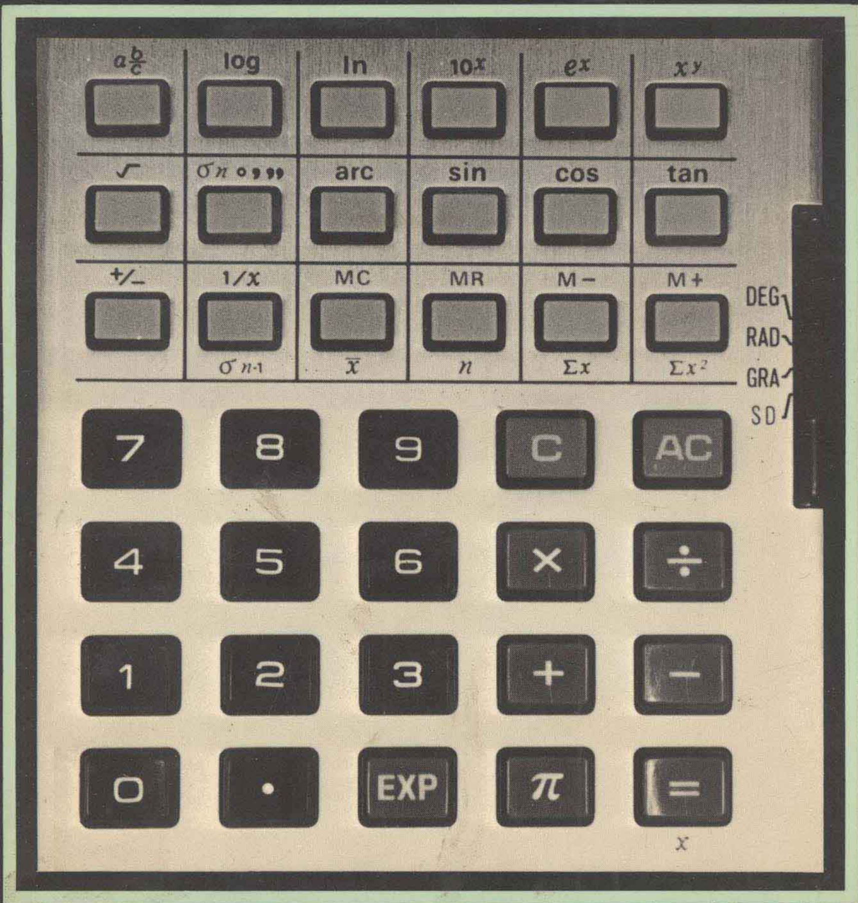
Mathematics for level-2 technicians

C.W. Schofield and D. Smethurst



Mathematics for level-2 technicians

C. W. Schofield

BSc, FIMA, FCSI, FRSH
Highlands College

and

D. Smethurst

BSc, MSc
Paddington College

Edward Arnold

First published 1979
by Edward Arnold (Publishers) Ltd
41 Bedford Square, London WC1B 3DQ

British Library Cataloguing in Publication Data

Schofield, Clarence Ward
Mathematics for level-2 technicians.
1. Shop mathematics
I. Title II. Smethurst, D
510'.2'46 TJ1165

ISBN 0-7131-3385-6

Text set in 10/11 pt IBM Press Roman, printed by photolithography, and bound in Great Britain, at The Pitman Press, Bath

Preface

In writing this book, we have aimed to provide a comprehensive student text which extends the fundamentals of mathematics covered in *Basic mathematics for technicians* by C. W. Schofield and relates mathematical theory to a wide range of practical work appropriate to engineering, construction, and science technicians.

We have based the content on the current level-2 standard units in mathematics issued by the Technician Education Council, and have also taken note of the new bank of objectives in mathematics (U78/911) developed by the joint TEC/BEC Committee for Mathematics and Statistics.

