Roger Penrose Collected Works

VOLUME 3

1976-1980

ROGER PENROSE COLLECTED WORKS

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PREFACE

The topics addressed, in these *Collected Works*, vary over quite an extensive range, the boundaries between their areas being sometimes quite blurred. It would have been hard to classify all these articles and arrange them according to individual topics. Moreover, my own mode of working has been, on the whole, to keep most of my different scientific or mathematical interests running concurrently. Accordingly, it has seemed appropriate not to attempt to group the papers together into separate tidy categories. To do so would have provided a somewhat misleading picture of my overall mode of thinking. Instead, the ordering of articles that has been adopted is essentially chronological, at least with regard to the date of publication. The resulting somewhat higgledy-piggledy appearance that we find in the sometimes abrupt juxtaposition of different topics is, perhaps, fairly illustrative of my actual mode of working.

There are two main kinds of drive that underlie these works. On the one hand, some of the articles have arisen mainly from purely mathematical considerations, which can be ultimately aesthetic or just matters of simple curiosity. On the other hand, motivations for many others of the articles came primarily from a strong desire to discover more about the underlying principles governing the workings of the physical world. Yet, one of the deep mysteries about what has been discovered about these principles is the great breadth of intrinsically appealing mathematics that is found to underlie the actual operation of the physics of the universe, so these two drives are found to relate closely to one another. The reader may perhaps note the distinct geometrical flavour of much of my own mathematics, evidencing the strong appeal that geometry exerts on me, an appeal that is not infrequently intertwined with desire for algebraic elegance. Indeed, I have found it remarkable how much of the workings of our universe appear to depend upon sophisticated and powerful geometry and algebra.

My own very geometrical perspectives, as opposed to purely computational ones, are perhaps made more obvious in the later volumes of this series than in the early ones, but this appearance is deceptive. Very early in my career as a Cambridge graduate student, I had developed a graphical notation which enabled me to carry out many kinds of calculation in an entirely diagrammatic way. For a long time I was rather secretive of this procedure, and in my earlier writings would take care to remove all trace of these visual origins. I do not doubt that my predilections for such geometric or algebraically elegant forms of understanding have influenced the specific choices of direction in which my endeavours towards furthering our understandings of the physics of the world have taken.

The focus of these directions has become more clarified as time has moved on, and has narrowed down to such a degree that there has emerged a somewhat disturbing repetitiveness in many of the later articles. A good number of these provide rather non-technical accounts. An example of such repetitiveness occurs with my descriptions of gravitational collapse to a black hole. There is, however, a gradually developing outlook on this issue, leading to an emphasis on the fundamental contrast in character between the space-time singularities that inevitably appear within black holes and the space-time singularity that constitutes the big bang, this contrast being central to the existence of a second law of thermodynamics in our universe. The same kind of repetitiveness applies to my frequently repeated claim of a need for a fundamental change in the basic laws of quantum theory, where only towards the end do my suggestions become particularly specific. Likewise, many mildly differing accounts are provided outlining the formalism and the basic ideas that underlie twistor theory. The later ones lead up to various viewpoints on the 'googly' problem, which has constituted the main obstruction to the finding of a full understanding of general relativity in twistor terms. Despite such repetition, it is my hope that there is a sufficient variety of emphasis and of historical development in such articles for it to have been worth while to include them all here.

Several colleagues have collaborated with me in writing the works presented here, and I can mention but only a few. Foremost, I have collaborated long and particularly fruitfully with Ezra T. Newman, especially with regard to the asymptotics of general relativity, and also with Wolfgang Rindler in the development of the formalism of 2-spinor calculus. Other collaborators who have played important roles in helping to develop the mathematics of discrete physics and twistor theory have been Erwin Kronheimer, Michael Eastwood, Ronny Wells, and Denny Hill. Among my collaborators, providing very significant inputs, who were former students in my research group, are Andrew Hodges, Paul Tod, Lane Hughston, George Sparling, Richard Ward, Nick Woodhouse, and Lionel Mason, but there were many others. Although not playing roles as actual collaborators, fundamental influences in my research have come from Dennis Sciama and Michael Atiyah.

