PERRY'S CHEMICAL ENGINEERS' HANDBOOK

SEVENTH EDITION

Section 9

Process Economics*

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INVESTMENT AND PROFITABILITY

ACCOUNTING AND COST CONTROL

MANUFACTURING-COST ESTIMATION

FIXED-CAPITAL-COST ESTIMATION

* The contribution of the late Mr. F. A. Watson, who was an author for the Sixth edition, is acknowledged.

Nomenclature and Units

Symbol	Definition	Units
a	Empirical constant in general equations	Various
A	Annual income or expenditure particularized by the subscript	\$/year
A_A	Annual allowances against tax other than for depreciation of fixed assets	\$/year
A_D	Annual writing down (depreciation) of fixed assets, allowable against tax	\$/year
(ATR)	Asset-turnover ratio defined by Eq. (9-131)	Dimensionless
b	Empirical constant in general equations	Various
b_c	Deviation from budgeted capacity	Dimensionless
B	Parametric constant in Eq. (9-204)	Dimensionless
c	Empirical constant in general equations	Various
c	Cost (or income) per unit of sales or production particularized by the subscript	\$/unit
c_B	Cost of base heat supply	\$/unit
c_D	Cost of heat energy delivered by a heat pump defined by Eq. (9-240)	\$/GJ
c_I	Cost of high-grade energy supplied to the compressor of a vapor compression heat pump	\$/GJ
c_L	Cost of labor per unit of production	\$/hour
c°	Standard cost particularized by the subscript	\$/hour
C	Cost particularized by the subscript	\$
C_{CT}	Installed cost of a cooling tower	\$
C_{DS}	Installed cost of a demineralized-water system	\$
$(C_{EQ})_{DEL}$	Delivered-equipment cost	\$
C_K	Capitalized cost of a fixed asset defined by Eq. (9-47)	\$
C_L	Cost of land and other nondepreciable assets	\$
C_{RS}	Installed cost of a refrigeration system	\$
C_{RW}	Installed cost of a river-water supply system	\$
C_{WS}	Installed cost of a water-softening system	\$
(CI)	Cost index as used in Eq. (9-246)	Dimensionless
$(COP)_A$	Actual coefficient of performance of a heat pump	Dimensionless
(CR)	Capital ratio defined by Eq. (9-134)	Year
(CRR)	Capital-rate-of-return ratio defined by Eq. (9-56)	Year
(CSR)	Contribution-sales ratio defined by Eq.(9-236)	Dimensionless
d	Empirical constant in general equations	Various
d	Symbol indicating differentiation	Dimensionless
(DR)	Debt ratio defined by Eq. (9-139)	Dimensionless
(DCFRR)	Discounted-cash-flow rate of return	Year^{-1}
e	Empirical constant in general equations	Various
e	Base of natural logarithms, 2.71828	Dimensionless
$\exp(a)$	Exponential function of a, e^a	Dimensionless
(EMIP)	Equivalent maximum investment period defined by Eq. (9-55)	Year
f_{AF}	Annuity future-worth factor, $i[(1+i)^n - 1]^{-1}$	Dimensionless
f_{AP}	Annuity present-worth factor, $f_{AF}(1+i)^n$	Dimensionless

Symbol	Definition	Units
f_d	Discount factor, $(1+i)^{-n}$	Dimensionless
f_i	Compound-interest factor, $(1+i)^n$	Dimensionless
f_k	Capitalized-cost factor, f_{AP}/i	Dimensionless
f_p	Piping-cost factor defined by Eq. (9-249)	Dimensionless
$f(x)$	Distribution function of x variously defined	Dimensionless
F	Future value of a sum of money	\$
F_n	Sum of f_d for Years 1 to n	Dimensionless
i	Interest rate per period, usually annual, often the cost of capital	Dimensionless
i_e	Effective interest rate defined by Eq. (9-111)	Dimensionless
i_m	Minimum acceptable interest rate defined by Eq. (9-107)	Dimensionless
i_r	Entrepreneurial-risk interest rate	Dimensionless
i'	Nominal annual interest rate	Dimensionless
I	Value of inventory particularized by the subscript	\$
k_n	Constants in Eq. (9-81)	Various
K	Effective value of the first unit of production	\$/unit, time/unit, etc.
$\ln(a)$	Logarithm to the base e of a	Dimensionless
$\log(a)$	Logarithm to the base 10 of a	Dimensionless
m	Number of interest periods due per year	Dimensionless
m	Number of units removed from inventory	Dimensionless
(MSF)	Measured-survival function defined by Eq. (9-106)	Dimensionless
n	Number of years, units, etc.	Dimensionless
N	Slope of the learning curve defined by Eq. (9-64)	Dimensionless
N	Number of inventory orders per year	Dimensionless
(NPV)	Net present value	\$
$p(x)$	Probability of the variable having the value x	Dimensionless
P	Present value of a sum of money	\$
P_a	Production time worked	Hour
P_b	Budgeted production	Standard hour
P_e	Production efficiency defined by Eq. (9-216)	Dimensionless
P_1	Level of productive activity defined by Eq. (9-217)	Dimensionless
P_s	Actual production rate	Standard hour
P'_s	Book value of asset at the end of year s'	\$
P_w	Budgeted working time	Hour
(PBP)	Payback period defined by Eq. (9-30)	Year
(PM)	Profit margin defined by Eq. (9-127)	Dimensionless
(PSR)	Profit-sales ratio defined by Eq. (9-235)	Dimensionless
q	Quantity defining the scale of operation	Various
Q_D	Process-heat-rate requirement	GJ/hour
r	Fraction of range of the independent variable	Dimensionless
R	Production rate	Units/year
R°	Standard production rate	Units/year
R_B	Breakeven production rate	Units/year

Nomenclature and Units (*Concluded*)