Our approach has been greatly influenced by the general availability of electronic calculators which, besides making much detail of arithmetical 'working out' unnecessary, has made an introductory section on calculators desirable, with references to their use in evaluations throughout.

Both of us wish to record our grateful thanks to Bob Davenport of Edward Arnold (Publishers) Ltd, whose careful editing and helpful suggestions have been responsible for many improvements. We are also indebted to many of our colleagues, whose willing assistance we have greatly appreciated.

C. W. Schofield
D. Smethurst

Key to TEC objectives

General objective	TEC unit and sections of this book							
	U75/012	U75/038	U75/039	U76/031	U76/032	U76/033	U76/060	U76/340
1	D5, D7, D8	D6	D6, D7, D8	B5, B6	C1	B3, B7	A1	*
2	D6	D7	D3	B6, B8	C2	B3	B1, B3	*
3	D1	D8	B3, B7	B3	B8	D2	B5, B7, D2	B4
4	D2	B7	C1	B7	E1, E2	B6, B7	B9	B4
5	E1, E2	B2, B5, B6	C2	E1		B8	D3	D1, D2
6	B5	D3	E1, E2	D5, D7		C1	D7	A1
7	B5, B6	D1, D4				B8, A1		B1, B3, B7
8		E1, E2				A1, A2		B2
9		A1				E1		B1
10						E3		B1
11						D5, D7, D8		B5, B6
12						D8		D5, D8
13						D6		D7
14						B10		D6
15								D5
16								D7, D9

*Revision of level-1 work covered in *Basic mathematics for technicians* by C. W. Schofield.

Contents

A Number

A1 Using a calculator

A1.1 Basic operations

With modern electronic calculators, all arithmetical calculations become simple press-button operations. Numbers are entered by pressing the appropriate button for each digit in the number. By international agreement, the digits from 1 to 9 are arranged in a 3 x 3 block with [7][8][9] on the top row, [4][5][6] on the centre row, and [1][2][3] on the lowest row, with the zero [0] either to the left or below. Simple calculators may have little more on the remaining buttons than the four basic operations of [x][÷][+][−] together with an equals [=], a cancel or clear [C] and either a decimal point [.] or an exponent [E].

The operating procedure varies according to the type of calculator, and you should refer to the instruction book to find the sequence appropriate to your particular machine.

You should practise using your calculator until you can operate it quickly and accurately. Try using it to check results obtained by using four-figure tables or a slide rule.

Warnings

1. Before starting any calculation, clear the machine (including any memory).
2. Limit your answers to a reasonable number of significant figures consistent with the accuracy of the input data.
3. If using a battery model, switch off your machine between calculations (to conserve battery life).
4. If the display begins to fade, recharge or replace the batteries or the calculations may be affected.

A1.2 Special function keys

Scientific calculators provide a range of special function keys, and in selecting a calculator it is necessary to choose one which has those functions most likely to be needed. Calculators vary in the way these functions are identified and used. Some of the more common key symbols are given below.

[%] A percentage key is useful for financial calculations but has limited application in scientific work.

[π] Gives the value of the constant 'pi' to whatever accuracy the register will allow (e.g. 3.141 592 7).

[x^2]

Pressing this key automatically squares whatever number has been entered.

[$\sqrt{x}$]

This square-root key is essential for many mathematical calculations.

[log]

The common-logarithm key gives logarithms to base 10.

[10^x]

This key raises 10 to the power of the number which has been entered. It can be regarded as a key for antilogarithms, i.e. the reverse of 'log'.

[ln]

Pressing this key causes the natural logarithm of the entered number to be computed and displayed.

[e^x]

This key raises the exponential e (approximately 2.718 281 8) to the power of the number which has been entered. It thus provides natural antilogarithms, but has other uses since e occurs in many formulae.

[sin] [cos] [tan]

These keys produce the appropriate trigonometric function for the angle entered. Some calculators are limited to angles from 0° to 90° but most have a greater range.

