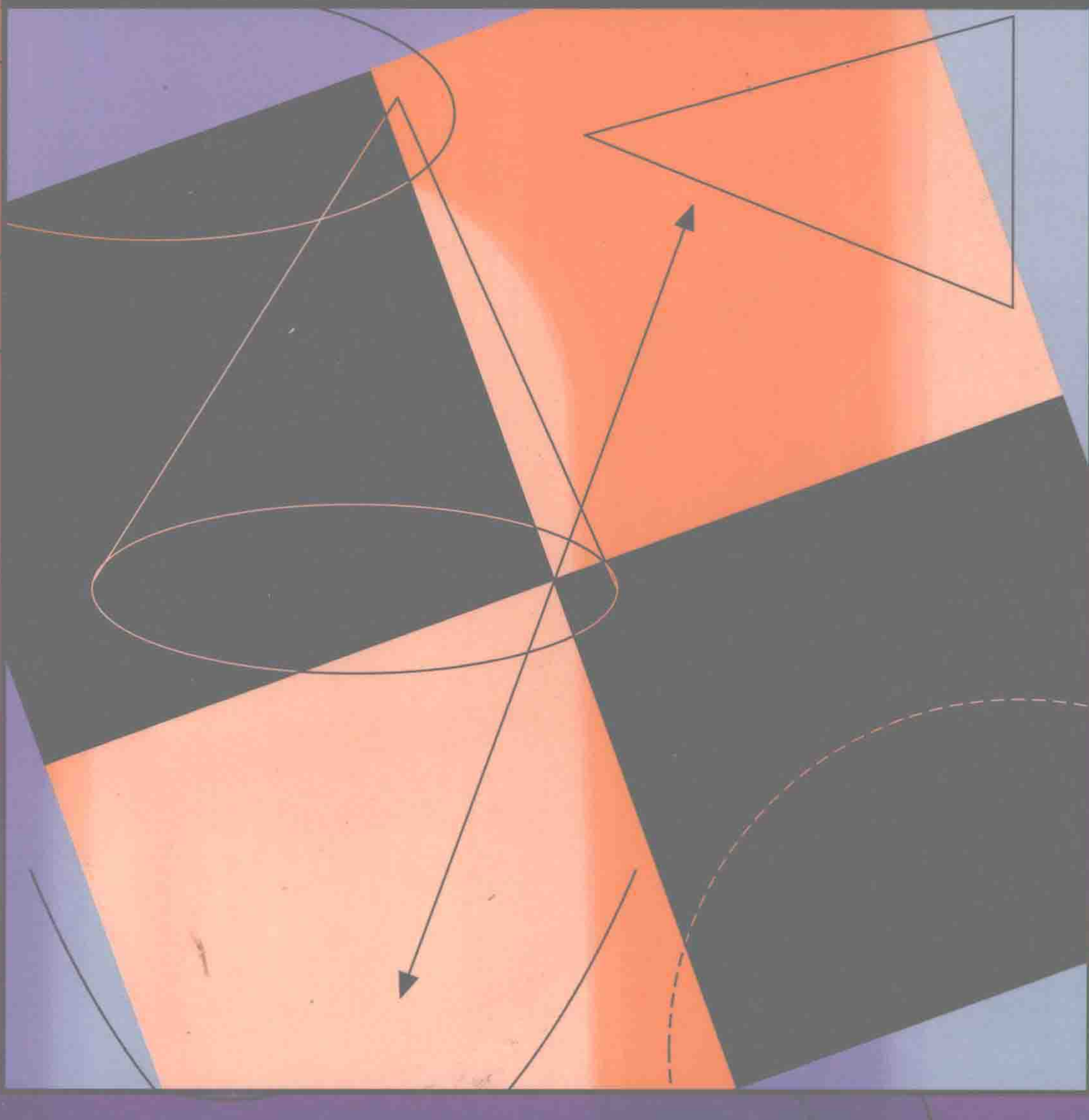
VideoText *Interactive*

ALGEBRA
A COMPLETE COURSE

WORKTEXT



TOM CLARK

ALGEBRA
A COMPLETE COURSE

Worktext

Tom Clark
Indiana University - Purdue University at Indianapolis

VideoText Interactive

Addison-Wesley Publishing Company

Reading, Massachusetts • Menlo Park, California • New York
Don Mills, Ontario • Wokingham, England • Amsterdam • Bonn
Sydney • Singapore • Tokyo • Madrid • San Juan • Milan • Paris

ISBN 0-201-88986-2
1 2 3 4 5 6 7 8 9 10- CRS-99989796

Table of Contents

Unit II – First Degree Relations with One Placeholder

Unit III – First Degree Relations with Two Placeholders

Unit VI – Algebraic Fractions

Unit VII – Relations of Rational Number Degree

Unit I – The Structure of Mathematics
Part A – Mathematics as a Language

LESSON 1 *Mathematical Parts of Speech*

Objective: To identify the five parts of mathematical speech and understand their use.

Important Terms:

Number Symbols – more commonly called ***numerals***, symbols used to represent quantities.

Operation Symbols – symbols such as + (add), – (subtract), × (multiply) and ÷ (divide), used to indicate ***action***.

