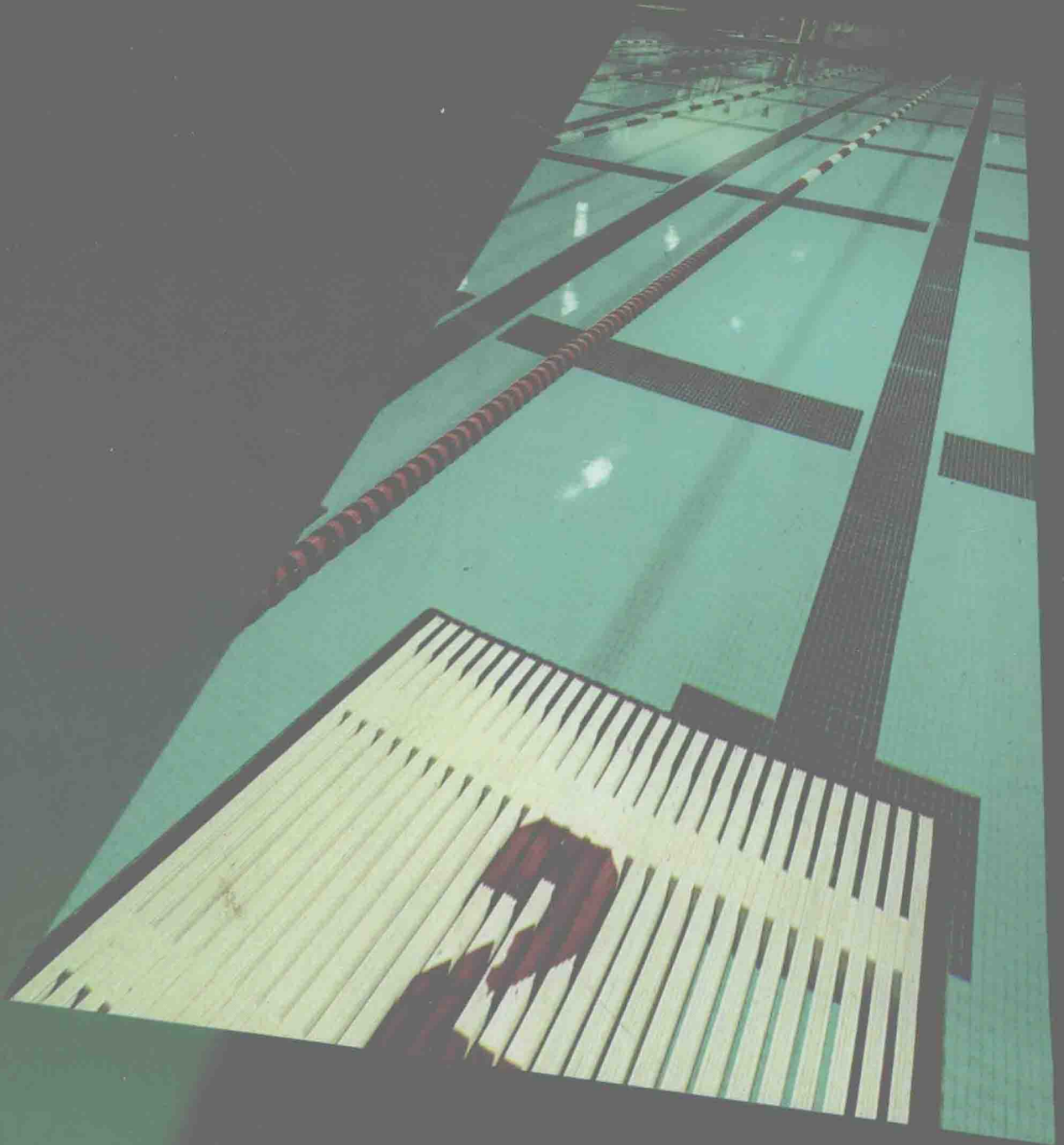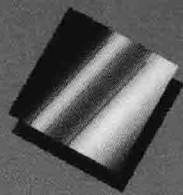# Bittinger Keedy

BASIC MATHEMATICS

SEVENTH EDITION

Basic Mathematics

SEVENTH EDITION

Marvin L. Bittinger
Indiana University—Purdue University at Indianapolis

Mervin L. Keedy
Purdue University

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Preface

This text is appropriate for a one-term course in arithmetic or prealgebra. It is the first in a series of texts that includes the following:

> Bittinger/Keedy: *Basic Mathematics*, Seventh Edition,
>
> Bittinger/Keedy: *Introductory Algebra*, Seventh Edition,
>
> Bittinger/Keedy: *Intermediate Algebra*, Seventh Edition.

What's New in the Seventh Edition?

