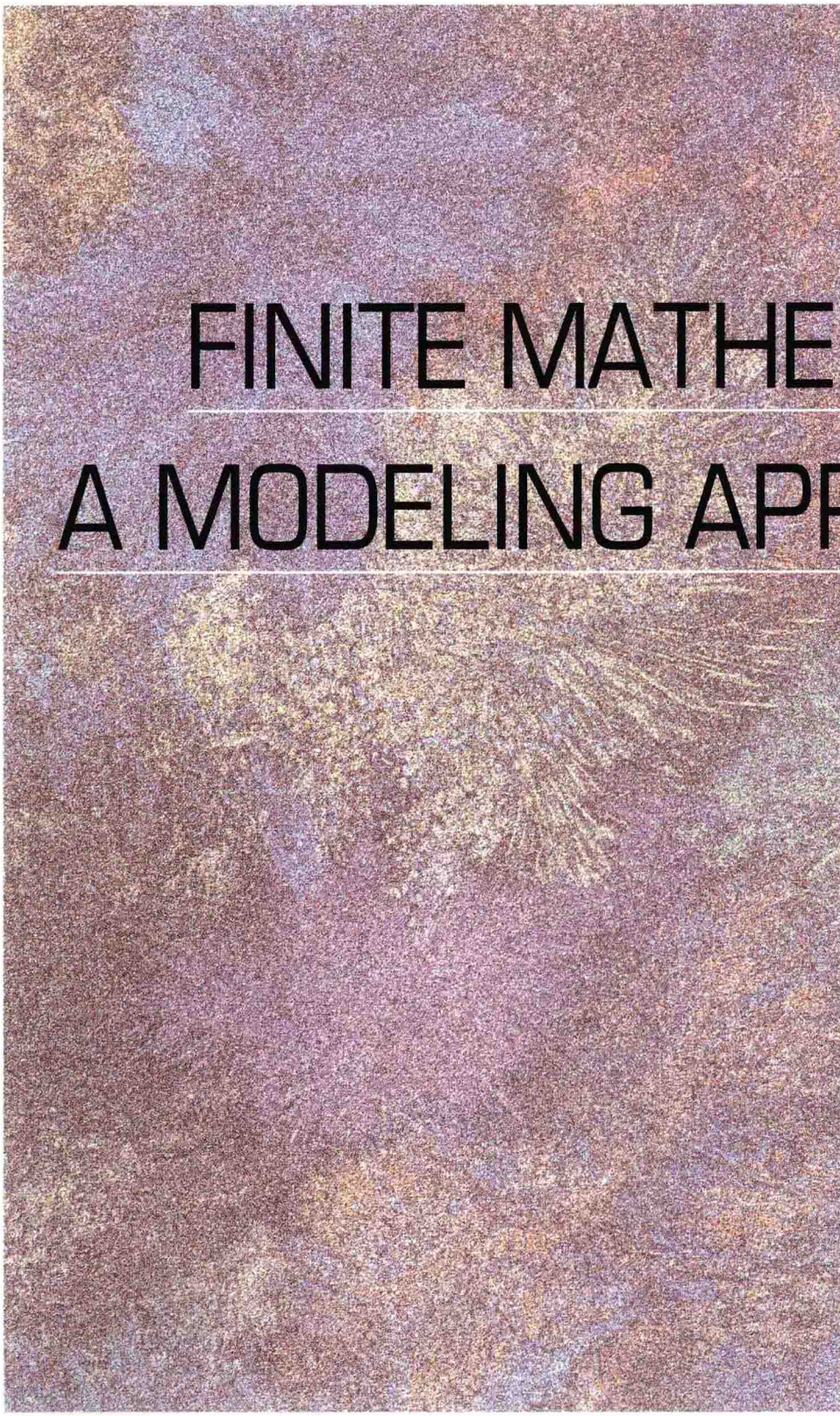


FINITE MATHEMATICS: A MODELING APPROACH

To the many wonderful teachers
who showed us why and how,
especially
Mary M. Brown
Mary Mannion
Nicholas J. Rose

Philosophy is written in this grand book—I mean the universe—which stands continually open to our gaze, but it cannot be understood unless one first learns to comprehend the language and interpret the characters in which it is written. It is written in the language of mathematics.