Symbol	Definition	Units
R_0	Scheduled production rate	Units/year
R_S	Sales rate	Units/year
(ROA)	Return on assets defined by Eq. (9-129)	Dimensionless
(ROE)	Return on equity defined by Eq. (9-130)	Dimensionless
(ROI)	Return on investment defined by Eq. (9-128)	Dimensionless
s	Scheduled number of productive years	
s'	Number of productive years to date	
s°	Sample standard deviation	Various
S	Scrap value of a depreciable asset	$
t	Fractional tax rate payable on adjusted income	Dimensionless
t_C	Time taken to construct plant	Years
t_{SU}	Time taken to start up plant	Years
T	Auxiliary variable defined by Eq. (9-92)	Various
U	Size of inventory order	Units
V	Variable cost of inventory order	$/unit
W	Power supplied at shaft of a heat pump	GJ/hour
x	General variable	
$\bar{x}$	Mean value of x	Various
X	Cumulative production from startup	Units
y	Cumulative probability	Dimensionless
y	Operating time of a heat pump	Hours/year
Y	Cumulative average cost, production time, etc.	$/unit, hour/unit, etc.
Y	Operating-labor rate in Eq. (9-204)	labor-hour/ton
$\bar{Y}$	Cumulative-average batch cost, etc.	$/unit, etc.
z	Standard score defined by Eq. (9-73)	Dimensionless
	Greek symbols	
α	Proportionality factor in Eq. (9-168)	Dimensionless
β	Proportionality factor in Eq. (9-171)	Dimensionless
β	Exponent in Eqs. (9-106) and (9-117)	Dimensionless
δ	Symbol indicating partial differentiation	Dimensionless
Δ	Symbol indicating a difference of like quantities	Dimensionless
$\dot{\eta}$	Contribution efficiency defined by Eq. (9-119)	Dimensionless
η	Margin of safety defined by Eq. (9-229)	Dimensionless
θ	Time taken to produce a given amount of product	Hour
σ	Population standard deviation	Various
Σ	Symbol indicating a sum of like quantities	Dimensionless
ϕ	Fractional increase in production rate	Dimensionless
ϕ_P	Parameter defined with Eq. (9-254)	Dimensionless
ψ	Parameter defined with Eq. (9-241)	Dimensionless
χ	Plant capacity in Eq. (9-204)	Tons/day
χ	Weight of product per unit of raw material	Dimensionless
	Subscripts	
A	Allowance against tax other than for capital depreciation	
BD	Depreciation allowance shown in company balance sheet	
BL	Within project boundary limits	
BOH	Budgeted overhead	
CF	Cash flow after payment of tax and expenses	
CI	Cash income after payment of expenses	
DCF	Discounted cash flow	
DME	Direct manufacturing expense	
FC	Fixed capital	
FE	Fixed expense	
FGE	Fixed general expense	
FIFO	On a first-in–first-out basis	
FIN	Financial-resources inventory	
FME	Fixed manufacturing expense	
FOH	Fixed overhead	
GE	General expense	
GP	Gross profit	
IME	Indirect manufacturing expense	
INV	Inventory	
IO	Inventory-orders cost	
IT	Income tax payable	
IW	Inventory working cost	
L	Labor-earnings index	
L	Lower-quartile value of the variable	
LIFO	Last-in–first-out basis	
max	Maximum value	
M	Median value of the variable	
ME	Manufacturing expense	
N	At agreed normal production rate	
NCI	Net cash income after payment of tax	
NOH	Overhead cost at agreed normal production rate	
NNP	Net profit after payment of tax	
NP	Net profit before payment of tax	
OH	Overhead cost	
P	Profit	
RM	Raw material	
s'	In the s'th productive year	
S	From sales and other income	
SAV	On a simple-average basis	
ST	Steel-price index	
$SVOH$	Semivariable overhead	
TC	Total capital	
TE	Total expense	
TFE	Total fixed expense	
TVE	Total variable expense	
U	Utilities	
U	Upper-quartile value of the variable	
VE	Variable expense	
VGE	Variable general expense	
VME	Variable manufacturing expense	
VOH	Variable overhead expense	
W	Weighted value	
WC	Working capital	
WAV	On a weighted-average basis	
1, 2, j, n	1st, 2d, jth, nth item, year, etc.	

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NOMENCLATURE

An attempt has been made to bring together most of the methods currently available for project evaluation and to present them in such a way as to make the methods amenable to modern computational techniques. To this end the practices of accountants and others have been reduced, where possible, to mathematical equations which are usually solvable with an electronic hand calculator equipped with scientific function keys. To make the equations suitable for use on high-speed computers an attempt has been made to devise a nomenclature which is suitable for machines using ALGOL, COBOL, or FORTRAN compilers. The number of letters and numbers used to define a variable has usually been limited to five. The letters are mnemonic in English wherever possible and are derived in two ways. First, when a standard accountancy phrase exists for a term, this has been abbreviated in capital letters and enclosed in parentheses, e.g., (ATR), for assets-to-turnover ratio; (DCFRR), for discounted-cash-flow rate of return. Clearly, the parentheses are omitted when the letter group is used to define the variable name for the computer. Second, a general symbol is defined for a type of variable and is modified by a mnemonic subscript, e.g., an annual cash quantity A_{TC}, annual total capital outlay, \$/year. Clearly, the symbols are written on one line when the letter group is used to define a variable name for the computer. In other cases, when well-known standard symbols exist, they have been adopted, e.g., z for the standard score as used in the normal distribution. Also, a, b, c, d, and e have been used to denote empirical constants and x and y to denote general variables where their use does not clash with other meanings of the same symbols.

The coverage in this section is so wide that nomenclature has sometimes proved a problem which has required the use of primes, asterisks, and other symbols not universally acceptable in the naming of computer variables. However, it is realized that each individual will program only his or her preferred methods, which will release some symbols for other uses. Also, it is not difficult to replace a forbidden symbol by an acceptable one; e.g., c_{RM} might be rendered CARM and P_S' as PSP by using A for asterisk and P for prime. For compilers which recognize only one alphabetical case, an extra prefix can be used to distinguish between uppercase and lowercase letters, for which purpose the letters U and L have been used only in a restricted way in the nomenclature.