Basic Mathematics, Seventh Edition, is a significant revision of the Sixth Edition, especially with respect to design, an all-new art program, pedagogy, and an enhanced supplements package. Its unique approach, which has been developed and refined over many years, is designed to help students both learn *and* retain mathematical skills. It is our belief that the Seventh Edition will *continue* to help today's students through pedagogical use of full color and updated applications. As part of *MathMax: The Bittinger/Keedy System of Instruction*, it is accompanied by an extremely comprehensive and well-integrated supplements package to provide maximum support for both instructor and student.

The style, format, and approach of the Sixth Edition have been strengthened in this new edition in a number of ways.

USE OF COLOR The text is now printed in an extremely functional use of full color, evident in striking new design elements and artwork on nearly every page of the text. The use of color has been carried out in a methodical and precise manner so that its use carries a consistent meaning, which enhances the readability of the text for both student and instructor.

For example, when perimeter is considered, figures have a red border to emphasize the perimeter. When area is considered, figures are outlined in black and screened with peach to emphasize the area. Similarly, when volume is considered, figures are three-dimensional, air-brushed blue.
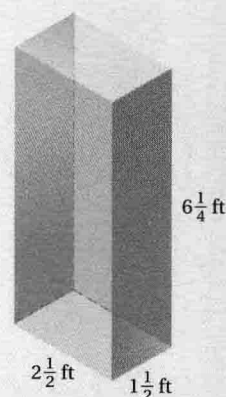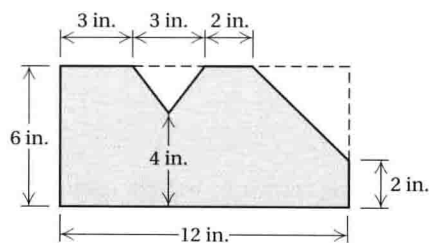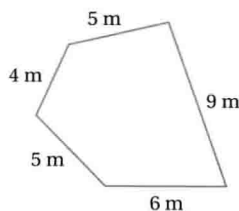
NEW ART PROGRAM   All art in both exposition and the answer section is new. The use of full color in the art program greatly enhances the learning process.

$23\frac{1}{3}$ ft

$18\frac{1}{2}$ ft
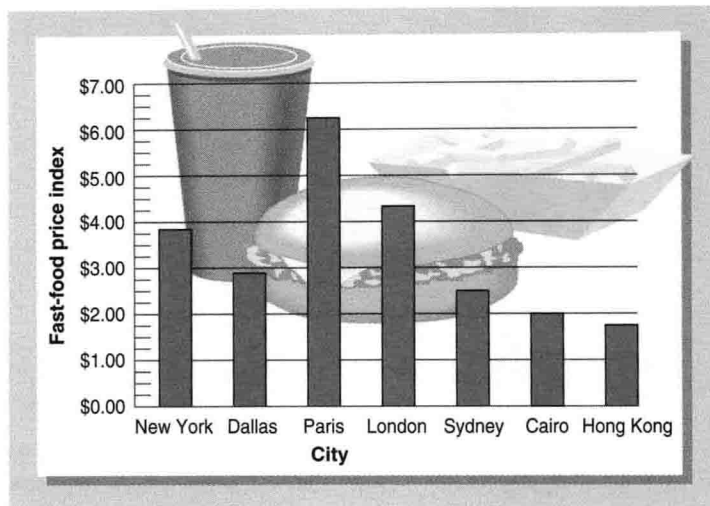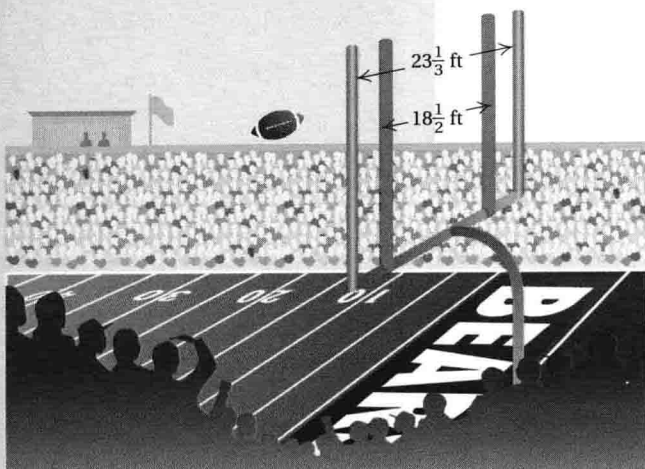
UPDATED APPLICATIONS   Extensive research has been done to make the Seventh Edition's applications even more up-to-date and realistic. Not only are 20% of the exercises new to this edition, but many are drawn from the fields of business and economics, life and physical sciences, social sciences, and areas of general interest such as sports and daily life. To encourage students to "see mathematics" around them every day, many applications use graphs and drawings similar to those found in today's newspapers and magazines.