GALILEO

Mathematics, rightly viewed, possesses not only truth, but supreme beauty.

BERTRAND RUSSELL

PREFACE

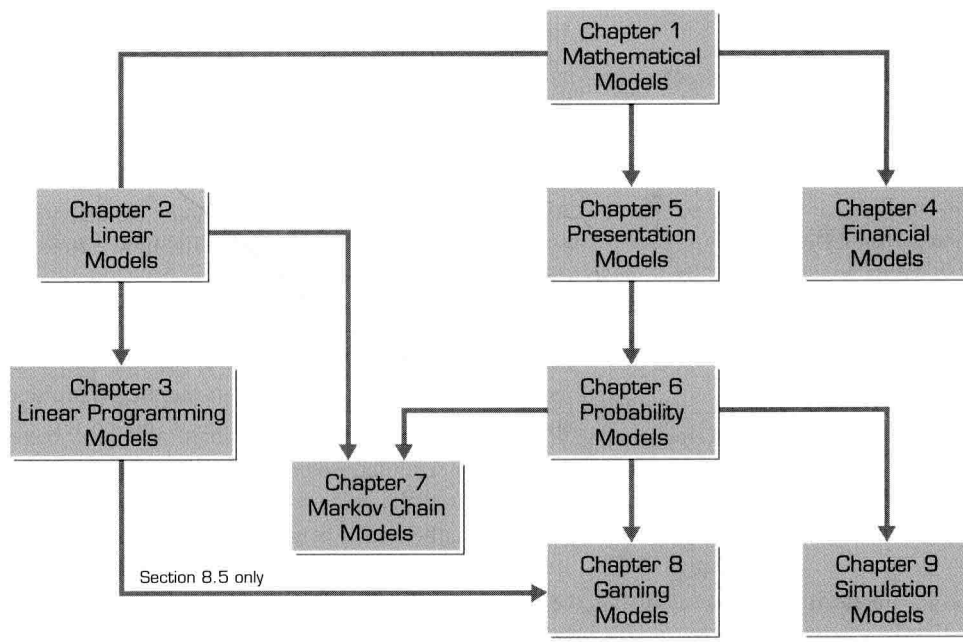
The world has changed dramatically in the last hundred years. As society became more complex, so too did its problems. Issues such as solid waste disposal, global warming, international finance, pollution, and nuclear proliferation are relatively new, and solutions to problems in these areas challenge even our best technology and most advanced mathematics.

New problems often require new solution techniques. To better understand the dynamics of intricate systems, analysts simplify them into more manageable parts through a process called modeling. They then study the model to gain a better understanding of the real system they hope to manage. Basic arithmetic skills are inadequate for much of this analysis, so new solution procedures have been developed in probability, linear programming, game theory, and simulation.

In this book, we introduce readers to some of the newest ideas in mathematical thought. We describe in great detail the process of mathematical modeling, and we develop techniques for analyzing models. The theme of this book is modeling, and each chapter focuses on a class of models in common use today. The intent is to develop an understanding of modern mathematics that will assist managers of systems—be they social, political, commercial, or ecological systems—with their decision-making processes. Through such understanding, both the beauty and relevance of mathematics becomes apparent.

In this book, new concepts are carefully illustrated in situations that are common to most readers. Academic transcripts, road maps, and organizational charts are used to illustrate models in Chapter 1. Matrices are introduced as inventory levels in a department store. Financial decisions begin with savings accounts and later extend to installment loans. Gaming models are developed for casino games, state lotteries, and sweepstakes.