It is, of course, impossible to allow for all possible variations of equation requirements and machine capability, but it is hoped that the nomenclature in the table presented at the beginning of the section will prove adequate for most purposes and will be capable of logical extension to other more specialized requirements.

INVESTMENT AND PROFITABILITY

In order to assess the profitability of projects and processes it is necessary to define precisely the various parameters.

Annual Costs, Profits, and Cash Flows To a large extent, accountancy is concerned with annual costs. To avoid confusion with other costs, annual costs will be referred to by the letter A.

The revenue from the annual sales of product A_S, minus the total annual cost or expense required to produce and sell the product A_{TE}, excluding any annual provision for plant depreciation, is the annual cash income A_{CI}:

$$A_{CI} = A_S - A_{TE} \tag{9-1}$$

Net annual cash income A_{NCI} is the annual cash income A_{CI}, minus the annual amount of tax A_{IT}:

$$A_{NCI} = A_{CI} - A_{IT} \tag{9-2}$$

Taxable income is $(A_{CI} - A_D - A_A)$, where A_D is the annual writing-down allowance and A_A is the annual amount of any other allowances. A distinction is made between the writing-down allowance permissible for the computation of tax due, the actual depreciation in value of an asset, and the book depreciation in value of that asset as shown in the company position statement. There is no necessary connection between these values unless specified by law, although the first two or all three are often assigned the same value in practice. Some governments give cash incentives to encourage companies to build plants in otherwise unattractive areas. Neither A_D nor A_A involves any expenditure of cash, since they are merely book transactions. The annual amount of tax A_{IT} is given by

$$A_{IT} = (A_{CI} - A_D - A_A)t \tag{9-3}$$

where t is the fractional tax rate. The value of t is determined by the appropriate tax authority and is subject to change. For most developed countries the value of t is about 0.35 or 35 percent.

The annual amount of tax A_{IT} included in Eq. (9-2) does not necessarily correspond to the annual cash income A_{CI} in the same year. The tax payments in Eq. (9-2) should be those actually paid in that year. In the United States, companies pay about 80 percent of the tax on estimated current-year earnings in the same year. In the United Kingdom, companies do not pay tax until at least 9 months after the end of the accounting period, which, for the most part, amounts to paying tax on the previous year's earnings. When assessing projects for different countries, engineers should acquaint themselves with the tax situation in those countries.

In modern methods of profitability assessment, cash flows are more meaningful than profits, which tend to be rather loosely defined. The net annual cash flow after tax is given by

$$A_{CF} = A_{NCI} - A_{TC} \tag{9-4}$$

where A_{TC} is the annual expenditure of capital, which is not necessarily zero after the plant has been built. For example, working capital, plant additions, or modifications may be required in future years.

The total annual expense A_{TE} required to produce and sell a product can be written as the sum of the annual general expense A_{GE} and the annual manufacturing cost or expense A_{ME}:

$$A_{TE} = A_{GE} + A_{ME} \tag{9-5}$$

Annual general expense A_{GE} arises from the following items: administration, sales, shipping of product, advertising and marketing, technical service, research and development, and finance.

The terms gross annual profit A_{GP} and net annual profit A_{NP} are commonly used by accountants and misused by others. Normally, both A_{GP} and A_{NP} are calculated before tax is deducted. Gross annual profit A_{GP} is given by

$$A_{GP} = A_S - A_{ME} - A_{BD} \tag{9-6}$$

where A_{BD} is the balance-sheet annual depreciation charge, which is not necessarily the same as A_D used in Eq. (9-3) for tax purposes. Net annual profit A_{NP} is simply

$$A_{NP} = A_{GP} - A_{GE} \tag{9-7}$$

Equation (9-7) can also be written as

$$A_{NP} = A_{CI} - A_{BD} \tag{9-8}$$

Net annual profit after tax A_{NNP} can be written as

$$A_{NNP} = A_{NCI} - A_{BD} \tag{9-9}$$

The relationships among the various annual costs given by Eqs. (9-1) through (9-9) are illustrated diagrammatically in Fig. 9-1. The top half of the diagram shows the tools of the accountant; the bottom half, those of the engineer. The net annual cash flow A_{CF}, which excludes any provision for balance-sheet depreciation A_{BD}, is used in two of the more modern methods of profitability assessment: the net-present-value (NPV) method and the discounted-cash-flow-rate-of-return (DCFRR) method. In both methods, depreciation is inherently taken care of by calculations which include capital recovery.

Annual general expense A_{GE} can be written as the sum of the fixed and variable general expenses:

$$A_{GE} = A_{FGE} + A_{VGE} \tag{9-10}$$

Similarly, annual manufacturing expense A_{ME} can be written as the sum of the fixed and variable manufacturing expenses:

$$A_{ME} = A_{FME} + A_{VME} \tag{9-11}$$

A variable expense is considered to be one which is directly proportional to the rate of production R_P or of sales R_S as is most appropriate to the case under consideration. Unless the variation in finished-product inventory is large when compared with the total production over the period in question, it is usually sufficiently accurate to consider R_P and R_S to be represented by the same-numerical-value R units of sale or production per year. A fixed expense is then considered to be one which is not directly proportional to R, such as overhead charges. Fixed expenses are not necessarily constant but may be sub-