Roger Penrose April 2010

BIOGRAPHY

Early years and influences

I was born in Colchester, Essex, UK on 8 August 1931. My father, Lionel Sharples Penrose, FRS, was a highly talented and intelligent man, with many interests. He, himself, had come from a Quaker family, his own father (James Doyle Penrose) having been a professional artist and his mother (Josephine Peckover) was from a well-to-do Quaker banking family. Lionel's primary professional interest was in the causes of hereditary mental disease. He was Galton Professor of Human Genetics at University College London from 1945 and, following retirement, Director of the Kennedy-Galton Institue, St Albans. He had a very considerable artistic talent and a keen interest in music, especially Mozart, but also Bach and other classical composers, dabbling a little in composition himself, particularly when there was a puzzle aspect in what was involved (where he once wrote a piece that could be played equally well upside down as right-way up). He liked making puzzles of various kinds in wood, and spent much of his later life designing wooden models illustrating self-reproduction. He was recognized as a fine composer of chess problems. He had a definite fondness for mathematics and was an innovative statistician. He loved explaining matters of science and mathematics. On one occasion, when I told him that my mathematics teacher at school had informed us that he would be explaining calculus the next day, Lionel immediately took me aside so as to be sure to be the first to teach me about the beauties of calculus. This made a great impression on me. Yet he was an extremely difficult person to communicate with on personal matters.

My mother, born Margaret Leathes, was a charming and caring woman with a very subtle mind. She had been Head Girl at Bedales school, and became medically qualified, but found she was not able to practice after her marriage. After Lionel died, in 1972, she married Max Newman FRS, a fascinating man, who had been Professor of Mathematics in Manchester and, as we later learned, a key figure at Bletchley Park during the war. Margaret's father, John Beresford Leathes, FRS, had been a distinguished professor of physiology, and her mother, a concert pianist, born Sara Mara Natanson from a Jewish family in Latvia, had disowned her family (then in St Petersberg), becoming completely secretive about her origins, and even about her maiden name. Margaret enjoyed geometry at school, but did not

communicate much on this topic with me. My older brother Oliver was very precocious, and I learnt a lot of physics and mathematics from him as I was growing up. He later became a distinguished mathematical physicist (statistical mechanics) and FRS. My younger brother Jonathan excelled at games, particularly chess, and later became British Chess Champion a record 10 times, and then a top correspondence chess player. My much younger sister Shirley Victoria, now Shirley Hodgson, became a highly regarded Professor of Cancer Genetics.

This atmosphere of science, mathematics, art, and music, and of puzzles and games, had been an important part of my upbringing. During the war years, the family lived in London, Ontario, Canada, where Lionel became Director of Psychiatric Research at the Ontario Hospital. We returned to England in 1945, where I attended University College School, London. As an undergraduate, I studied Mathematics at University College London, from 1951 to 1954; then I went to St John's College Cambridge to do research in algebraic geometry, initially under William Hodge and subsequently John Todd. In my same year, under Hodge, was Micheal Atiyah, who was an inspirational, though somewhat intimidating colleague. Frank Adams and Christopher Zeemen (see Chapter 13) were contemporaries who also influenced me, and I became good friends with Hans Liebeck and Douglas Munn (see Chapter 3), who were close research-student colleagues at St John's.

Yet, the colleague who had the greatest influence on the development of my research during this period, and for many years later, was the cosmologist Dennis Sciama. Even before studying at Cambridge, I had a chance meeting with him during a visit to my brother Oliver in Cambridge in which I raised a question that had worried me in relation to the inspiring series of BBC radio talks given by Fred Hoyle in 1951 (The nature of the universe. A series of broadcast lectures). On the strength of this encounter (at which I used a space-time diagram with tilted light cones to make my point), Dennis had evidently formed the opinion that it would be worth while to interest me more in physics, even trying, later, to persuade me to switch my topic of research to cosmology. I was then too committed to my many puremathematical activities, but I learnt a great deal of physics from Dennis, and of the excitements involved in striving to uncover secrets of the universe. Another influential colleague was Felix Pirani, from whom I learnt much about developments in general relativity.

Pivotal were the superb lecture courses of Bondi, on general relativity, and Dirac, on quantum mechanics. On the pure-mathematical side, I recall particularly an elegant information-packed algebra course by Philip Hall. Lectures by Steen on mathematical logic turned out to influence me strongly towards my later viewpoint concerning the relation between computation and conscious understanding (e.g. Chapters 169, 184, 186).