Chapter 1 introduces the concept of a model and develops the rudimentary skills of modeling, from identifying important factors to establishing relationships between those factors. Great care is taken to explain the process of modeling, because skill in modeling is central to all that follows. Chapters 2 through 9 each develop a particular category of models: linear models, linear programming models, financial models, presentation models, probability models, Markov chain models, gaming models, and simulation models.



The progression of chapters is logically consistent, although there is great flexibility in the order in which topics can be studied. Of course, some topics by necessity must follow others, as shown in the preceding diagram. While Section 5.1 on sets and Section 5.4 on tree diagrams are essential to Chapter 6 on probability, the other sections in Chapter 5 can be skipped if one wants to get to probability topics quickly. It is also possible to move directly from Chapter 6 to Chapter 9 on simulation.

The order of topics can be changed to suit individual goals. Financial models can be addressed at any point after Chapter 1. Combinatorics and Bernoulli trials appear at the end of Chapter 6, because we prefer to introduce all the concepts of probability first, without complicating those concepts with the strictly mechanical procedures for counting. Others may prefer to move these two sections to the end of Chapter 5, following tree diagrams, or even to the end of Chapter 8.

The focus on modeling, the inclusion of simulation, and the sources for many of our examples and problems are unique to our presentation. To emphasize the relevance of mathematics to everyday concerns, we have taken the information for many of our problems from newspapers, magazines, professional journals, government agencies, the General Social Survey, and books such as the Economists Book of Vital World Statistics.

LEARNING AIDS

To make this book accessible to the largest possible audience, we have included a variety of learning aids.

Highlight boxes

Highlight boxes are interspersed throughout the text to showcase particularly important material. These boxes emphasize new ideas, useful procedures, and notable models.

Illustrative examples

New methods are coupled with a large number of completely worked-out problems. We are convinced that such examples, when presented in detail, are essential to promoting real understanding, and we use such examples liberally—there are over three hundred fully developed examples in this book.

Visual aids

Over four hundred figures, tables, and photographs are included to help readers visualize the material discussed in this book. Reprints from newspapers and magazines show the relevance of certain models to everyday life.

Chapter keys

At the end of each chapter, a chapter summary in the form of chapter keys provides a concise review. These summaries list new terminology, important concepts, and key formulas and procedures.

Chapter tests

Each chapter ends with a set of problems that readers can use to test themselves on the material in that chapter. The even-numbered problems in each set can be used as one test, and the odd-numbered problems as a second test.

EXERCISES

Because mathematics is not a spectator sport, one must experience mathematics to understand it. Therefore, this book is rich in problems, containing over 2,400 exercises. The exercises are separated into six distinct categories, each designed to support a different type of skill mastery or cognitive learning.

Improving Skills consists of problems on manipulation. Readers are asked to apply mechanical operations to very basic information. The purpose of these problems is to develop competency in the basic steps required for various models.

Creating Models asks readers to construct mathematical models from descriptive problems. Its function is to develop the modeling capabilities of the student.

Exploring in Teams encourages readers to expand their horizons and develop material beyond what is explicitly covered in the book. The problems are exploratory and are therefore ideal for group learning environments where the dynamics of a team can help forge new understandings.

Exploring with Technology directs students to use existing technologies, particularly graphing calculators and electronic spreadsheets, to explore the relationships in this book in greater depth. Technology can enhance understanding, and the problems in this section are designed to do just that. When used indiscriminately, however, technology can obscure underlying concepts. Therefore, we do not ask students to use calculators or computers in those sections where the use of technology is inappropriate.

Reviewing Material is designed for review and reinforcement of material from previous chapters. These problems first appear in Chapter 2, where they are limited to a single problem from Chapter 1 per section. In each section of Chapter 3 there are two problems dealing with topics from each of the previous two chapters. There are three problems in each section of Chapter 4 and four problems in each section of Chapter 5. From Chapters 6 through 9, the number of review problems in each section is capped at four. Review problems are never the focus of a section, so the number of such problems in each section is strictly limited.