CRITICAL THINKING   Each chapter now ends with a set of Critical Thinking exercises, which includes Calculator Connections, Extended Synthesis Exercises, and Exercises for Thinking and Writing (all described more fully below).

THE SCIENTIFIC CALCULATOR   Instruction and exercises for the scientific calculator are now covered in many different locations. A calculator icon ▦ highlights exercises that lend themselves to practice with a calculator. Calculator Connection exercises occur in the Critical Thinking sections at the end of each chapter. (These exercises occasionally cover calculator procedures, but for the most part provide critical thinking exercises using a calculator.) Last, there is a special new appendix on calculator keystroke instruction, prepared with the assistance of Rheta Beaver of Valencia Community College.

EXPANDED TREATMENT OF ORDER OF OPERATIONS   The coverage of order of operations has been enhanced in this edition to include the use of fractional notation.

The Bittinger/Keedy System of Instruction

Following are distinctive features of the Bittinger/Keedy System of Instruction that work to ensure learning success for developmental math students.

试读结束，需要全本PDF请购买 www.ertongbook.com

CAREFUL DEVELOPMENT OF CONCEPTS We have divided each section into discrete and manageable learning objectives. Within the presentation of each objective, there is a careful buildup of understanding through a series of developmental examples. These examples enable students to thoroughly understand the mathematical concepts involved at each step.

FOCUS ON UNDERSTANDING Throughout the text, we present the appropriate mathematical rationale for a topic, rather than simply listing rules and procedures. For example, when simplifying fractional notation, we remove factors of 1 rather than cancel (although cancellation is mentioned with appropriate cautions). This method helps prevent student errors in simplification. The notion of multiplying by 1 is a theme that is carried out throughout the book and the series, providing a rationale for many other procedures, such as the conversion of mixed numerals to fractional notation and the development of the cross-product method for comparing fractions, as well as the understanding behind unit conversions in Chapters 9 and 10.

As another example, finding the least common multiple (LCM) is developed sequentially, through a series of methods, rather than through the presentation of just one rule. Also, the understanding of rounding decimal notation is enhanced by using the number line.

PROBLEM SOLVING We include real-life applications and problem-solving techniques to motivate students and encourage them to think about how mathematics can be used in their everyday life. The basis for problem solving is a five-step process (*Familiarize, Translate, Solve, Check,* and *State*) established early in the text and used henceforth.

Learning Aids

INTERACTIVE WORKTEXT APPROACH The pedagogy of this text is designed as an interaction between the student and the exposition, annotated examples, margin exercises, and exercise sets. This approach provides students with a clear set of learning objectives, involves them with the development of the material, and provides immediate and continual reinforcement.

Section objectives are keyed by letter not only to appropriate objectives of the section, but also to exercises in the exercise sets and answers to review exercises and test questions, so that students can easily find appropriate review material if they are unable to work a particular exercise.

Numerous *margin exercises* throughout the text provide immediate reinforcement of the concepts covered in each section.

FOR EXTRA HELP Many valuable study aids accompany this text. Below each list of section objectives are references to appropriate videotape, audiotape, and tutorial software programs, to make it easy for the student to find the correct support materials. The text exercises that appear on the videotapes are listed in an index at the back of the text as well as in the Instructor's Resource Guide.

EXERCISE SETS The exercises are paired, meaning that each even-numbered exercise is very much like the odd-numbered one that precedes it. Answers to the odd-numbered exercises are given at the back of the book, whereas those for the even-numbered exercises are not. This

provides the instructor with many options. If an instructor wants the student to have answers, the odds are assigned. If an instructor wants the student to be able to practice (as on a test) with no answers, the evens are assigned. Thus each exercise set serves as two exercise sets. If an instructor wants the student to have all the answers, a complete answer book is available.

OPPORTUNITIES FOR CRITICAL THINKING   In response to the recommendations of both instructors and educational organizations, we provide many opportunities for students to synthesize concepts, verbalize mathematics, and think critically.

Synthesis Exercises at the end of most exercise sets require students to synthesize learning objectives from the section being studied and preceding sections in the book.

Critical Thinking exercise sets occur at the end of each chapter. These exercise sets provide further opportunity for critical thinking by providing three types of exercises:

- *Calculator Connections* review keystrokes and provide exercises for a scientific calculator.

- *Extended Synthesis Exercises* call for students to further synthesize objectives from the chapter being studied and preceding chapters, thereby building critical-thinking skills.

- *Exercises for Thinking and Writing* encourage students to both think and write about key mathematical ideas in the chapter.

SKILL MAINTENANCE   A well-received feature of preceding editions, the Skill Maintenance exercises have been enhanced by the inclusion of 50% more exercises in this edition. They occur at the end of most exercise sets. Although these exercises can review any objective of preceding chapters, they tend to focus on four specific objectives, called *Objectives for Review.* These objectives are listed at the beginning of each chapter and are covered in each *Summary and Review* and *Chapter Test* at the end of each chapter. The Objectives for Review are also included in a consistent manner in the Printed Test Bank that the instructor uses for testing.

