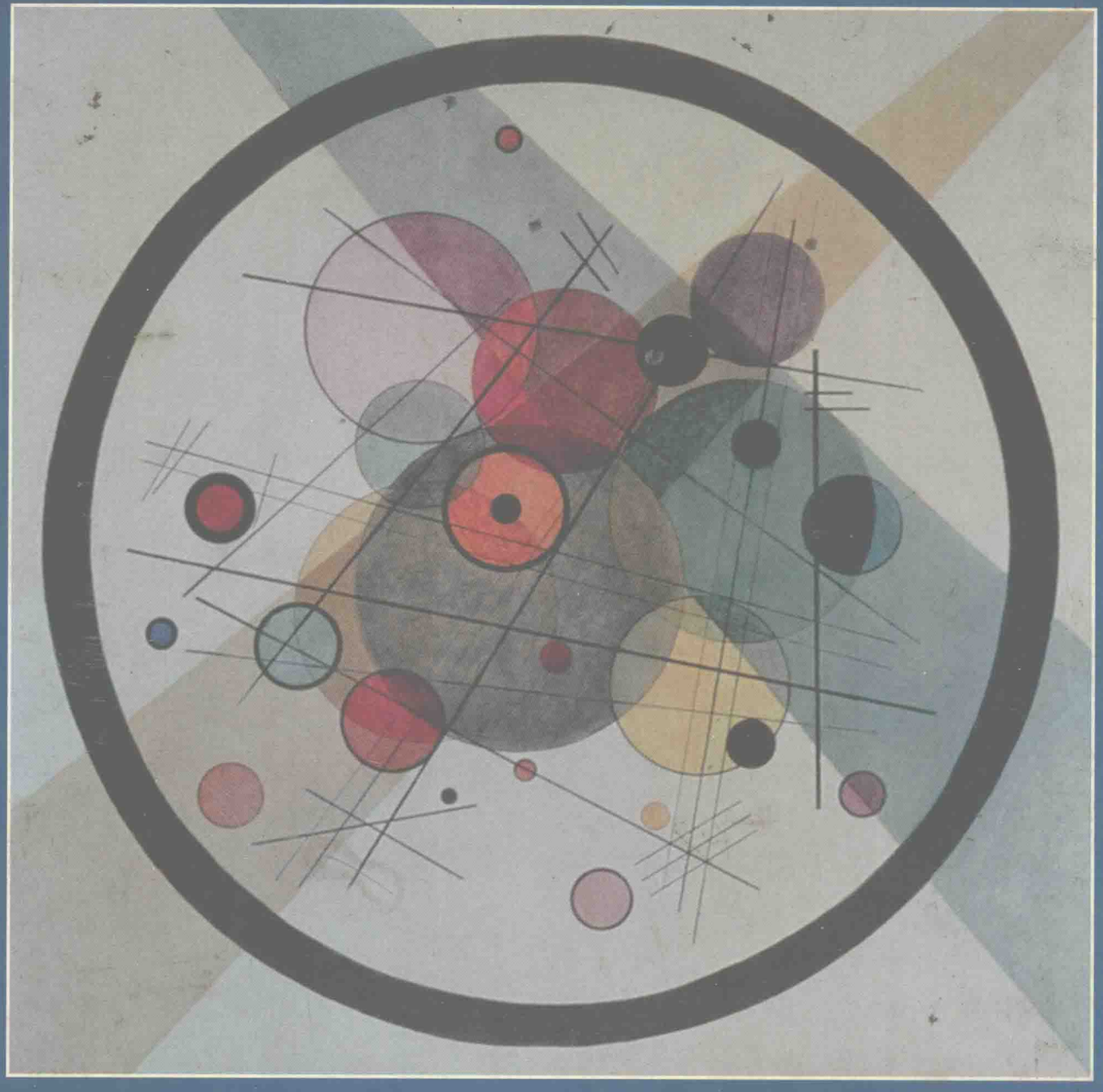
Elementary Algebra

Mugridge



ELEMENTARY

Algebra

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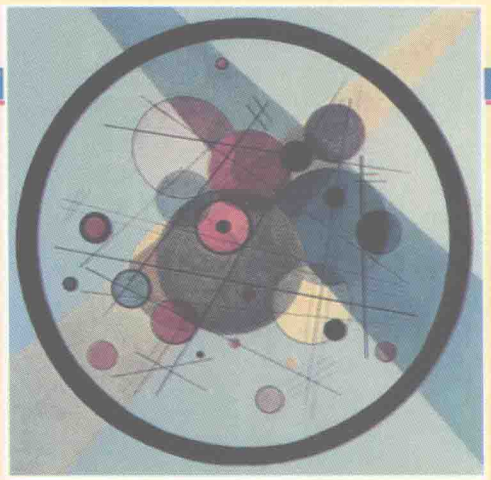

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ELEMENTARY Algebra

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This book is dedicated to my children,
Lyn and Jason

Preface

This book is intended for students who have no background in elementary algebra, but who have had some experience with arithmetic. Each concept is introduced by example before the definition is given, and the student is encouraged to become an active participant. A complete pedagogical system that is designed to motivate the student and make mathematics more accessible includes the following features:

Key features

Chapter Overviews Each chapter begins with an Overview to introduce the student to the material and demonstrate its relevance to the real world.

Objectives Each section contains Objectives to help the student focus on skills to be learned in the chapter.

Examples with Solutions Approximately 700 examples with complete, worked-out solutions are provided; explanations are highlighted in blue print. Some of the examples contain steps that can be done mentally once the student has achieved a certain level of proficiency (although performing these steps mentally may not be possible at first); this logo 

calls the student's attention to the highlighted parts of the equation. A blue triangle indicates the end of the solution.

Common Error Boxes Wherever appropriate, students are shown errors that are commonly made. The correct approach is then illustrated and explained.

Strategies After developing a mathematical technique, such as factoring a trinomial, a summary of the technique is given as a strategy. This will provide the student with a reference when doing problems from the exercise set.

Calculator Crossovers Many of the examples are followed by problems that are specifically designed to be solved with a calculator. These problems are similar to the preceding examples and illustrate the role of the calculator in solving problems. Calculator Crossovers are set off by a calculator logo for easy reference. Calculator Crossovers may be used at the instructor's discretion.