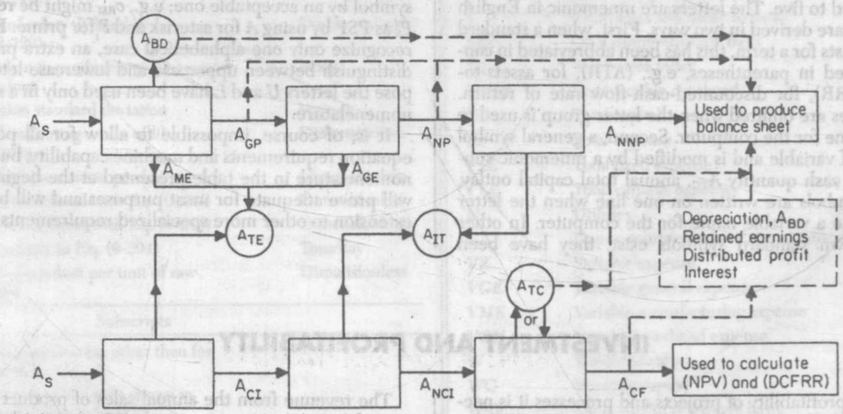

FIG. 9-1 Relationship between annual costs, annual profits, and cash flows for a project. A_{BD} = annual depreciation allowance; A_{CF} = annual net cash flow after tax; A_{CI} = annual cash income; A_{GE} = annual general expense; A_{GP} = annual gross profit; A_{IT} = annual tax; A_{ME} = annual manufacturing cost; A_{NCI} = annual net cash income; A_{NNP} = annual net profit after taxes; A_{NP} = annual net profit; A_S = annual sales; A_{TC} = annual total cost; (DCFRR) = discounted-cash-flow rate of return; (NPV) = net present value.

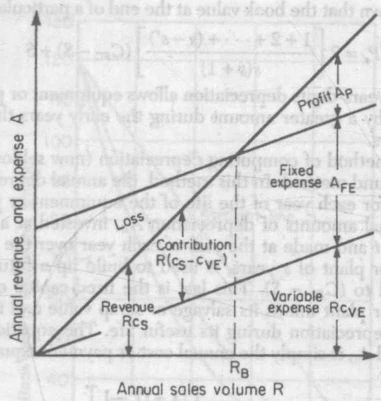

FIG. 9-2 Conventional breakeven chart.

ject to stepwise variation at different levels of production. Some authors consider such steps as included in a semivariable expense, which is less amenable to mathematical analysis than the above division of expenses.

Contribution and Breakeven Charts These can be used to give valuable preliminary information prior to the use of the more sophisticated and time-consuming methods based on discounted cash flow. If the sales price per unit of sales is c_S and the variable expense is c_{VE} per unit of production, Eq. (9-7) can be rewritten as

$$A_{NP} = R(c_S - c_{VE}) - A_{FE} \qquad (9\text{-}12)$$

where $R(c_S - c_{VE})$ is known as the annual contribution. The net annual profit is zero at an annual production rate

$$R_B = A_{FE}/(c_S - c_{VE}) \qquad (9\text{-}13)$$

where R_B is the breakeven production rate.

Breakeven charts can be plotted in any of the three forms shown in Figs. 9-2, 9-3, and 9-4. The abscissa shown as annual sales volume R is also frequently plotted as a percentage of the designed production or sales capacity R_0. In the case of ships, aircraft, etc., it is then called the percentage utilization. The percentage margin of safety is defined as $100(R_0 - R_B)/R_0$.

A decrease in selling price c_S will decrease the slope of the lines in Figs. 9-2, 9-3, and 9-4 and increase the required breakeven value R_B for a given level of fixed expense A_{FE}.

Capital Costs The total capital cost C_{TC} of a project consists of the fixed-capital cost C_{FC} plus the working-capital cost C_{WC}, plus the cost of land and other nondepreciable costs C_L:

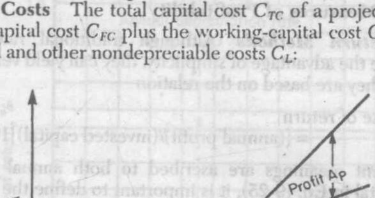

FIG. 9-3 Breakeven chart showing fixed expense as a burden cost.

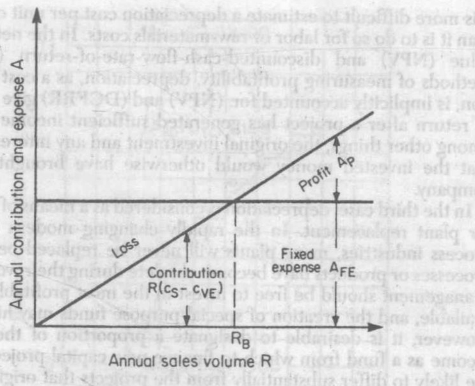

FIG. 9-4 Breakeven chart showing relationship between contribution and fixed expense.

$$C_{TC} = C_{FC} + C_{WC} + C_L \qquad (9\text{-}14)$$

The project may be a complete plant, an addition to an existing plant, or a plant modification.

The working-capital cost of a process or a business normally includes the items shown in Table 9-1. Since working capital is completely recoverable at any time, in theory if not in practice, no tax allowance is made for its depreciation. Changes in working capital arising from varying trade credits or payroll or inventory levels are usually treated as a necessary business expense except when they exceed the tax debt due. If the annual income is negative, additional working capital must be provided and included in the A_{TC} for that year. The value of land and other nondepreciables often increases over the working life of the project. These are therefore not treated in the same way as other capital investments but are shown to have made a (taxable) profit or loss only when the capital is finally recovered.

Working capital may vary from a very small fraction of the total capital cost to almost the whole of the invested capital, depending on the process and the industry. For example, in jewelry-store operations, the fixed capital is very small in comparison with the working capital. On the other hand, in the chemical-process industries, the working capital is likely to be in the region of 10 to 20 percent of the value of the fixed-capital investment.

Depreciation The term "depreciation" is used in a number of different contexts. The most common are:

1. A tax allowance
2. A cost of operation
3. A means of building up a fund to finance plant replacement
4. A measure of falling value

In the first case, the annual taxable income is reduced by an annual depreciation charge or allowance which has the effect of reducing the annual amount of tax payable. The annual depreciation charge is merely a book transaction and does not involve any expenditure of cash. The method of determining the annual depreciation charge must be agreed to by the appropriate tax authority.

In the second case, depreciation is considered to be a manufacturing cost in the same way as labor cost or raw-materials cost. However, it is more difficult to estimate a depreciation cost per unit of product than it is to do so for labor or raw-materials costs. In the net-present-value (NPV) and discounted-cash-flow rate-of-return (DCFRR) methods of measuring profitability, depreciation, as a cost of operation, is implicitly accounted for. NPV and DCFRR are measures of return after a project has generated sufficient income to repay, among other things, the original investment and any interest charges that the investors would otherwise have brought into the company.