The *Summary and Review* at the end of each chapter provides an extensive set of review exercises along with a list of important formulas and properties covered in that chapter.

We also include a *Cumulative Review* at the end of each chapter but the first, which reviews material from all preceding chapters.

At the back of the text are answers to all end-of-chapter review exercises, together with section and objective references, so that students know exactly what material to restudy if they miss a review exercise.

TESTING   The following assessment opportunities exist in the text.

The *Diagnostic Pretest,* provided at the beginning of the text, can place students in the appropriate chapter for their skill level by identifying familiar material and specific trouble areas.

Chapter Pretests can then be used to place students in a specific section of the chapter, allowing them to concentrate on topics with which they have particular difficulty.

Chapter Tests allow students to review and test comprehension of chapter skills, as well as the four Objectives for Review from earlier chapters. Answers to all Chapter Test questions are found at the back of the book, along with appropriate section and objective references.

Supplements for the Instructor

TEACHER'S EDITION

The Teacher's Edition is a specially bound version of the student text with answers to all exercises in the margins, the exercise sets, and the chapter tests printed in a special color. It also includes answers to all the Critical Thinking exercises at the back of the text.

INSTRUCTOR'S SOLUTIONS MANUAL

The Instructor's Solutions Manual by Judith A. Penna contains brief worked-out solutions to all even-numbered exercises in the exercise sets.

INSTRUCTOR'S RESOURCE GUIDE

The Instructor's Resource Guide contains the following:

- Conversion Guide.
- Extra practice exercises (with answers) for some of the most difficult topics in the text.
- Answers to the Critical Thinking exercises.
- Number lines and grids that can be used as transparency masters for teaching aids and for test preparation.
- Indexes to the videotapes, audiotapes, and tutorial software that accompany the text.
- Instructions for using the Math Hotline.
- Essays on setting up learning labs and testing centers, together with a directory of learning lab coordinators who are available to answer questions.

PRINTED TEST BANK

Prepared by Donna DeSpain, the Printed Test Bank is an extensive collection of alternate chapter test forms, including the following:

- 4 alternate test forms for each chapter, with questions in the same topic order as the objectives presented in the chapter.
- 4 alternate test forms for each chapter, modeled after the Chapter Tests in the text.
- 3 alternate test forms for each chapter, designed for a 50-minute class period.
- 2 multiple-choice test forms for each chapter.
- 2 cumulative review tests for each chapter (with the exception of Chapter 1).
- 8 alternate forms of the final examination, 3 with questions organized by chapter, 3 with questions scrambled, as in the cumulative reviews, and 2 with multiple-choice questions.

ANSWER BOOK

The Answer Book contains answers to all exercises in the exercise sets in the text. Instructors may make quick reference to all answers or have quantities of these booklets made available for sale if they want students to have all the answers.

COMPUTERIZED TESTING: OMNITEST^3

Addison-Wesley's algorithm-driven computerized testing system for Macintosh and DOS computers features a brand-new graphical user interface for the DOS version and a substantial increase in the number of test items available for each chapter of the text.

The new graphical user interface for DOS is a Windows look-alike. It allows users to choose items by test item number or by reviewing all the test items available for a specific text objective. Users can choose the exact iteration of the test item they wish to have on their test or allow the computer to generate iterations for them. Users can also preview all the items for a test on screen and make changes to them during the preview process. They can control the format of the test, including the appearance of the test header, the spacing between items, and the layout of the test and the answer sheet. In addition, users can now save the exact form of the test they have created so that they can modify it for later use. Users can also enter their own items using Omnitest^3's WYSIWYG editor, and have access as well to a library of preloaded graphics.

Both the DOS and Macintosh versions of Omnitest^3 for *Basic Mathematics* contain over 2000 items; 1000 of these are algorithm-driven—capable of generating hundreds of alternative versions. Omnitest^3 for *Basic Mathematics* features at least one algorithm-driven multiple-choice and free-response item for *each* text objective, as well as a selection of static items. Many objectives are covered by *several* multiple-choice and free-response algorithm-based items—the coverage is comparable to the exercise coverage in the text's Summary and Review sections. Each chapter also includes a selection of Thinking and Writing questions.

Omnitest^3 also includes preloaded chapter tests, cumulative tests, and tests designed to parallel state competency examinations to make building your own tests easier than ever before!

COURSE MANAGEMENT AND TESTING SYSTEM

InterAct Math Plus for Windows (available from Addison-Wesley) combines course management and on-line testing with the features of the basic tutorial software (see "Supplements for the Student") to create an invaluable teaching resource. Consult your Addison-Wesley representative for details.