Learning Advantages These hints occur in Chapters 1 through 5 to give students additional help; they immediately precede the end-of-section exercises. For more detailed explanations on studying algebra, I recommend the pamphlet *How to Study Mathematics* by James Margenau and Michael Sentlowitz (The National Council of Teachers of Mathematics, Inc., Reston, Virginia, 1977, from which the Learning Advantages are adapted).

Exercises Graded by level of difficulty, the exercises are carefully constructed so that there is odd-even pairing. In addition, each exercise problem is related to an example from the section. The numbers used in the examples and exercises have been carefully selected so that the student becomes confident in using fractions and decimals in addition to the integers. The numbers, however, do not become so complicated as to "turn off" the student or teacher. All computations can be done without the use of a calculator.

Beginning with Chapter 2, each exercise set contains **Review Exercises** to help the student understand and remember concepts studied in earlier sections. The **Enrichment Exercises** at the end of each section are challenging problems that the more capable students will be able to solve. The Answer Section in the back of the book contains answers to the odd-numbered exercises and the Review Exercises and all of the Enrichment Exercises.

Applications Whenever appropriate, word problems have been included so that the student constantly practices translating word phrases and statements into mathematical equations. The word problems are realistic and are culled from applications in geometry, physics, business, economics, and psychology. The structure of problem solving is emphasized throughout the text, beginning with the simple conversion of word phrases into mathematical expressions. Many concepts introduced in early chapters are repeated using different applications.

Chapter Summary and Review Each chapter concludes with a comprehensive review which includes definitions and strategies learned in the chapter. All terms are keyed to the section number for easy reference. Marginal examples are included to further illustrate the concepts reviewed.