[arc]

With calculators with this key, inverse trigonometrical functions are obtained by following this key with the appropriate sine, cosine, or tangent key.

[$\sin^{-1}$] [$\cos^{-1}$] [$\tan^{-1}$]

These keys provide an alternative way of obtaining inverse trigonometrical functions. Thus either arcsin x or $\sin^{-1} x$ can be used to obtain the angle of which the sine is x.

[INV]

This more general 'INVERSE' key can be used to replace the 'arc' key when inverse trigonometrical functions are required, but it can also be used to give antilogarithms from the logarithm keys and may have other uses.

1/x	This reciprocal key gives the value of $1 \div x$. It can also be used in combination with the keys for the three basic trigonometrical ratios to give cosecant, secant, and cotangent.
x^y or y^x	This power key raises the first number entered to the power of a second number.
$\sqrt[x]{y}$ or $y^{1/x}$	This root key enables the xth root of any positive value of y to be found.
EXP or EE	This exponent entry key enables numbers to be entered in scientific notation, i.e. in the standard form $x \times 10^n$. The number 0.000 123 is displayed as 1.23 − 04.
+/−	This key is used for changing the sign of the number entered.
x!	This factorial key gives the value of $x(x-1)(x-2)\ldots(1)$ for integral values of x.
STO STO 1	The number in the display is stored in a cleared memory when this key is used. Some calculators have more than one memory location, and the user can specify which memory register a particular number is to be stored in.
M+	Pressing this key adds the displayed number to the memory.
M−	Subtracts the display from the memory.
RCL or RM	The recall-memory key – content of the memory store to be displayed.
CM	This key clears the memory, and it is advisable to develop a habit of using it every time a fresh calculation is begun.
$x \leftrightarrow$M	Exchanges the displayed number for the number stored in the memory.

[(and)]

Keys for brackets or parentheses are very useful in the evaluation of complicated formulae. They permit storage of intermediate results while working out subsections of the calculation.

A1.3 Multiple-function keys

A pocket calculator will rarely have room for more than about forty keys if they are to be reasonably spaced and easy to use. To keep down the number of keys, many manufacturers of scientific calculators allocate second or even third functions to several of the keys. A special 'function' key (often marked 'F' or '2nd') is provided to select which of the uses of a multifunction key is intended. For example, the functions e^x and 10^x may share a single key. Pressing this key will give e^x for the number in the display directly. To find 10^x the 'function' key is pressed first. For example,

1.25 [e^x] displays 3.4903 i.e. $e^{1.25} = 3.4903$

1.25 [F][e^x] displays 17.7828 i.e. $10^{1.25} = 17.7828$

The second-function symbols are usually marked on the body of the calculator next to the appropriate key, while the first-function symbol is marked on the key itself. Many mistakes are made by inadvertently obtaining the wrong function with these keys.

A1.4 Algebraic entry

Most calculators today operate in a way which allows algebraic functions to be entered in a logical sequence similar to the order in which we would write them down on paper. For example, the problem of adding 8 to 12 and subtracting 6 is normally written as

$$8 + 12 - 6 = 14$$

and we enter it as

8 [+] 12 [−] 6 [=]

The answer 14 will appear in the display as soon as we press the 'equals' key.

A1.5 Sequence of operations

Certain calculator functions have precedence over others and will be completed as soon as the function key is pressed. For example, if we clear the calculator and press the sequence of keys

2 [x] 3 [ln]

the display will show 1.0986. This is the value of ln 3 and *not* the value of ln (2 x 3). Pressing the 'equals' key will now give 2.1972, which is 2 ln 3. Other functions on the calculator such as 'sin', 'cos', 'tan', 'log', 'e^x', '$1/x$', and 'x^2' also operate immediately on the number in the display regardless of other operations previously entered and not yet carried out.

The system of precedence of functions is called a *hierarchy*, and you should check the calculator handbook to see how your machine operates in this respect.