Recommending Action asks readers to recommend, in writing, actions for addressing particular problems that are presented in writing to the problem solver. While problem solvers generally use mathematical techniques to analyze systems, ultimately any recommendation must be communicated in an acceptable form (usually, in writing).

PEDAGOGICAL INITIATIVES

Group learning, writing across curriculum, and enabling technologies are now common features of many mathematics courses. This book supports all three of these innovations.

Group learning

Any problem or set of problems in the exercise sections can be assigned to students for solution by a team. However, the problems in *Exploring in Teams* are particularly good for this purpose. These problems take students into unchartered waters where group dynamics provide an effective medium for exploration and discovery.

Writing across curriculum

Any problem or set of problems in the exercises can be assigned as part of a report to be typed and submitted. However, the problems in *Recommending Action* are designed expressly for this purpose. They require the reader to synthesize the material in each section into a memo to help someone else address a problem.

Enabling technologies

It is assumed that all students have access to simple calculators that perform the basic arithmetic operations, raise numbers to a power, and create random numbers. Raising

numbers to nonintegral powers is essential for dealing with financial models in Chapter 4; a simple random number generator is needed for the simulation models in Chapter 9; and the ability to do basic arithmetic operations quickly and accurately is assumed throughout the book. No other technology is needed within the main body of the text or for most of the problems. The only exceptions are the problems in *Exploring with Technology*, which exploit the power of a graphing calculator, electronic spreadsheet, or linear programming software to enhance basic understanding and, on occasion, to further develop some of the concepts introduced in the book.

SUPPLEMENTS

Solutions Manuals

The *Instructor's Solutions Manual* provides complete solutions to all *Exploring in Teams* and *Recommending Action* problems and to the even-numbered *Improving Skills*, *Exploring with Technology*, and *Creating Models problems*. The *Student's Solutions Manual* contains complete solutions to all *Reviewing Material* and *Testing Yourself* problems and to odd-numbered *Improving Skills*, *Exploring with Technology*, and *Creating Models* problems. Both manuals were prepared by Alison Paradise of the University of Puget Sound.

Test Bank

Leslie Cobar, of the University of New Orleans, wrote the Test Bank which contains approximately 100 test questions per chapter, of which 70% are multiple choice.

TI-82 Graphing Calculator Manual

Written by Maureen Dion of San Joaquin Delta College, this manual discusses the various keys and commands on the TI-82 and illustrates their usage through examples that are either taken from the text or similar to problems in the text. Chapter organization follows that of the Bronson/Bronson text. An introductory section provides instructions on how to use the TI-82 to do basic computations. An appendix highlights some of the TI-85 features that are either different from or additional to the TI-82.

Explorations in Finite Mathematics Software

This IBM format software package, created by David Schneider of the University of Maryland, contains a wide selection of utilities for topics such as Gaussian elimination, matrix operations, graphical and simplex methods for linear programming problems, simple and compound interest, loan, and annuity analysis, and much more. All routine are menu driven. Matrix routines use and display rational numbers, and matrices can be saved and printed. Software is accompanied by a manual containing instructions and additional student exercises.

Transparency Masters

Transparencies of 100 figures and tables from the text are available.