Review Exercise Set These exercises are included at the end of the chapter and represent all types of problems to ensure that the student has attained a level of proficiency and is comfortable proceeding to the next chapter. All exercises are keyed to sections so the student can refer to the text for assistance. Answers to all of these exercises can be found in the back of the book.

Pedagogical Use of Color This text uses color in the figures and to highlight the various pedagogical features throughout the text. Multiple colors are useful, for example, to distinguish between two lines that are graphed simultaneously, or to highlight important statements. The complete color system is described in more detail on page xvii.

An overview of the book

The main thrust of this book is to enable students to develop algebraic skills to be used to solve word problems. This is the primary aim of each chapter; we start with developing a skill and then use it to solve word problems. In addition, many geometric problems are included. A chapter-by-chapter overview follows:

Chapter 1 Operations and variables

Since the foundation of algebra is arithmetic, it is reviewed comprehensively in this chapter, starting with addition, subtraction, multiplication, and division of fractions. Next, we review basic symbols and order of operations. The set of real numbers is examined along with the subsets of natural, whole, rational, and irrational numbers. Then, we review the four basic operations on real numbers: addition, subtraction, multiplication, and division. In Section 1.6, we study variables and variable expressions. We then introduce the important concept of writing verbal statements as algebraic expressions. In the final section we deal with the properties of real numbers.

Chapter 2 Linear equations and inequalities

In this chapter, algebraic expressions are used to construct linear equations and inequalities. In Sections 2.2 and 2.3, we investigate properties of equality that will be used to solve linear equations, which constitutes Section 2.4. The skill of translating word problems to mathematics and applying these skills to word problems that require solving linear equations is developed. In the two sections that follow, we solve linear inequalities, and, in the final section, we solve word problems using formulas from business (simple interest, cost-revenue-profit) as well as from geometry and trigonometry (area and volume formulas of regions and solids, and the sum of the three angles of a triangle being 180°).

Chapter 3 Exponents and polynomials

In this chapter we study multiplication properties with exponents and simplify expressions using $a^m a^n = a^{m+n}$ and $(a^m)^n = a^{mn}$. Next, we investigate further exponent rules for division properties with exponents. We use these rules to introduce scientific notation. The remainder of the chapter deals with the algebra of polynomials, beginning with multiplying and dividing monomials. In Section 3.4, we introduce polynomials, discussing the types of polynomials, the degree of a polynomial, and how to simplify polynomials. The next two sections cover the addition, subtraction, and multiplication of polynomials.

Chapter 4 Factoring

Continuing our study of polynomials, we develop techniques to factor them in this chapter. Coverage includes factoring of monomials, general trinomials, special polynomials such as the difference of two squares, perfect square trinomials, and the sum or difference of two cubes. We then develop factoring by grouping and state a strategy for general factoring. These techniques are then used to solve quadratic equations that are factorable, which, in turn, leads to applications of quadratic equations. The applications include using the Pythagorean Theorem, solving geometric problems, and understanding applications to business.

Chapter 5 Rational expressions

In the first section, we define rational expressions, evaluate them, and determine where a rational expression is undefined. Next, we multiply and divide rational expressions, putting the answer in lowest terms. In Section 5.3 we develop the concept of least common denominator, which will be used to add and subtract rational expressions. Complex fractions are discussed in the next section, where we develop methods to simplify complex fractions into rational expressions. We then solve equations containing rational expressions and introduce formulas that involve rational expressions such as the total resistance formula. Ratio and proportion are covered, showing how proportions are used to solve problems involving similar triangles. The final topic of the chapter is applications of rational expressions in which direct and inverse variation, along with various applications of these concepts, are developed.

Chapter 6 Linear equations and their graphs

In this chapter, we start the investigation of the connection between analytic geometry and its applications, beginning by plotting points in a plane and then developing linear equations and their graphs. In Section 6.3 we find the slope of a line given two points on the line and the slope of a line from the graph. The difference between zero slope and undefined slope is explained, and the geometric meaning of positive and negative slope is discussed. In the next section, we find the equation of a line given (a) the slope and y-intercept, (b) the slope and a point on the line, or (c) two points on the line. In Section 6.5 graphing linear inequalities and using linear inequalities in applications are illustrated. Many applications of linear equations are included, such as linear depreciation, linear cost-revenue-profit, and linear supply and demand equations. In the last section we discuss relations and functions, establishing the vertical line test to determine when a relation is also a function. The domain and range of a function are defined, and the $f(x)$ notation is introduced.