Supplements for the Student

STUDENT'S SOLUTIONS MANUAL

The Student's Solutions Manual by Judith A. Penna contains completely worked-out solutions with step-by-step annotations for all the odd-numbered exercises in the exercise sets in the text. It may be purchased by your students from Addison-Wesley Publishing Company.

"MATH MAKES A DIFFERENCE" VIDEOTAPES

"Math Makes a Difference" is new to this edition of *Basic Mathematics*. It is a complete revision of the existing series of videotapes, based on extensive input from both students and instructors. "Math Makes a Difference"

features a team of mathematics teachers who present comprehensive coverage of each section of the text:

Marvin Bittinger, *Indiana University—Purdue University at Indianapolis*

Carilynn Bouie, *Chattanooga State Technical Community College*

Michael Butler, *College of the Redwoods*

Patricia Cleary, *University of Delaware*

Bettyann Daley, *University of Delaware*

Barbara Johnson, *Indiana University—Purdue University at Indianapolis*

Joanne Peeples, *El Paso Community College*

Anita Polk-Conley, *Chattanooga State Technical Community College*

Since the format is a lecture to a group of students, each videotape is interactive and engaging. Lecturers use odd-numbered exercises from the text as examples—these are listed in the videotape indexes in the Instructor's Resource Guide and at the back of the text. Icons 📼 at the beginning of each section reference the appropriate videotape number.

A complete set of "Math Makes a Difference" videotapes is free to qualifying adopters.

AUDIOTAPES

The audiotapes are designed to lead students through the material in each text section. Bill Saler explains solution steps to examples, cautions students about common errors, and instructs them at certain points to stop the tape and do exercises in the margin. He then reviews the margin exercise solutions, pointing out potential errors. Icons 🎧 at the beginning of each section reference the appropriate audiotape number.

The audiotapes are free to qualifying adopters.

THE MATH HOTLINE

Prepared by Larry A. Bittinger, the Math Hotline is open 24 hours a day at 1-800-333-4227 so that students can obtain detailed hints for exercises. Exercises covered include all the odd-numbered exercises in the exercise sets, with the exception of the Skill Maintenance and Synthesis exercises.

INTERACT MATH TUTORIAL SOFTWARE

InterAct Math Tutorial Software, new to this edition of *Basic Mathematics,* has been developed and designed by professional software engineers working closely with a team of experienced developmental math teachers.

InterAct Math Tutorial Software includes exercises that are linked one-to-one with the odd-numbered exercises in the textbook and require the same computational and problem-solving skills as their companion exercises in the text. Each exercise has an example and an interactive guided solution that are designed to involve students in the solution process and to help them identify precisely where they are having trouble. In addition, the software recognizes common student errors and provides students with appropriate customized feedback.

With its sophisticated answer recognition capabilities, InterAct Math Tutorial Software recognizes appropriate forms of the same answer for any kind of input. It also tracks student activity and scores for each section, which can then be printed out. Icons at the beginning of each section ⊞ reference the appropriate disk number.

Available for both DOS-based and Macintosh computers, the software is free to qualifying adopters.

We, your authors, have committed ourselves to writing a usable, understandable, accomplishable, error-free book that will extend the student's knowledge and enjoyment of mathematics. Students and instructors will undoubtedly have many general impressions and attitudes that form during their semester or two in a mathematics course. To help us to continually improve the text and to support the instructor's goals, we invite correspondence from both students and instructors to:

Marv Bittinger and Mike Keedy
c/o Marv Bittinger
3011 Whispering Trail
Carmel, IN 46033

Acknowledgments

The Steps to Success

The following six pages show you how to use *Basic Mathematics* to maximize understanding while making studying easier.

Use the chapter opener to begin your work with the chapter.

1

3

Addition and Subtraction: Fractional Notation

INTRODUCTION

In this chapter, we consider addition and subtraction using fractional notation. Also discussed are addition, subtraction, multiplication, and division using mixed numerals. We then apply all these operations to the use of rules for order of operations and to problem solving.

• •

The chapter introduction provides an overall view of the chapter's content.

AN APPLICATION

A bicycle wheel makes $66\frac{2}{3}$ revolutions per minute. It rotates for 12 min. How many revolutions does it make?

THE MATHEMATICS

Let n = the total number of revolutions. The problem then translates to this equation:

$$66\frac{2}{3} \cdot 12 = n.$$

This is a mixed numeral.

Read the Objectives for Review, which list four objectives from preceding chapters that will be reinforced for skill maintenance in this chapter and its test.

2

OBJECTIVES FOR REVIEW

The review sections to be tested in addition

[2.7b] Divide and simplify using fra
[.4a]   Solve problems involving pro
[.4], [7.4b] Solve basic percent problems
          Solve applied percent problem

Pretest: Chapter

In Questions 1–3, find (a) the averag

1. 46, 50, 53, 55

4. A car traveled 660 mi in 12 hr. What was the average number of miles per hour?

5. To get a C in chemistry, Delia must average 70 on four tests. Scores on the first three tests were 68, 71, and 65. What is the lowest score that she can make on the last test and still get a C?