In the third case, depreciation is considered as a means of providing for plant replacement. In the rapidly changing modern chemical-process industries, many plants will need to be replaced because the processes or products have become obsolete during their working life. Management should be free to choose the most profitable projects available, and the creation of special-purpose funds may hinder this. However, it is desirable to designate a proportion of the retained income as a fund from which to finance new capital projects. These are likely to differ substantially from the projects that originally generated the funds.

In the fourth case, a plant or a piece of equipment has a finite useful life. The primary reason for the decrease in value is the decrease in future life and the consequent decrease in the number of years for which income will be earned. At the end of its life, the equipment may have some scrap or salvage value.

TABLE 9-1 Working-Capital Costs

Raw materials for plant startup
Raw-materials, intermediate, and finished-product inventories
Cost of handling and transportation of materials to and from stores
Cost of inventory control, warehouse, associated insurance, security arrangements, etc.
Money to carry accounts receivable (i.e., credit extended to customers) less accounts payable (i.e., credit extended by suppliers)
Money to meet payrolls when starting up
Readily available cash for emergencies
Any additional cash required to operate the process or business

it is more difficult to estimate a depreciation cost per unit of product than it is to do so for labor or raw-materials costs. In the net-present-value (NPV) and discounted-cash-flow-rate-of-return (DCFRR) methods of measuring profitability, depreciation, as a cost of operation, is implicitly accounted for. (NPV) and (DCFRR) give measures of return after a project has generated sufficient income to repay, among other things, the original investment and any interest charges that the invested money would otherwise have brought into the company.

In the third case, depreciation is considered as a means of providing for plant replacement. In the rapidly changing modern chemical-process industries, many plants will never be replaced because the processes or products have become obsolete during their working life. Management should be free to invest in the most profitable projects available, and the creation of special-purpose funds may hinder this. However, it is desirable to designate a proportion of the retained income as a fund from which to finance new capital projects. These are likely to differ substantially from the projects that originally generated the income.

In the fourth case, a plant or a piece of equipment has a limited useful life. The primary reason for the decrease in value is the decrease in future life and the consequent decrease in the number of years for which income will be earned. At the end of its life, the equipment may be worth nothing, or it may have a salvage or scrap value S. Thus a fixed-capital cost C_{FC} depreciates in value during its useful life of s years by an amount that is equal to $(C_{FC} - S)$. The useful life is taken from the startup of the plant.

On the basis of straight-line depreciation, the average annual amount of depreciation A_D over a service life of s years is given by

$$A_D = (C_{FC} - S)/s \tag{9-15}$$

The book value after the first year P_1 is given by

$$P_1 = C_{FC} - A_D \tag{9-16}$$

The book value at the end of a specified number of years s' is given by

$$P_{s'} = C_{FC} - s'A_D \tag{9-17}$$

The principal use of a particular depreciation rate is for tax purposes. The permitted annual depreciation is subtracted from the annual income before the latter is taxed. The basis for depreciation in a particular case is a matter of agreement between the taxation authority and the company, in conformity with tax laws.

Other commonly used methods of computing depreciation are the declining-balance method (also known as the fixed-percentage method) and the sum-of-years-digits method.

On the basis of declining-balance (fixed-percentage) depreciation, the book value at the end of the first year is given by

$$P_1 = C_{FC}(1 - r) \tag{9-18}$$

where r is a fraction to be agreed with the taxation authority.

The book value at the end of specified number of years s' is given by

$$P_{s'} = C_{FC}(1 - r)^{s'} \tag{9-19}$$

When the fraction r is chosen to be $2/s$, i.e., twice the reciprocal of the service life s, the method is called the double-declining-balance method.

The declining-balance method of depreciation allows equipment or plant to be depreciated by a greater amount during the earlier years than during the later years. This method does not allow equipment or plant to be depreciated to a zero value at the end of the service life.

On the basis of sum-of-years-digits depreciation, the annual amount of depreciation for a specified number of years s' for a plant of fixed-capital cost C_{FC}, scrap value S, and service life s is given by

$$A_{Ds'} = \left(\frac{s - s' + 1}{1 + 2 + 3 + \cdots + s}\right)(C_{FC} - S) \tag{9-20}$$

Equation (9-20) can also be rewritten in the form

$$A_{Ds'} = \left[\frac{2(s - s' + 1)}{s(s + 1)}\right](C_{FC} - S) \tag{9-21}$$

It can be shown that the book value at the end of a particular year s' is

$$P_{s'} = 2\left[\frac{1 + 2 + \cdots + (s - s')}{s(s + 1)}\right](C_{FC} - S) + S \tag{9-22}$$

The sum-of-years-digits depreciation allows equipment or plant to be depreciated by a greater amount during the early years than during the later years.

A fourth method of computing depreciation (now seldom used) is the sinking-fund method. In this method, the annual depreciation A_D is the same for each year of the life of the equipment or plant. The series of equal amounts of depreciation A_D, invested at a fractional interest rate i and made at the end of each year over the life of the equipment or plant of s years, is used to build up a future sum of money equal to $(C_{FC} - S)$. This last is the fixed-capital cost of the equipment or plant minus its salvage or scrap value and is the total amount of depreciation during its useful life. The equation relating $(C_{FC} - S)$ and A_D is simply the annual cost or payment equation, written either as

$$C_{FC} - S = A_D\left[\frac{(1 + i)^s - 1}{i}\right] \tag{9-23}$$

or

$$C_{FC} - S = \frac{A_D}{f_{AF}} \tag{9-24}$$

where f_{AF} is the annuity future-worth factor given by

$$f_{AF} = i/[(1 + i)^s - 1]$$

In the sinking-fund method of depreciation, the effect of interest is to make the annual decrease of the book value of the equipment or plant less in the early than in the later years with consequent higher tax due in the earlier years when recovery of the capital is most important.