Relation Symbols – symbols such as = (is equal to), > (is greater than), and < (is less than), used to show ***comparisons***, as well as ***combinations*** such as ≥ (is greater than or equal to), ≥ (is less than or equal to), and the negations of all of these (≠, ≯, ≮, ≰), among others.

Grouping Symbols – symbols such as ( ) (parentheses), [ ] (brackets), and { } (braces), used to show ***groupings***.

Placeholder Symbols – more commonly called ***variables***, symbols such as *a, b, c* (letters of the alphabet) and ❐ (empty boxes), used to hold the place of a number until the number has been identified.

Example 1: Tell which of the following are relation symbols:

a. $17\frac{1}{2}$ **b.** ≠ **c.** > **d.** ÷

Solution:

a. $17\frac{1}{2}$ is not a relation symbol. It is a number symbol meaning seventeen and one-half.

b. ≠ is a relation symbol meaning "is ***not*** equal to."

c. > is a relation symbol meaning "is greater than."

d. ÷ is not a relation symbol. It is an operation symbol meaning "divide."

Lesson 1 – Exercises:

Tell what part of mathematical speech each of the following is, and state in words what each means.

1. c	**2.** [ ]	**3.** $3.\overline{45}$	**4.** $+$
5. $=$	**6.** 63,231	**7.** $\geq$	**8.** $\sqrt{5}$
9. $-$	**10.** { }	**11.** 10^4	**12.** $\div$
13. $\nless$	**14.** ( )	**15.** $\frac{2}{3}$	**16.** 41.765
17. $\times$	**18.** n	**19.** $\neq$	**20.**

Part A – Mathematics as a Language

LESSON 2 Mathematical Expressions

Objective: To identify the four types of mathematical expressions and understand what to do with each.

Important Terms:

Closed Phrase – any mathematical expression which contains neither a relation symbol nor a placeholder symbol. For example, $2(5-1)$ is a closed phrase.

Open Phrase – any mathematical expression which contains a placeholder symbol but does not contain a relation symbol. For example, $6 + [3 - n]$ is an open phrase.

Closed Sentence – any mathematical expression which contains a relation symbol but does not contain a placeholder symbol. For example, $7(4-2) = 13$ is a closed sentence.

Open Sentence – any mathematical expression which contains both a relation symbol and a placeholder symbol. For example, $9 + 2[n-4] > 12$ is an open sentence.

Example 1: Identify the following mathematical expressions by looking for relation symbols and placeholder symbols.

a. $17 - x$ **b.** $16 - (2 \cdot 5) \geq 6$ **c.** $\frac{12+3}{2}$ **d.** $20 = 4(n - 1)$

Solution:

a. **Open Phrase** – Because there is no relation symbol, this expression is just a phrase. But there is a placeholder symbol, so the expression is open.

b. **Closed Sentence** – Because there is a relation symbol, this expression is a sentence. However, there is no placeholder symbol, so the expression is closed.

c. **Closed Phrase** – Because there is no relation symbol, this expression is just a phrase. Neither is there a placeholder symbol, so the expression is closed.

d. **Open Sentence** – There is a relation symbol, so the expression is a sentence. There is also a placeholder symbol, so the expression is open.

Example 2: Take the appropriate action for each of the following expressions. For open phrases, use a domain of $\{0, 1, 2\}$. For open sentences, use a replacement set of $\{4, 5, 6\}$.

a. $17 - x$ **b.** $16 - (2 \cdot 5) \geq 6$ **c.** $\frac{12+3}{2}$ **d.** $20 = 4(n - 1)$

Solution:

a. Since this is an open phrase, the appropriate action is substitution from a domain and evaluation to a range.

Using $\{0, 1, 2\}$, we substitute and evaluate:

$$17-(0) \rightarrow 17$$
$$17-(1) \rightarrow 16$$
$$17-(2) \rightarrow 15$$

So the range is $\{17, 16, 15\}$.

Example 2 cont'd:

b. This expression is a closed sentence, so what we can do is tell if it is true or false.

$$\begin{aligned} 16-(2\cdot 5) &\geq 6 \\ 16-10 &\geq 6 \\ 6 &\geq 6 \end{aligned}$$

Since $6 = 6$ and that is one acceptable condition, the expression is true.

c. Since this is a closed phrase, what we can do is evaluate it.

$$\frac{12+3}{2}=\frac{15}{2}=7\tfrac{1}{2}$$

Its value is $7\frac{1}{2}$.

d. This is an open sentence, so the appropriate action is to substitute from a replacement set and obtain a solution set of values that make the expression true. Using $\{4, 5, 6\}$ we substitute and determine truth or falsehood.

$20 = 4([4]-1)$	$20 = 4([5]-1)$	$20 = 4([6]-1)$
$= 4(3)$	$= 4(4)$	$= 4(5)$
$= 12$	$= 16$	$= 20$
FALSE	FALSE	TRUE

So our solution set is $\{6\}$.

Lesson 2 – Exercises:

Tell whether each of the following expressions is an open phrase, closed phrase, open sentence, or closed sentence.