6. The following data show the percentage of women selecting a particular reason for exercising. Make a circle graph to show the data.

   Health: 51%
   Lose weight: 38%
   Relieve stress: 11%

7. The following table shows the comparison of the cost of a $100,000 life insurance policy for female smokers and nonsmokers at certain ages.

   a) How much does it cost a female smoker, age 32, for insurance?
   b) How much does it cost a female nonsmoker, age 32, for insurance?
   c) How much more does it cost a female smoker, age 35, than a nonsmoker at the same age?

8. Using the data in Exercise 7, draw a vertical bar graph showing the cost of insurance for a female smoker at various ages. Use age on the horizontal scale and cost on the vertical scale.

9. Using the data in Exercise 7, draw a line graph showing the cost of insurance for a female smoker at various ages. Use age on the horizontal scale and cost on the vertical scale.

The line graph at the right shows the relationship between blood cholesterol level and risk of coronary heart disease.
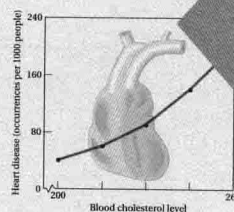
10. At what cholesterol level is the risk highest?

11. About how much higher is the risk at 260 than at 200?

| LIFE INSURANCE: FEMALE | | |
AGE	COST (SMOKER)	COST (NONSMOKER)
31	$294	$170
32	298	172
33	302	176
34	310	178
35	316	182

An application drawn from a chapter example or exercise immediately shows how a key chapter concept applies to the real world.

Take the Chapter Pretest to assess your own strengths and weaknesses in the upcoming material.
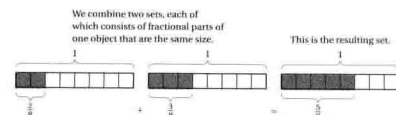
3

Each section of the chapter is designed as an interaction between you and the written explanations, the annotated examples, the margin exercises, and the exercise set.

Read the objectives listed in the margin and keyed to the text.

Objectives provide an instant outline of the section.

Important definitions, rules, and procedures are highlighted in boxes.

As you study the examples, note the detailed annotations and color highlights that help you on your way.

You are encouraged to do the margin exercises as you work through the material.

The videotape, audiotape, and software references provide extra help for each section.

Answers to the margin exercises are given at the back of the book.

4

6

3.2 Addition

a LIKE DENOMINATORS

Addition using fractional notation corresponds to combining or putting like things together, just as addition with whole numbers does. For example,

We combine two sets, each of which consists of fractional parts of one object that are the same size.

This is the resulting set.

2 eighths + 3 eighths = 5 eighths,

or $2 \cdot \frac{1}{8} + 3 \cdot \frac{1}{8} = 5 \cdot \frac{1}{8}$,

or $\frac{2}{8} + \frac{3}{8} = \frac{5}{8}$.

Do Exercise 1.

To add when denominators are the same,
a) add the numerators,
b) keep the denominator, and
c) simplify, if possible.

$$\frac{2}{6} + \frac{5}{6} = \frac{2 + 5}{6} = \frac{7}{6}$$

EXAMPLES Add and simplify.

1. $\frac{2}{4} + \frac{1}{4} = \frac{2 + 1}{4} = \frac{3}{4}$ No simplifying is possible.

2. $\frac{11}{6} + \frac{3}{6} = \frac{11 + 3}{6} = \frac{14}{6} = \frac{2 \cdot 7}{2 \cdot 3} = \frac{2}{2} \cdot \frac{7}{3} = 1 \cdot \frac{7}{3} = \frac{7}{3}$ Here we simplified.

3. $\frac{3}{12} + \frac{5}{12} = \frac{3 + 5}{12} = \frac{8}{12} = \frac{4 \cdot 2}{4 \cdot 3} = \frac{4}{4} \cdot \frac{2}{3} = 1 \cdot \frac{2}{3} = \frac{2}{3}$

Do Exercises 2–4.

b ADDITION USING THE LCD: DIFFERENT DENOMINATORS

What do we do when denominators are different? We try to find a common denominator. We can do this by multiplying by 1. Consider adding $\frac{1}{6}$ and $\frac{3}{4}$. There are several common denominators that can be obtained. Let's look at two possibilities.

OBJECTIVES

After finishing Section 3.2, you should be able to:

a Add using fractional notation when denominators are the same.

b Add using fractional notation when denominators are different, by multiplying by 1 to find the least common denominator.

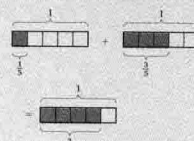c Solve problems involving addition with fractional notation.

FOR EXTRA HELP

TAPE 6   TAPE 5A   MAC: 3A
                         IBM: 3A

1. Find $\frac{1}{5} + \frac{3}{5}$.

Add and simplify.