It is preferable not to think of annual depreciation as a contribution to a fund to replace equipment at the end of its life but as part of the difference between the revenue and the expenditure, which difference is tax-free.

Some of the preceding methods of computing depreciation are not allowed by taxation authorities in certain countries. When calculating depreciation, it is necessary to obtain details of the methods and rates permitted by the appropriate authority and to use the information provided.

Figure 9-5 shows the fall in book value with time for a piece of equipment having a fixed-capital cost of \$120,000, a useful life of 10 years, and a scrap value of \$20,000. This fall in value is calculated by using (1) straight-line depreciation, (2) double-declining depreciation, and (3) sum-of-years-digits depreciation.

Traditional Measures of Profitability

Rate-of-Return Methods Although traditional rate-of-return methods have the advantage of simplicity, they can yield very misleading results. They are based on the relation

$$\text{Percent rate of return} = [(\text{annual profit})/(\text{invested capital})]100 \tag{9-25}$$

Since different meanings are ascribed to both annual profit and invested capital in Eq. (9-25), it is important to define the terms precisely. The invested capital may refer to the original total capital investment, the depreciated investment, the average investment, the current value of the investment, or something else. The annual profit may refer to the net annual profit before tax A_{NP}, the net annual profit after tax A_{NNP}, the annual cash income before tax A_{CI}, or the annual cash income after tax A_{NCI}.

The fractional interest rate of return based on the net annual profit after tax and the original investment is

$$i = A_{NNP}/C_{TC} \tag{9-26}$$

which can be written in terms of Eq. (9-9) as

$$i = (A_{NCI}/C_{TC}) - (A_{BD}/C_{TC}) \tag{9-27}$$

where A_{BD} is the balance-sheet annual depreciation. The main disadvantage of using Eq. (9-27) is that the fractional depreciation rate

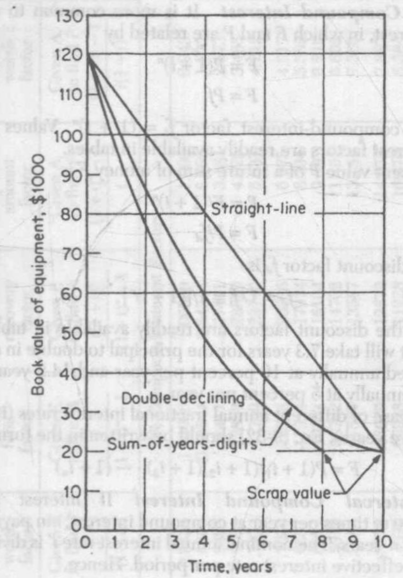

FIG. 9-5 Book value against time for various depreciation methods.

A_{BD}/C_{TC} is arbitrarily assessed. Its value will affect the fractional rate of return considerably and may lead to erroneous conclusions when making comparisons between different companies. This is particularly true when making international comparisons.

Figures 9-6, 9-7, and 9-8 show the effect of the depreciation method on profit for a project described by the following data:

Net annual cash income after tax $A_{NCI} = \$25{,}500$ in each of 10 years

Fixed-capital cost $C_{FC} = \$120{,}000$

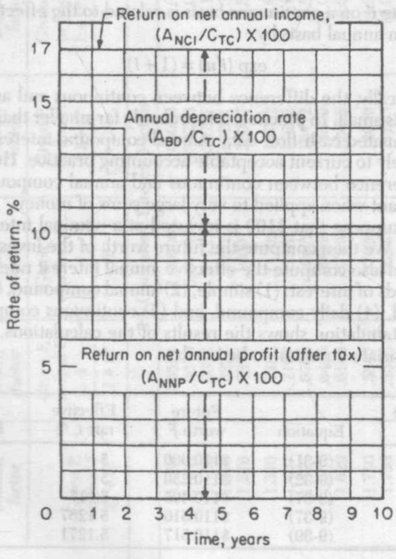

FIG. 9-6 Effect of straight-line depreciation on rate of return for a project. A_{BD} = annual depreciation allowance; A_{NCI} = annual net cash income after tax; A_{NNP} = annual net profit after payment of tax; C_{TC} = total capital cost.

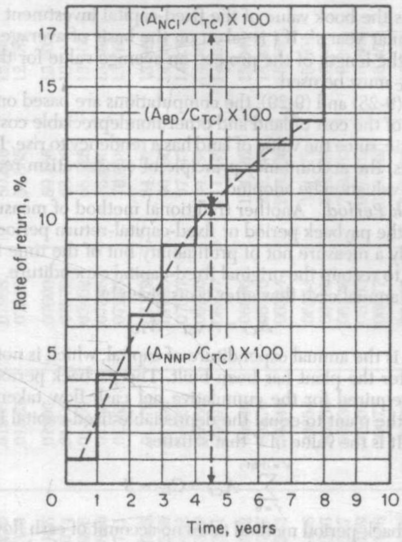

FIG. 9-7 Effect of double-declining depreciation on rate of return for a project.

Estimated salvage value of plant items $S = \$20{,}000$

Working capital $C_{WC} = \$10{,}000$

Cost of land $C_L = \$20{,}000$

In Eq. (9-27), i can be taken either on the basis of the net annual cash income for a particular year or on the basis of an average net annual cash income over the length of the life of the project. The equations corresponding to Eq. (9-26) based on depreciated and average investment are given respectively as follows:

$$i = A_{NNP}/(P_x + C_{WC} + C_L) \qquad (9\text{-}28)$$

and

$$i = 2A_{NNP}/(C_{FC} + S + 2C_{WC} + 2C_L) \qquad (9\text{-}29)$$

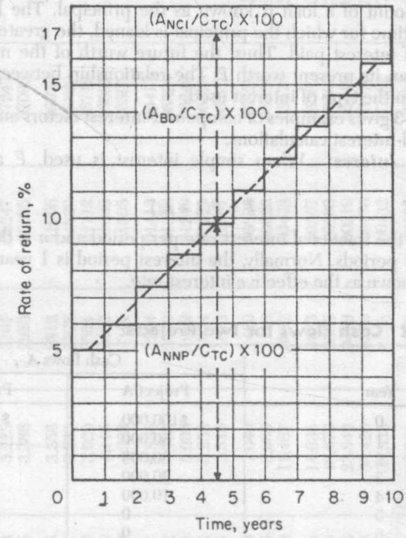

FIG. 9-8 Effect of sum-of-years-digits depreciation on rate of return for a project.

where $P_{s'}$ is the book value of the fixed-capital investment at the end of a particular year s'. If i is taken on the basis of average values for A_{NNP} over the length of the project, an average value for the working capital C_{WC} must be used.