1. $6+8(3)$

2. $m+2$

3. $5-z=2$

4. $8-2^3=4-2$

5. $\dfrac{12+8}{5}>2\cdot 2$

6. w^2-w

7. $9y$

8. $\dfrac{15+18}{3}\neq 5+6$

9. $3k+11<17$

10. $2\cdot 4-1$

In the following exercises, take the appropriate mathematical action with each expression. For open phrases, use a domain of $\{0, 1, 2\}$. For open sentences, use a replacement set of $\{4, 5, 6\}$.

1. $6 + 8(3)$

2. $m + 2$

3. $5 - z = 2$

4. $8 - 2^3 = 4 - 2$

5. $\frac{12+8}{5} > 2 \cdot 2$

6. $w^2 - w$

7. $9y$

8. $\frac{15+18}{3} \neq 5 + 6$

9. $3k + 11 < 17$

10. $2 \cdot 4 - 1$

Part A – Mathematics as a Language

LESSON 3 Translation of Mathematical Symbols

Objective: To translate English expressions into expressions with mathematical symbols.

Example 1: Translate the following English phrases into phrases with mathematical symbols.

a. The sum of m and 9 decreased by the product of 6 and y.

b. The quotient of the cube of x and the square of z.

Solution:

a. $$\underbrace{\text{the sum of } m \text{ and } 9}_{(m+9)} \quad \underbrace{\text{decreased by}}_{-} \quad \underbrace{\text{the product of 6 and } y}_{6 \cdot y}$$

So $(m + 9) - 6y$ is the desired phrase.

b. $$\underbrace{\text{the quotient of}}_{} \quad \underbrace{\text{the cube of } x}_{x^3} \quad \underbrace{\text{and}}_{\div} \quad \underbrace{\text{the square of } z}_{z^2}$$

So $x^3 \div z^2$ or $\frac{x^3}{z^2}$ is the desired phrase.

Example 2: Translate the following English sentences into sentences with mathematical symbols.

a. Six less than twice a number is 14.

b. 58 is greater than the sum of 5 times a number and 18.

Solution: **a.** $\underbrace{\text{six less than}}_{-6}\ \underbrace{\text{twice a number}}_{2n}\ \underbrace{\text{is}}_{=}\ \underbrace{14}_{14}$

So $2n - 6 = 14$ is the desired sentence.

Note that "six less than" means "subtract 6," so this must be placed to the **right** of $2n$.

b. $\underbrace{58}_{58}\ \underbrace{\text{is greater than}}_{>}\ \underbrace{\text{the sum of}}_{}\ \underbrace{\text{5 times a number}}_{5m}\ \underbrace{\text{and}}_{+}\ \underbrace{18}_{18}$

So $58 > 5m + 18$ is the desired sentence.

Lesson 3 – Exercises:

Translate the following English phrases into phrases with mathematical symbols.

1. The sum of q and 6, increased by 4.

2. The product of m and n, divided by 11.

3. 27 more than the total of x and y.

4. The difference between p and 8.

5. The sum of a and b, divided by 6.

6. a to the power of 6, increased by 12.

7. The product of 10 and z, decreased by the product of 8 and x

8. y multiplied by 6, increased by the product of 5 and z.

9. The cube of y, subtracted from the product of 5 and y.

10. The quantity x plus 5, times the quantity x minus 8.

11. The square of the difference of y and 9, divided by the product of 4 and t.

12. The difference between c and d, subtracted from the sum of q and r.

13. The sum of 6 and b, combined with the total of 7 and c.

14. The difference between the cube of v and the cube of w, divided by the square of e.

15. The square of h divided by b less than the cube of j.

16. 9 more than 4 times a number is equal to 55.

17. The result of 8 times a number decreased by 20 is the same as the sum of 4 times the number and 20.

18. Twice a number is 28.

19. 6 times a number decreased by 12 equals the sum of 3 times the number and 8.

20. 150 is the same as 6 times a number decreased by 30.

21. The difference between 25 and 4 times a number is less than 100 more than 3 times the number.

22. 15 is the difference between one-fourth of a number and 12.

23. Three times a number increased by 20 is greater than 56.

24. 14 more than 10 times a number is the same as the difference between 100 and 4 times the number.

25. The sum of 5 times a number and 30 is less than or equal to 50.

Unit I – The Structure of Mathematics
Part B – Further Investigation of Number Symbols

LESSON 1 The Development of Our Number System

Objective: To identify the different types of numbers and understand how they are related to each other.

Important Terms:

Well-Defined Operation – an operation which satisfies the conditions of ***Existence*** (must get an answer), ***Uniqueness*** (must get only one answer for each number combination), and ***Closure*** (answer must be in the set we are working in).

Natural Numbers – the numbers we use to count objects. The complete set of natural numbers is shown as {1, 2, 3, …}, usually called "the set N." These numbers are also called counting numbers.

Whole Numbers – the natural numbers combined with the number 0. The complete set of whole numbers is shown as {0, 1, 2, 3, …} and usually called "the set W."

Integers – the whole numbers combined with all of their opposites. The complete set is shown as {…, ⁻3, ⁻2, ⁻1, 0, 1, 2, 3, …}.