Chapter 7 Systems of equations

In this chapter, we continue to investigate the connection between analytic geometry and its applications. We show that with a system of two equations in two variables we are able to study more complex applications. We start our study of systems of equations by finding solutions by graphing, particularly in the following two cases: (a) when the system has no solutions and (b) when a system has infinitely many solutions. In the next two sections we use two algebraic methods for solving a system: the elimination method and the substitution method. Systems of equations are used to solve word problems, and a strategy for solving word problems using two variables is given, which is then applied to word problems including the break-even point discussion from economics. The equilibrium point for supply and demand equations is also discussed. In the final section, the technique of graphing linear inequalities is developed and then used to solve word problems using systems of linear inequalities. Many of the problems show the initial steps to solve linear programming problems in two variables by the graphing method.

Chapter 8 Roots and radicals

We return to the concept of exponents and properties such as $a^m a^n = a^{m+n}$, $(a^m)^n = a^{mn}$, and $a^{-n} = 1/a^n$, where m and n are integers. In this chapter, we define numbers raised to rational powers, connecting this concept with taking the nth root of a number. In Section 8.2, we simplify radical expressions, and in the next two sections we study the algebra of radical expressions. In Section 8.5, radical equations and word problems using radical equations are solved; the connection is then made between taking roots and rational exponents. This chapter concludes by defining a complex number in terms of i and covering the algebra of complex numbers.

Chapter 9 Quadratic equations

One of the main goals of this book is developing techniques for solving equations. In the last chapter we return to the problem of solving the general quadratic equation $ax^2 + bx + c = 0$, where the trinomial may not be factorable. We begin with a discussion of using the square root method for solving quadratic equations and then develop the completing of the square method. The quadratic formula is developed in Section 9.3 and then used to find any complex solutions to quadratic equations. Applications of quadratic equations are discussed, including geometric and work problems. In the last section we show how to graph quadratic equations by finding the coordinates of the vertex and the line of symmetry; we then determine if a quadratic equation has a highest or lowest point and find the coordinates of this point.

Ancillary package

The following supplements are available to the students to accompany this text:*

Student Solutions Manual and Study Guide (Linda Holden, Bloomington University) Contains step-by-step solutions to one fourth of the problems in the exercise sets (every other odd-numbered problem) in addition to providing the student with a short summary of the important concepts in each chapter.

Videotapes A complete set of videotapes (18 hours) has been created and scripted by the author. Keyed to the text, the videotapes explain, using computer graphics, examples with corresponding practice problems. The student can participate by stopping the tape to work the practice problems on his or her own and then checking the solutions by continuing the tape. The tapes are available in VHS format and include a list of topics and the amount of time spent on each section.

One-to-One Interactive Software (George W. Bergeman, Northern Virginia Community College) Available for both Apple and IBM, this software allows students to test their skills and to pinpoint and correct weak areas. One-to-One

*For the instructor we have an Instructor's Manual with solutions to all the exercises not included in the Student Solutions Manual; Prepared Tests with six tests for each chapter, as well as midterm and final examinations and a Diagnostic test; a Computerized Test Bank for the Apple II, IBM, and Macintosh computers; and a Printed Test Bank containing tests generated from the Computerized Test Bank.

presents worked examples and displays annotated, step-by-step solutions to problems answered incorrectly. Students may also choose to see the solutions to problems answered correctly and to view a partial solution if they need help to begin solving a problem. The software includes two useful review capabilities: Missed Problems Review and Disk Review. As the student works, One-to-One keeps track of problems that are answered incorrectly. When the work on a topic is completed, the students are given the option of reviewing all problems answered incorrectly. In addition, the Disk Review feature provides a quick and efficient review of all the topics on the disk.