In Eqs. (9-28) and (9-29), the computations are based on unchanging values of the cost of land and other nondepreciable costs C_L. This is unrealistic, since the value of land has a tendency to rise. In such circumstances, the accountancy principle of conservatism requires that the lowest valuation be adopted.

Payback Period Another traditional method of measuring profitability is the payback period or fixed-capital-return period. Actually, this is really a measure not of profitability but of the time it takes for cash flows to recoup the original fixed-capital expenditure.

The net annual cash flow after tax is given by

$$A_{CF} = A_{NCI} - A_{TC} \tag{9-4}$$

where A_{TC} is the annual expenditure of capital, which is not necessarily zero after the plant has been built. The payback period (PBP) is the time required for the cumulative net cash flow taken from the startup of the plant to equal the depreciable fixed-capital investment $(C_{FC} - S)$. It is the value of s' that satisfies

$$\sum_{s'=0}^{s'=(\text{PBP})} A_{CF} = C_{FC} - S \tag{9-30}$$

The payback-period method takes no account of cash flows or profits received after the breakeven point has been reached. The method is based on the premise that the earlier the fixed capital is recovered, the better the project. However, this approach can be misleading.

Let us consider projects A and B, having net annual cash flows as listed in Table 9-2. Both projects have initial fixed-capital expenditures of \$100,000. On the basis of payback period, project A is the more desirable since the fixed-capital expenditure is recovered in 3 years, compared with 5 years for project B. However, project B runs for 7 years with a cumulative net cash flow of \$110,000. This is obviously more profitable than project A, which runs for only 4 years with a cumulative net cash flow of only \$10,000.

Time Value of Money A large part of business activity is based on money that can be loaned or borrowed. When money is loaned, there is always a risk that it may not be returned. A sum of money called interest is the inducement offered to make the risk acceptable. When money is borrowed, interest is paid for the use of the money over a period of time. Conversely, when money is loaned, interest is received.

The amount of a loan is known as the principal. The longer the period of time for which the principal is loaned, the greater the total amount of interest paid. Thus, the future worth of the money F is greater than its present worth P. The relationship between F and P depends on the type of interest used.

Table 9-3 gives examples of compound-interest factors and example compound-interest calculations.

Simple Interest When simple interest is used, F and P are related by

$$F = P(1 + ni) \tag{9-31}$$

where i is the fractional interest rate per period and n is the number of interest periods. Normally, the interest period is 1 year, in which case i is known as the effective interest rate.

TABLE 9-2 Cash Flows for Two Projects

Year	Cash flows A_{CF} Project A	Project B
0	\$100,000	\$100,000
1	50,000	0
2	30,000	10,000
3	20,000	20,000
4	10,000	30,000
5	0	40,000
6	0	50,000
7	0	60,000
$\sum A_{CF}$	\$ 10,000	\$110,000
Payback period (PBP)	3 years	5 years

Annual Compound Interest It is more common to use compound interest, in which F and P are related by

$$F = P(1 + i)^n \tag{9-32}$$

or

$$F = Pf_i \tag{9-33}$$

where the compound-interest factor $f_i = (1 + i)^n$. Values for compound-interest factors are readily available in tables.

The present value P of a future sum of money F is

$$P = F/(1 + i)^n \tag{9-34}$$

or

$$F = P/f_d \tag{9-35}$$

where the discount factor f_d is

$$f_d = 1/f_i = 1/[(1 + i)^n]$$

Values for the discount factors are readily available in tables which show that it will take 7.3 years for the principal to double in amount if compounded annually at 10 percent per year and 14.2 years if compounded annually at 5 percent per year.

For the case of different annual fractional interest rates $(i_1, i_2, \ldots, i_n$ in successive years), Eq. (9-32) should be written in the form

$$F = P(1 + i_1)(1 + i_2)(1 + i_3) \cdots (1 + i_n) \tag{9-36}$$

Short-Interval Compound Interest If interest payments become due m times per year at compound interest, mn payments are required in n years. The nominal annual interest rate i' is divided by m to give the effective interest rate per period. Hence,

$$F = P[1 + (i'/m)]^{mn} \tag{9-37}$$

It follows that the effective annual interest i is given by

$$i = [1 + (i'/m)]^m - 1 \tag{9-38}$$

The annual interest rate equivalent to a compound-interest rate of 5 percent per month (i.e., $i'/m = 0.05$) is calculated from Eq. (9-38) to be

$$i = (1 + 0.05)^{12} - 1 = 0.796, \text{ or } 79.6 \text{ percent/year}$$

Continuous Compound Interest As m approaches infinity, the time interval between payments becomes infinitesimally small, and in the limit Eq. (9-37) reduces to

$$F = P \exp(i'n) \tag{9-39}$$

A comparison of Eqs. (9-32) and (9-39) shows that the nominal interest rate i' on a continuous basis is related to the effective interest rate i on an annual basis by

$$\exp(i'n) = (1 + i)^n \tag{9-40}$$

Numerically, the difference between continuous and annual compounding is small. In practice, it is probably far smaller than the errors in the estimated cash-flow data. Annual compound interest conforms more closely to current acceptable accounting practice. However, the small difference between continuous and annual compounding may be significant when applied to very large sums of money.