A1.6 Parentheses and memories

Calculators vary in the way they evaluate a chain of operations involving +, −, x, and ÷ functions. To find (3 x 2) + (4 x 5) = 26, for example, may be a straightforward task on a calculator using the *sums-of-products* precedence. On such a machine we simply use the key sequence

3 [x] 2 [+] 4 [x] 5 [=] 26

Many other machines will give an answer of 50 with this sequence, i.e. [(3 x 2) + 4] x 5. To obtain the correct value using a machine of this type we use parentheses or the memory as follows:

3 [x] 2 [+] [(] 4 [x] 5 [)] [=] 26

or 3 [x] 2 [=] [STO 1] 4 [x] 5 [=] [+] [RCL 1] [=] 26

As a general rule, to avoid mistakes it is advisable to complete separately the evaluation of any expressions within parentheses, just as we would do to solve the problem manually.

A1.7 Checking

Some sort of check is always advisable and, depending on the type of problem, one of the following three ways of checking should be possible.

i) Use amended figures to simplify the calculation and provide a rough guide to the magnitude of the answer to be expected. Thus, in dividing 20.78 by 4.09 we note that 20 divided by 4 would give us 5, so we expect an answer around this figure. Using the calculator gives us 5.08.
ii) Use an alternative mechanism for the calculation, such as four-figure tables or a slide rule. This is good practice at this stage as well as a means of gaining a reasonably accurate answer.
iii) Use an alternative sequence for the calculation on the machine, or reverse it and work back from the answer to the original input. Thus if we have found $(1.36)^4 = 3.421$ we should be able to show that $\sqrt[4]{3.421} = 1.36$.

A1.8 Significant figures

When using a calculator, there is a temptation to read answers from the display and to copy the figures down in full on the assumption that what is shown on the display is accurate to the number of figures produced by the calculator. To illustrate the danger, let us consider a practical example in which we wish to determine the area of a rectangle drawn on a map and measured as 20 mm x 25 mm, given that the accuracy of measurement was to the nearest millimetre. This implies that the dimensions actually lie between

19.5 mm x 24.5 mm and 20.5 mm x 25.5 mm, giving an area somewhere between 477.75 mm^2 and 522.75 mm^2. This shows that the actual area is better expressed as 500 ± 23 mm^2 or as 500 mm^2 ± $4\frac{1}{2}$%. Now this example is rather an extreme case, but it does illustrate that the accuracy of any final answer is dependent upon the accuracy of the input data. If the reading on an ammeter or the volume of liquid delivered by a burette can be determined with an accuracy of the order of 1%, then any subsequent calculation based upon such readings cannot be taken to have a greater accuracy.

It is apparent from such instances that it would be absurd to perform a calculation based on practical data and then give an answer to as many as eight significant figures (which is the display capacity of many pocket calculators). In most cases it would be realistic to limit such an answer to three significant figures, bearing in mind that the third cannot be guaranteed. Where a more accurate figure is essential, the accuracy of the input data must be increased. This may mean, for example, that instead of measuring a tube diameter by caliper methods we may have to use a travelling microscope to obtain the required accuracy. It is generally desirable to limit any final answer to the same number of significant figures as the input data, but as many figures as possible should be retained throughout the intermediate stages of a calculation.

A1.9 Evaluations

Your calculator will help you in the arithmetical calculations arising from your work in other technical subjects and also in evaluation from formulae.

Example Find the value of $4\pi r^2$ when $r = 0.39$ m.

Most calculators will allow the evaluation of this expression using a logical sequence of operations from left to right:

		Display
i)	enter 4	4
ii)	press the multiplication key	4
iii)	press the π key	3.141 592 7
iv)	press the multiplication key	12.566 370 6
v)	enter the value of r	0.39
vi)	square using the 'x^2' key	0.1521
vii)	press the '=' key to obtain the result	1.911 345 0

The answer 1.9 m^2 is written down, corrected to the number of significant figures given for the initial value of r (two significant figures). Note that it is necessary to add the appropriate units to the numerical answer.