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I want to thank the dedicated staff at Saunders College Publishing for their tireless energy throughout the past three years: Bob Stern, Senior Mathematics Editor, with his impeccable sense of timing; Ellen Newman, Developmental Editor, who spent long hours, that frequently shortened the evening, on the book; and Janet Nuciforo, Project Editor, who guided the book to a successful conclusion with her expertise in converting the manuscript to a polished full-color book.

Larry R. Mugridge
Kutztown University

Pedagogical Use of Color

The various colors in the text figures are used to improve clarity and understanding. Many figures with three-dimensional representations are shown in various colors to make them as realistic as possible. Color is used in those graphs where different lines are being plotted simultaneously and need to be distinguished.

In addition to the use of color in the figures, the pedagogical system in the text has been enhanced with color as well. We have used the following colors to distinguish the various pedagogical features:

PROPERTY

STRATEGY

CALCULATOR CROSSOVER

DEFINITION

COMMON ERROR

RULE

Table No.	Table title
Col. head	*Col. head*

Introducing the Book

The **Overview** introduces the material to the student and demonstrates its relevance to the real world.

Overview

We continue our study of polynomials. In the last chapter, we multiplied polynomials using the distributive property as well as the FOIL method and special product rules. In this chapter, we reverse our thinking and develop techniques to factor polynomials. Factoring polynomials will enable us to solve certain equations, which will lead to solving more word problems.

Objectives help the student focus on skills to be learned in the chapter.

4.1

Factoring monomials

OBJECTIVES

- *To determine if a number is prime or composite*
- *To factor a number into primes*
- *To factor a monomial*
- *To factor out the greatest common monomial from a polynomial*

In the last chapter, we developed methods of multiplying polynomials. The reverse of multiplication is called *factoring*. In this chapter, we will study methods to factor polynomials.

We start our discussion with factoring real numbers. If the number 84 is written as a product of other numbers, then these numbers are called **factors** of 84. For example, since $84 = 2 \cdot 42$, the numbers 2 and 42 are each factors of 84. Also,

$$84 = 2 \cdot 6 \cdot 7 \qquad 84 = \left(\frac{1}{3}\right)(252) \qquad 84 = (-4)(-3)(7)$$