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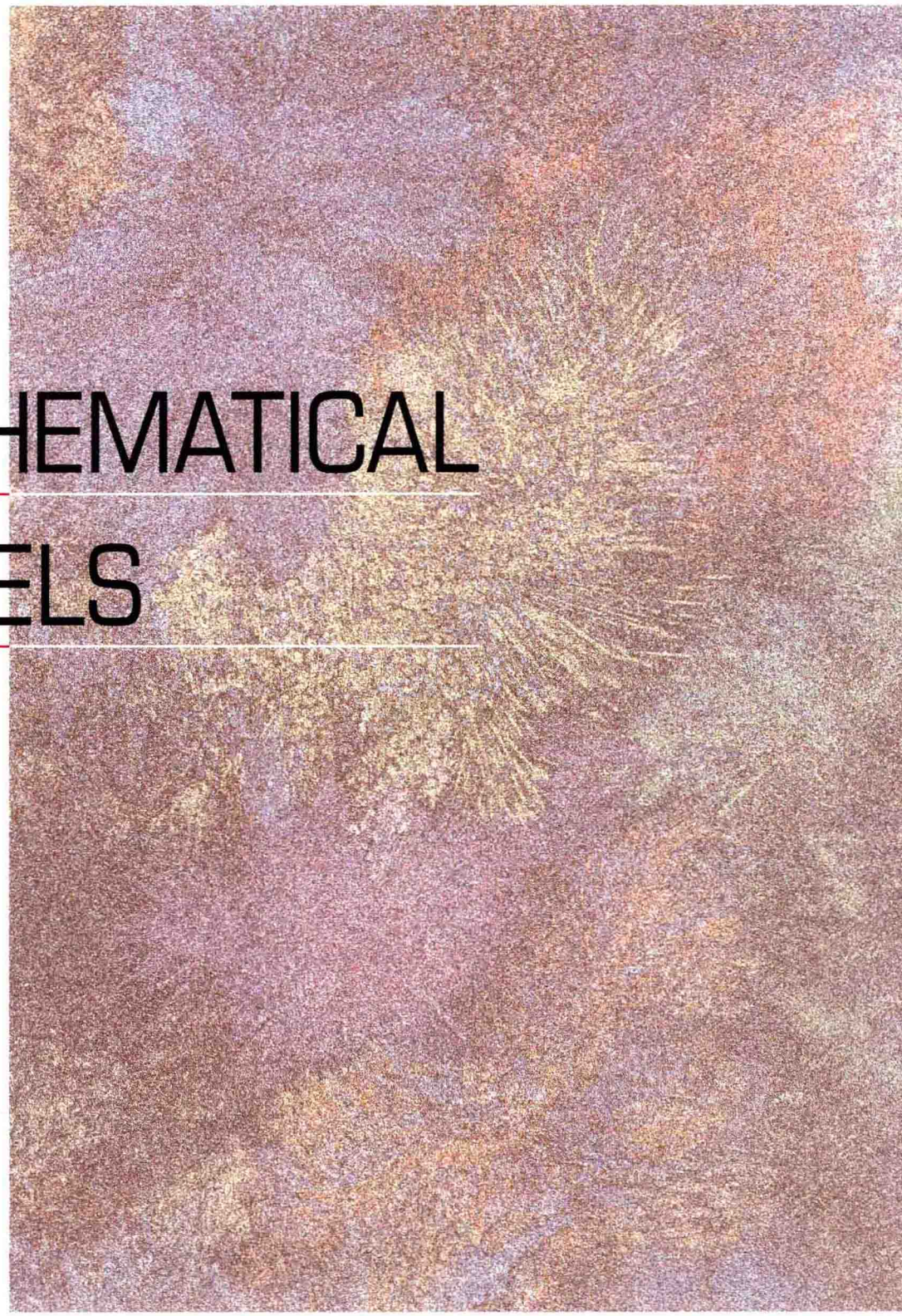
MATHEMATICAL MODELS



1.1 THE MODELING PROCESS

There are two reasons for studying mathematics: First, mathematics provides a framework for solving real-world problems, and this, in turn, helps people prosper. Second, mathematics itself is an art and science anchored on reason and logic. In a world often filled with irrational behavior and haphazard events, the lure of an ordered world governed solely by rational thought has an irresistible appeal to some—the mathematicians.

For most people, the appeal of mathematics is directly proportional to its usefulness in solving problems. Many individuals are employed to solve problems—problems in government, economics, ecology, and commerce, to mention just a few areas. As the world becomes more complex, so do its problems, and any advantage gained in the ability to solve some of those problems is an advantage to be cherished.

Years ago, the world was simpler and so was the mathematics needed to be effective. Proficiency in arithmetic was often sufficient. Such skills are still essential, but they are not adequate for solving many of today's problems. One approach to this situation is to develop more sophisticated mathematical techniques; a second approach is to simplify problems so that they can be solved with existing methodologies. We shall develop both strategies in this book.