Degrees

1952 BSc Mathematics (Spec) (1st Cl), University College, London

1957 PhD (Cantab), St John's College, Cambridge

Employment

1956	NRDC London (research on numerical analysis with computers); later consultant		
1956-1957	Assistant Lecturer, Bedford College, London		
1957–1960	Research Fellow, St John's College, Cambridge		
1959-1961	NATO Research Fellow, Princeton, Syracuse, Cornell Universities		
1961–1963	Research Associate, King's College, London		
1963-1964	Visiting Associate Professor, University of Texas, Austin, Texas		
1964-1966	Reader (Applied Mathematics), Birkbeck College, London		
1966–1967	Visiting appointments: Yeshiva, Princeton, Cornell, Chicago Universities; lecturer Battelle Inst., Seattle		
1967-1973	Professor (Applied Mathematics), Birkbeck College, London		
1969–1970	Visiting professor, Yeshiva New York, Princeton, and Cornell Universities		
1973-1998	Rouse Ball Professor (Mathematics), Oxford; 1998– Emeritus		
1978	Miller Fellow, Univ. of California, Berkeley		
1982–1987	Lovett Professor (Mathematics), Rice Univ., Houston, Texas, USA (Part-time)		
1987–1993	Distinguished Professor (Physics, Mathematics), Syracuse Univ. USA (Part-time)		
1993–	Francis and Helen Pentz Distinguished Professor (Physics, Mathematics; part-time)		
1998-2001	Gresham Professor (Geometry), Gresham College, London		

Honours Prizes include 1971 Heinemann Prize; 1972 Fellow of Royal Society; 1978 Plenary Lecture, International Congress of Mathematicians, Helsinki; 1985 Royal Medal (Royal Society);1988 Wolf Prize (physics, with Hawking); 1990 Albert Einstein Medal; 1994 knighted (services to science); 1994 Polish Academy of Sciences; 1998 US National Academy of Sciences; 2000 Order of Merit; 2004 de Morgan Medal (London Mathematical Society); 2001 Royal Irish Academy (Science); 2008 Copley Medal (Royal Society).

Marriages

1958	Joan Isabel Wedge; divorced 1980
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1988 Vanessa Dee Thomas

Children

1963	Christopher Shaun
1964	Toby Nicholas
1966	Eric Alexander
2000	Maxwell Sebastian

Honorary degrees

1992	University of New Brunswick, Canada
1993	University of Surrey
1994	University of Bath
1995	University of London
1996	University of Glasgow
1996	University of Essex
1997	University of St Andrews
1998	University of Santiniketan, India
1998	Open University
2002	University of Southampton
2004	University of Waterloo, Canada
2005	University of Leuven, Belgium
2005	Warsaw University
2005	University of Athens
2006	University of York

Books

1972	Techniques of Differential Topology in Relativity (NSF and SIAM)
1984 and 1986	Spinors and Space-Time (Volumes 1 and 2; Cambridge University Press; with Rindler)
1989	The Emperor's New Mind (Oxford University Press 1989)
1994	Shadows of the Mind (Oxford University Press)
1996	The Nature of Space and Time (Princeton University Press; with Hawking)
1997	The Large, the Small and the Human Mind (Cambridge University Press)
2004	The Road to Reality (Jonathan Cape)

INTRODUCTION

Important new realizations concerning twistor theory came to our Oxford twistor group during the period covered by this volume. There were two main sources from earlier work, but then the ideas came together to give us a deeper understanding of what is involved, providing us with a new perspective on the way that mathematical features of the complex geometry of twistor theory relate to actual physical fields.

One of these sources came from the striving to find a good definition of total angular momentum for an asymptotically flat space-time. The problem was to find a satisfactory intrinsic way to define the notion of an asymptotic rotation which is 'supertranslation-free', following a burst of outgoing gravitational radiation (see Chapter 26). In the absence of such a definition, the normal procedures (e.g. using Noether's theorem) do not seem to work. In flat Minkowski space M, a genuine rotation is recognized asymptotically by the fact that it leaves invariant a family of 'good' (i.e. supertranslation-free) cross-sections ('cuts') of null conformal infinity h, these being intersections of ordinary light cones with h. When gravitational radiation is present, the characterization of such a 'light cone' as a null hypersurface with a proper vertex does not work, but one can attempt a different characterization that works for M, namely that the good cuts have the property that the asymptotic outgoing shear σ vanishes all along the cut. This would have been feasible were it not for the fact that σ is a *complex* quantity, and there is not, in general, the realnumber freedom to arrange for $\sigma = 0$ over the entire S^2 that represents the cut. Ted Newman then proposed what turned out to be a remarkably insightful idea, namely to allow a complex supertranslation to be applied, so that the cut becomes a complex (holomorphic) cross-section of the complexified null infinity $\mathbb{C}h^+$. The space of Newman's good cuts, using a curious (but, on reflection, natural) definition of complex metric that he introduced, was a complex-Riemannian 4-space that he referred to as \mathcal{H} -space (' \mathcal{H} ' standing for 'Heaven'—i.e. 'where the good Cohens [cones] go'). This is described in Chapter 97. When I finally realized what Ted was actually doing, it became evident to me that his 'good cuts', in this sense, must be represented as holomorphic curves (Riemann spheres) in asymptotic twistor space (see Chapter 52). It also turned out that \mathcal{H} -space is automatically 'empty'—i.e. Ricci-flat¹ and, with my own definitions of chirality, etc., anti-self-dual. In twistor language, this was essentially the 'non-linear graviton construction' asserted in Chapters 67 and 71 and made explicit in Chapter 70.