A1.10 Tabulating values

When it is necessary to make repeated evaluations from the same formula, it is usually helpful to draw up a table showing how the values obtained from the formula relate to the values selected for the variable.

Example 1 Draw up a table of values of y corresponding to values of x from 0 to +3 at intervals of 0.05 for the equation $y = 0.78\sqrt{x} + 0.42$.

The sequence of operations on the calculator is

x [$\sqrt{x}$] [×] .78 [+] .42 [=]

i.e.
- i) enter the next value of x
- ii) press the square-root key
- iii) press the multiplication key
- iv) press the decimal key followed by numbers 7 and 8
- v) press the addition key
- vi) press the decimal key followed by numbers 4 and 2
- vii) press the 'equals' key to obtain the result.

The following table of results is obtained:

x	0	0.50	1.00	1.50	2.00	2.50	3.00
y	0.42	0.97	1.20	1.38	1.52	1.65	1.77

Some calculators are *programmable*, which means that a sequence of operations like that above can be stored in the calculator and the program followed through automatically for each value of x entered in turn.

Example 2 Evaluate Ae^{bx} when $x = 1, 4, 10$, given that $A = 1.6$ and $b = \frac{1}{5}$.

For $1.6e^{x/5}$ the calculator sequence is

x [÷] 5 [=] e^x [×] 1.6 [=]

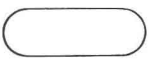
Substituting $x = 1$ yields 1.95

$x = 4$ yields 3.65

$x = 10$ yields 11.82

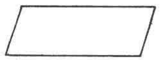
A1.11 Flow charts

Flow charts are used extensively by computer programmers, who need to establish the best sequence of operations to complete a task. Before writing a program for the computer, they break down the calculation into small steps and arrange them into a suitable order on a diagram, using the following symbols:

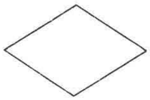


Used to show a terminal point, i.e. start and stop.

Used when arithmetical operations are to be carried out.



This symbol denotes input or output of numbers.



A decision symbol, determining which alternative path is to be followed.

Symbols are connected by flow lines to indicate the sequence of operations An arrow on the flow line shows the direction.

Flow charts can be used to prepare a problem for calculator solution, particularly when repeated calculations are necessary and a programmable calculator is being used.

Example A graph of the curve $y = 2x^3 - x^2 + x$ is to be drawn for values of x between $x = 0$ and $x = 3$ in steps of 0.2. Draw a flow chart to show how a table of suitable y values could be obtained from a calculator.

The flow chart is shown in fig. A1.1.

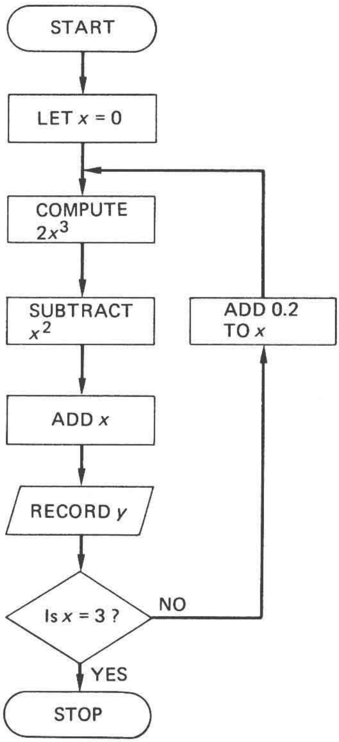


Fig. A1.1

Exercise A1

Use a calculator to evaluate each of the following expressions:

1 $3972 + 4165 + 2971 + 385$
2 $2.763 + 0.512 + 11.37 + 7.961$
3 $4734 + 5296 - 3488$
4 $10.356 - 1.983 - 2.497$
5 283×471
6 $2.735 \times 0.625 \times 1.024$
7 $74.35 \div 11.59$
8 $(6.594)^2$
9 $1/2.151$
10 $47.1 \times (0.763)^2$
11 5% of £7360
12 2% of $375
13 $\dfrac{21.73 \times 4.812}{37.16}$
14 $(2.594)^2 - (1.762)^2$
15 $(0.968)^3$
16 $(1.6754)^4$
17 $\dfrac{41.78}{2.421} - \dfrac{0.736}{0.566}$
18 $1/(0.877)^2$
19 $e^{0.2}$
20 $e^{-0.16}$
21 Find the value of πr^2 when $r = 0.518$.
22 Evaluate $2\pi\sqrt{(l/g)}$ when $l = 3.46$ and $g = 9.81$.
23 Given that $1/f = 1/u + 1/v$, find the value of f if $u = 40$ and $v = 160$.
24 If $V = \frac{4}{3}\pi r^3$, find V when $r = 2.4$.
25 Evaluate $\sqrt{s(s-a)(s-b)(s-c)}$ given that $s = \frac{1}{2}(a+b+c)$ when $a = 4.2$, $b = 3.4$, $c = 2.4$.
26 Using the formula $\dfrac{1}{R} = \dfrac{1}{r_1} + \dfrac{1}{r_2} + \dfrac{1}{r_3}$, find R when $r_1 = 0.820$, $r_2 = 1.05$, and $r_3 = 1.76$.
27 Use the formula $N = \sqrt{\dfrac{(15D)^5 H}{L}}$ to find the value of N when $D = 1.58$, $H = 51.2$, $L = 3.4$.
28 Use a calculator to evaluate

a) $(1.61 \times 2.45) + (3.57 \times 4.11)$
b) $(11.33 + 2.66) \times (2.18 + 9.67)$
c) $\dfrac{3.22}{(6.1 \times 1.33)} - \dfrac{2.7}{(8.55 \times 0.32)}$
d) $\dfrac{(14.62 - 10.26)}{(9.03 + 1.17)}$

29 Draw a flow chart to show how the voltage v across a capacitor increases as t varies between 0 s and 100 s, finding values of v every 5 seconds from the formula $v = E(1 - e^{-t/RC})$, where E, R, and C are constants.

A2 Naperian logarithms

These are logarithms with base e (where e = 2.7183). They are called Naperian or natural logarithms to distinguish them from common logarithms with base 10.

A2.1 Defining logarithms

We define the logarithm of a to the base b as follows:

if $\quad a = b^x \quad$ then $\quad x = \log_b a$

Thus

if $\quad 25 = e^x \quad$ then $\quad x = \log_e 25$

Conversely, if we are given a logarithmic relationship we can use the same definition to obtain an indicial equation. For example

$$11 = \log_x 8$$

$$x^{11} = 8$$

Note that, since $b^0 = 1$, $\log_b 1 = 0$; and, since $b^1 = b$, $\log_b b = 1$.

A2.2 Laws of logarithms

i) $\quad \log_b MN = \log_b M + \log_b N$

We can show this is true by using our definition of a logarithm.

Let $\quad \log_b M = x \quad$ and $\quad \log_b N = y$

then $\quad b^x = M \quad$ and $\quad b^y = N$

$\therefore \quad MN = b^x \times b^y = b^{x+y}$

$\therefore \quad \log_b MN = x + y = \log_b M + \log_b N$

ii) $\quad \log_b \dfrac{M}{N} = \log_b M - \log_b N$

Again we let $\quad \log_b M = x \quad$ and $\quad \log_b N = y$

then $\quad \dfrac{M}{N} = \dfrac{b^x}{b^y} = b^{x-y}$

From the definition of a logarithm,

$$\log_b \frac{M}{N} = x - y = \log_b M - \log_b N$$

iii) $\quad \log_b N^a = a \log_b N$

Let $\quad \log_b N = x \quad$ and so $\quad N = b^x$