A model is a representation of a particular situation.

The technique of replacing a complex problem with a simpler one is called *modeling*. A *model* is a representation of a particular situation. The college transcript in Figure 1.1 is a model of a person's academic achievement, the map in Figure 1.2 is a model of a road network, and the organizational chart in Figure 1.3 is a model of a company's management structure. Many situations, especially dynamic ones that change with time, can be modeled by mathematical equations.

Very few models are ever complete; that is, models do not reveal every aspect of the situation they represent. A college transcript does not indicate the ease with which the grades were achieved, a road map does not include weather conditions, and an organizational chart does not describe the personalities of the people filling the positions.

FIGURE 1.1
A college transcript as a model of academic achievement.

PERMANENT RECORD

Sem. End Mo. Yr.	Cat. No.	Course Title	Grade
-12/94	82EL102*	ENGL SEC LANG II	A
-12/94	88MG505	MGMT THRY & PRAC	B+
-12/94	88CS525	INTR COMP SCIENCE	A
-12/94	88CS550	BUSI PROG TECH	A
-		4PT SEM GPR 3.82	12.0
-		4PT CUM GPR 3.77	9.0
-05/95	87AC610	MGRL ACCT APPL	A
-05/95	87FN506	FIN ANALYSIS	A
-05/95	88CS603	COMPUTER ARCHITECTURE	A
-05/95	88CS720	MGMT INFO SYS	A
		4PT SEM GPR 4.00	12.0
		4PT CUM GPR 3.90	21.0

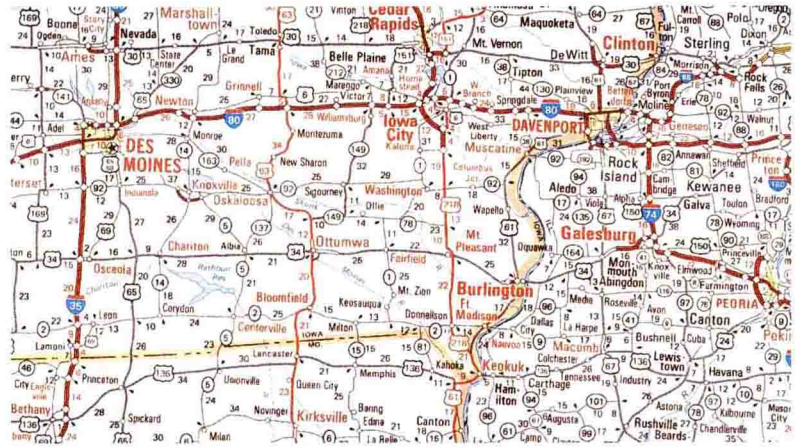
EXAMPLE 1

A patient's medical record in a doctor's office is a model of that patient's health. List some of the factors that this model would include and some that it may neglect.

Solution

This model would include the patient's vital medical statistics, such as age, weight, and blood pressure, from the patient's last physical examination. It should contain the patient's

FIGURE 1.2
A map as a model of a road network.



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medical history, including major illnesses and operations, as well as relevant family history on genetically linked diseases. It would also list the patient's allergies and reactions to medicines and contain copies of all reports from laboratory tests and recommended consultations with other doctors.

The model may not include the patient's financial condition, which could be a cause of stress or the current state of the patient's relationships (antagonistic, distant, harmonious) with friends and family members. It might also neglect behavior patterns, such as a passion for tanning, that could affect health, and it would not include information the patient omitted either purposefully or through forgetfulness. In addition, the model will not include information on emerging conditions that are not yet noticeable, such as microscopic tumors.

Models are built for a particular purpose that often requires representing only part of a situation. A flowchart for a computer program (see Figure 1.4) may be an adequate model

FIGURE 1.3
An organizational chart as a model of corporate structure.