¹ I was away at a conference in Italy, I believe, when I had persuaded myself that I could demonstrate this fact, and I puzzled the people in the conference office by getting them to send Ted a telegram with the cryptic message 'HEAVEN IS EMPTY'.

The other source of input came from the pure mathematicians whom I consulted about the issue of when one could be assured that the appropriate holomorphic curves actually exist as required for the construction. I initially consulted Isadore Singer about this, while I was away in the USA and he gave me an appropriate criterion. However, the language in which he phrased it involved the partial differential equations of a Dolbeault formulation, about which I could form little intuition, so I tried again with Michael Atiyah after my return to Oxford, and his approach—more along Čech lines, and involving the projective geometry that I was familiar with—was more to my liking and I was then able to comprehend the Kodaira theorems that were needed for the non-linear graviton construction (Chapters 70 and 97). This led to a further educational sessions with Atiyah involving my whole research group, and we began to realize how many of the puzzling aspects of twistor theory could be understood in terms of complex sheaf cohomology and complex-manifold theory. Michael Eastwood joined us and our association with him proved invaluable.

Very soon after I had found this non-linear graviton construction, Richard Ward, who was a graduate student with us at the time, quickly saw how to adapt these kinds of idea so as to find a corresponding type of twistor construction for (anti-)self-dual electromagnetism, which he rapidly generalized so that the general (anti)-self dual Yang-Mills field could be correspondingly obtained. Michael Atiyah again got involved and the subject very much took off. In relation to these developments for non-linear fields, we realized the importance of educating ourselves in many of the underlying concepts needed for the proper understanding of complex manifolds, and we were enormously fortunate that Michael Atiyah was at hand in Oxford to explain these things to us. Most important were basic notions of holomorphic sheaf cohomology, and it gradually became clear, as a result of these conversations that it was in terms of such notions that many of the puzzling features of twistor contour integrals were to be understood. The theory of integrable systems gained much from these developments, most specifically from Richard Ward's construction (see Ward, R.S. (1977) On self-dual gauge fields, Phys. Lett. 61A, 81-2; Mason, L.J. and Woodhouse, N.M.J. (1996) Integrability, Self-Duality, and Twistor Theory, Oxford University Press, Oxford).

During this period of fruitful activity in twistor theory, made largely by members of our Oxford research group, but a number of others were involved also, we founded the informal publication *Twistor Newsletter* (often abbreviated $\mathbb{T}\mathbb{N}$), produced at the Mathemtical Institute, Oxford. Roughly two volumes appeared per year, with articles, often handwritten, of varying degrees of technicality, seriousness, readability, reliability, and depth. These continued until shortly after my official retirement in 1998. A good proportion of the articles have been reprinted in four publications, with titles [Further] Advances in Twistor theory, published by Pitman, Pergamon, and Chapman and Hall. Often, members of our group, or other contributors, would be referred to in the articles merely by their initials, so to help in identifying these people, here is a partial list, with the last name spelled out: AHelfer, APHodges, APilato, BDBramson, DHill, EJDunne, GAJSparling, GBurnett-Stuart, KPTod, LJMason, LPHughston, MASinger, MFAtiyah, MGEastwood, NJeffaryes, NJHitchin, NMJWoodhouse, RJBaston, RMBryant, ROWells, RPenrose, TNBailey, TsouS-T, VDThomas, WSShaw, ZPerjés.

A paper of mine was finally published after some 15 years, as a 'golden oldie' (Chapter 114). This was concerned with the characteristic initial value problem, both in flat space-time (where I had employed a generalization of the Kirchoff–D'Adhemar integral formula that I had previously found) and for general relativity. The idea of exact sets described in Chapter 12 was central to the arguments

Finally, a mention should be made of the 2-piece aperiodic tile shape sets described in Chapter 111.

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