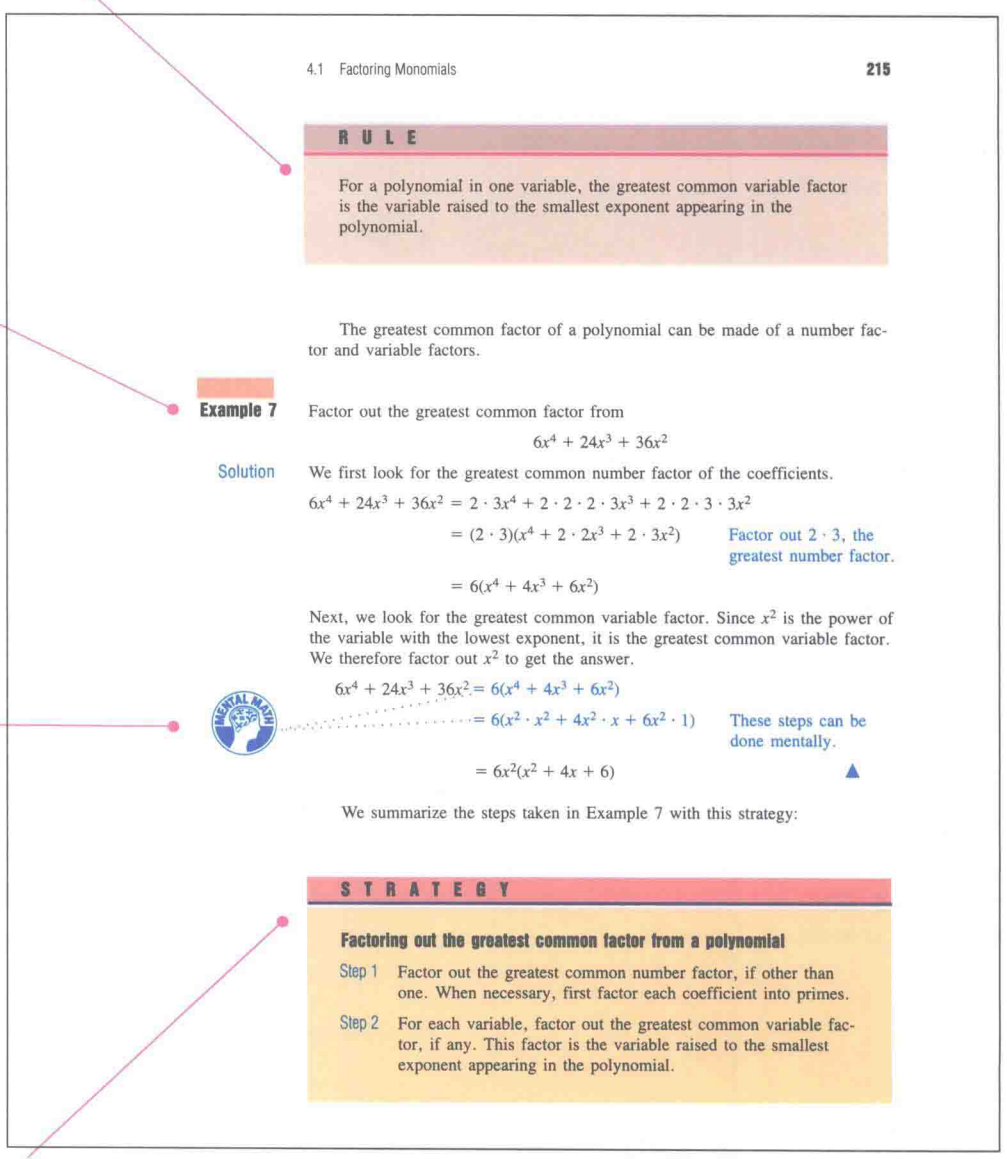
So there are many ways of factoring a number. We will be interested in factors that are *prime numbers*.

Rules are highlighted with a light and medium brown screen.

Examples provide complete worked-out solutions; each example begins with a light brown bar and ends with a blue triangle.

This logo alerts students to operations or steps that can be performed mentally.

Strategies are highlighted with a gold and orange screen.



RULE

For a polynomial in one variable, the greatest common variable factor is the variable raised to the smallest exponent appearing in the polynomial.

The greatest common factor of a polynomial can be made of a number factor and variable factors.

Example 7 Factor out the greatest common factor from

$$6x^4 + 24x^3 + 36x^2$$

Solution We first look for the greatest common number factor of the coefficients.

$$6x^4 + 24x^3 + 36x^2 = 2 \cdot 3x^4 + 2 \cdot 2 \cdot 2 \cdot 3x^3 + 2 \cdot 2 \cdot 3 \cdot 3x^2$$

$$= (2 \cdot 3)(x^4 + 2 \cdot 2x^3 + 2 \cdot 3x^2)$$ Factor out $2 \cdot 3$, the greatest number factor.

$$= 6(x^4 + 4x^3 + 6x^2)$$

Next, we look for the greatest common variable factor. Since x^2 is the power of the variable with the lowest exponent, it is the greatest common variable factor. We therefore factor out x^2 to get the answer.

$$6x^4 + 24x^3 + 36x^2 = 6(x^4 + 4x^3 + 6x^2)$$

$$= 6(x^2 \cdot x^2 + 4x^2 \cdot x + 6x^2 \cdot 1)$$ These steps can be done mentally.

$$= 6x^2(x^2 + 4x + 6)$$ ▲

We summarize the steps taken in Example 7 with this strategy:

STRATEGY

Factoring out the greatest common factor from a polynomial

Step 1 Factor out the greatest common number factor, if other than one. When necessary, first factor each coefficient into primes.

Step 2 For each variable, factor out the greatest common variable factor, if any. This factor is the variable raised to the smallest exponent appearing in the polynomial.