2. $\frac{1}{3} + \frac{2}{3}$

3. $\frac{5}{12} + \frac{1}{12}$

4. $\frac{9}{16} + \frac{3}{16}$

Answers on page A-2

3.2 ADDITION

143

Exercise Sets provide for a wealth of practice with chapter concepts.
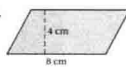
5

Exercises are keyed to objectives in the text.

Exercise Set 9.5

[a] Find the area.

1.
4 cm
8 cm

2.
4 cm
4 cm

3.
8 in.
←——15 in.——→

4.
18 yd
18 yd

5.
6 ft
8 ft
20 ft

6.
5 yd
4 yd
10 yd

7.
8 m
8 m

8.
7 mm
←———10 mm———→

9.   4.5 in.

10.
7.25 m
12 m

11.
←2.3 cm→

13.
9 cm
18 cm
24 cm

14.
13 mi
9 mi
19 mi
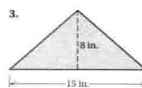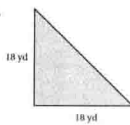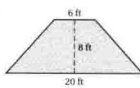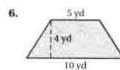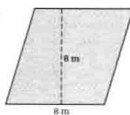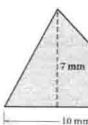
ANSWERS

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____

The Student's Solutions Manual and the Math Hotline are available for immediate help with the exercises.

ANSWERS

9. _____
10. _____
11. _____
12. _____
13. _____
14. _____
15. _____
16. _____
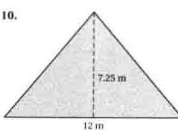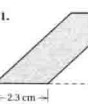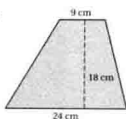17. _____
18. _____
19. _____
20. _____

9. When a tree 8 m high casts a shadow 5 m long, how long a shadow is cast by a person 2 m tall?

10. How high is a flagpole that casts a 42-ft shadow at the same time that a $5\frac{1}{2}$-ft woman casts a 7-ft shadow?

11. How high is a tree that casts a 27-ft shadow at the same time that a 4-ft fence post casts a 3-ft shadow?

12. How high is a tree that casts a 32-ft shadow at the same time that an 8-ft light pole casts a 9-ft shadow?

SKILL MAINTENANCE

13. A student has $34.97 to spend for a book at $49.95, a CD at $14.88, and a sweatshirt at $29.95. How much more money does the student need to make these purchases?

14. Divide: $80.892 \div 8.4$.

Multiply.

15. 8.4×80.892
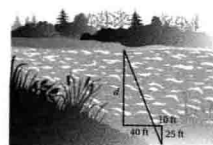
16. 0.01×274.568

17. 100×274.568

18. 0.002×274.568

SYNTHESIS

19. Find the distance across the river. Assume that the ratio of d to 25 ft is the same as the ratio of 40 ft to 10 ft.

20. To measure the height of a hill, a string is drawn tight from level ground to the top of the hill. A 3-ft yardstick is placed under the string, touching it at point P, a distance of 5 ft from point G, where the string touches the ground. The string is then detached and found to be 120 ft long. How high is the hill?

120 ft
5 ft
3 ft
G

d
10 ft
40 ft   25 ft
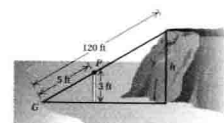
Skill Maintenance exercises at the end of most exercise sets review objectives from earlier chapters, especially the Objectives for Review that will be tested on the chapter test.

Synthesis exercises help you to synthesize objectives and provide insight into the material.

THE STEPS TO SUCCESS

xxi

7

CRITICAL THINKING

CALCULATOR CONNECTION

Many calculators have the capability of using fractional notation and mixed numerals for computations, giving answers in such notation as well. The following exercises assume that you are using such a calculator.

1. In the sum below, a and b are digits. Find a and b.

$$\frac{a}{17} + \frac{1b}{23} = \frac{35a}{391}$$

2. Consider only the numbers 2, 3, 4, and 5. Assume each is placed in a blank in the following.

$$\frac{\blacksquare}{\blacksquare} + \frac{\blacksquare}{\blacksquare} = ?$$

What placement of the numbers in the blanks yields the largest sum?

3. Consider only the numbers 3, 4, 5, and 6. Assume each can be placed in a blank in the following.

$$\blacksquare + \frac{\blacksquare}{\blacksquare} \cdot \blacksquare = ?$$

What placement of the numbers in the blanks yields the largest number?

4. Use a standard calculator. Arrange the following in order from largest to smallest.

$$\frac{3}{4}, \frac{17}{21}, \frac{13}{15}, \frac{7}{9}, \frac{15}{17}, \frac{13}{12}, \frac{19}{22}$$

EXTENDED SYNTHESIS EXERCISES

1. a) Simplify each of the following, using fractional notation for your answers.

$$\frac{1}{1 \cdot 2}$$

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3}$$

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4}$$

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \frac{1}{4 \cdot 5}$$

b) Look for a pattern in your answers to part (a). Then find the following without carrying out the computations.

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \frac{1}{4 \cdot 5} + \frac{1}{5 \cdot 6}$$
$$+ \frac{1}{6 \cdot 7} + \frac{1}{7 \cdot 8} + \frac{1}{8 \cdot 9} + \frac{1}{9 \cdot 10}$$
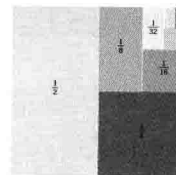
2. Each of the following represents a portion of the total area of the square shown below.

$$\frac{1}{2}$$

$$\frac{1}{2} + \frac{1}{4}$$

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8}$$

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}$$

a) Find each of the areas by simplifying the sums. Use fractional notation for your answers.

b) Look for a pattern in your answers to part (a). Make a conjecture about the total area of the square.

3. Yuri and Olga are orangutans who perform in a circus by riding bicycles around a circular track. It takes Yuri 6 min to make one trip around the track and Olga 4 min. Suppose they start at the same point and then complete their act when they again reach the same point. How long is their act?

4. The students in a math class can be organized into study groups of 8 each such that no students are left out. The same class of students can also be organized into groups of 6 such that no students are left out.

a) Find some class sizes for which this will work.

b) Find the smallest such class size.

5. Find r if

$$\frac{1}{r} = \frac{1}{100} + \frac{1}{150} + \frac{1}{200}.$$

(continued)

CRITICAL THINKING

6. *Estimation with fractions.* A fraction is very close to 0 when the numerator is very small compared to the denominator. For example, 0 is an estimate for $\frac{2}{17}$. A fraction is very close to $\frac{1}{2}$ when the numerator is about half the denominator. For example, $\frac{1}{2}$ is an estimate for $\frac{11}{23}$. A fraction is very close to 1 when the numerator is very close to the denominator. For example, 1 is an estimate for $\frac{37}{38}$ or for $\frac{43}{41}$. Estimate each of the following as either 0, $\frac{1}{2}$, or 1.

a) $\frac{2}{47}$ b) $\frac{4}{5}$ c) $\frac{1}{13}$ d) $\frac{7}{8}$

e) $\frac{6}{11}$ f) $\frac{10}{13}$ g) $\frac{7}{15}$ h) $\frac{1}{16}$

i) $\frac{7}{100}$ j) $\frac{5}{9}$ k) $\frac{19}{20}$ l) $\frac{5}{12}$

7. Find a number for the blank so that the fraction is close to but greater than $\frac{1}{2}$. Answers can vary.

a) $\frac{\blacksquare}{11}$ b) $\frac{\blacksquare}{8}$ c) $\frac{\blacksquare}{23}$ d) $\frac{\blacksquare}{35}$

e) $\frac{10}{\blacksquare}$ f) $\frac{7}{\blacksquare}$ g) $\frac{8}{\blacksquare}$ h) $\frac{51}{\blacksquare}$

8. Find a number for the blank so that the fraction is close to but less than 1. Answers can vary.

a) $\frac{7}{\blacksquare}$ b) $\frac{11}{\blacksquare}$ c) $\frac{13}{\blacksquare}$ d) $\frac{27}{\blacksquare}$

e) $\frac{\blacksquare}{15}$ f) $\frac{\blacksquare}{9}$ g) $\frac{\blacksquare}{18}$ h) $\frac{\blacksquare}{100}$

9. Estimate each of the follo... or as a mixed numeral wh... is $\frac{1}{2}$.

a) $2\frac{7}{8}$

c) $12\frac{5}{6}$

10. Estimate the sum.

a) $\frac{4}{5} + \frac{7}{8}$ b) $\frac{1}{12} + \frac{7}{15}$

c) $\frac{2}{3} + \frac{7}{13} + \frac{5}{9}$ d) $\frac{8}{9} + \frac{4}{5} + \frac{11}{12}$

e) $\frac{3}{100} + \frac{1}{10} + \frac{11}{1000}$ f) $\frac{23}{24} + \frac{37}{39} + \frac{51}{50}$

EXERCISES FOR THINKING AND WRITING

1. Explain why $2\frac{1}{4} \cdot 3\frac{2}{5} \neq 6\frac{2}{20}$.

2. Explain why $5 \cdot 3\frac{2}{5} \neq (5 \cdot 3) \cdot \left(5 \cdot \frac{2}{5}\right)$.

3. Discuss the role of least common multiples in adding and subtracting with fractional notation.

4. Find a real-world situation that fits the equation

$$2 \cdot 15\frac{3}{4} + 2 \cdot 28\frac{5}{8} = 88\frac{3}{4}.$$