Let us suppose that \$100 is invested at a nominal interest rate of 5 percent. We then compute the future worth of the investment after 2 years and also compute the effective annual interest rate for the following kinds of interest: (1) simple, (2) annual compound, (3) monthly compound, (4) daily compound, and (5) continuous compound. The following tabulation shows the results of the calculations, along with the appropriate equation to be used:

Interest type	Equation	Future worth F	Effective rate i, %	Equation
1	(9-31)	\$110.000	5	(9-31)
2	(9-32)	\$110.250	5	(9-38)
3	(9-37)	\$110.495	5.117	(9-38)
4	(9-37)	\$110.516	5.1267	(9-38)
5	(9-39)	\$110.517	5.1271	(9-38)

When computing the effective annual rate for continuous compounding, the first term of Eq. (9-38), $[1 + (i'/m)]^m$, approaches $e^{i'}$ as m approaches infinity.

TABLE 9-3 Compound Interest Factors*

(For examples demonstrating use see end of table.)

	Single payment		Uniform annual series				Single payment		Uniform annual series				
	Compound-amount factor	Present-worth factor	Sinking-fund factor	Capital-recovery factor	Compound-amount factor	Present-worth factor	Compound-amount factor	Present-worth factor	Sinking-fund factor	Capital-recovery factor	Compound-amount factor	Present-worth factor	
n	Given P, to find F $(1+i)^n$	Given F, to find P $\frac{1}{(1+i)^n}$	Given F, to find A $\frac{i}{(1+i)^n-1}$	Given P, to find A $\frac{i(1+i)^n}{(1+i)^n-1}$	Given A, to find F $\frac{(1+i)^n-1}{i}$	Given A, to find P $\frac{(1+i)^n-1}{i(1+i)^n}$	Given P, to find F $(1+i)^n$	Given F, to find P $\frac{1}{(1+i)^n}$	Given F, to find A $\frac{i}{(1+i)^n-1}$	Given P, to find A $\frac{i(1+i)^n}{(1+i)^n-1}$	Given A, to find F $\frac{(1+i)^n-1}{i}$	Given A, to find P $\frac{(1+i)^n-1}{i(1+i)^n}$	n
	5% Compound Interest Factors						6% Compound Interest Factors						
1	1.050	0.9524	1.00000	1.05000	1.000	0.952	1.060	0.9434	1.00000	1.06000	1.000	0.943	1
2	1.103	.9070	0.48780	0.53780	2.050	1.859	1.124	.8900	0.48544	0.54544	2.060	1.833	2
3	1.158	.8638	.31721	.36721	3.153	2.723	1.191	.8396	.31411	.37411	3.184	2.673	3
4	1.216	.8227	.23201	.28201	4.310	3.546	1.262	.7921	.22859	.28859	4.375	3.465	4
5	1.276	.7835	.18097	.23097	5.526	4.329	1.338	.7473	.17740	.23740	5.637	4.212	5
6	1.340	.7462	.14702	.19702	6.802	5.076	1.419	.7050	.14336	.20336	6.975	4.917	6
7	1.407	.7107	.12282	.17282	8.142	5.786	1.504	.6651	.11914	.17914	8.394	5.582	7
8	1.477	.6768	.10472	.15472	9.549	6.463	1.594	.6274	.10104	.16104	9.897	6.210	8
9	1.551	.6446	.09069	.14069	11.027	7.108	1.689	.5919	.08702	.14702	11.491	6.802	9
10	1.629	.6139	.07940	.12950	12.578	7.722	1.791	.5584	.07587	.13587	13.181	7.360	10
11	1.710	.5847	.07039	.12039	14.207	8.306	1.898	.5268	.06679	.12679	14.972	7.887	11
12	1.796	.5568	.06283	.11283	15.917	8.863	2.012	.4970	.05928	.11928	16.870	8.384	12
13	1.886	.5303	.05646	.10646	17.713	9.394	2.133	.4688	.05296	.11296	18.882	8.853	13
14	1.980	.5051	.05102	.10102	19.599	9.899	2.261	.4423	.04758	.10758	21.015	9.295	14
15	2.079	.4810	.04634	.09634	21.579	10.380	2.397	.4173	.04296	.10296	23.276	9.712	15
16	2.183	.4581	.04227	.09227	23.657	10.838	2.540	.3936	.03895	.09895	25.673	10.106	16
17	2.292	.4363	.03870	.08870	25.840	11.274	2.693	.3714	.03544	.09544	28.213	10.477	17
18	2.407	.4155	.03555	.08555	28.132	11.690	2.854	.3503	.03236	.09236	30.906	10.828	18
19	2.527	.3957	.03275	.08275	30.539	12.085	3.026	.3305	.02962	.08962	33.760	11.158	19
20	2.653	.3769	.03024	.08024	33.066	12.462	3.207	.3118	.02718	.08718	36.786	11.470	20
21	2.786	.3589	.02800	.07800	35.719	12.821	3.400	.2942	.02500	.08500	39.993	11.764	21
22	2.925	.3418	.02597	.07597	38.505	13.163	3.604	.2775	.02305	.08305	43.392	12.042	22
23	3.072	.3256	.02414	.07414	41.430	13.489	3.820	.2618	.02128	.08128	46.996	12.303	23
24	3.225	.3101	.02247	.07247	44.502	13.799	4.049	.2470	.01968	.07968	50.816	12.550	24
25	3.386	.2953	.02095	.07095	47.727	14.094	4.292	.2330	.01823	.07823	54.865	12.783	25
26	3.556	.2812	.01956	.06956	51.113	14.375	4.549	.2198	.01690	.07690	59.156	13.003	26
27	3.733	.2678	.01829	.06829	54.669	14.643	4.822	.2074	.01570	.07570	63.706	13.211	27
28	3.920	.2551	.01712	.06712	58.403	14.898	5.112	.1956	.01459	.07459	68.528	13.406	28
29	4.116	.2429	.01605	.06605	62.323	15.141	5.418	.1846	.01358	.07358	73.640	13.591	29
30	4.322	.2314	.01505	.06505	66.489	15.372	5.743	.1741	.01265	.07265	79.058	13.765	30
31	4.538	.2204	.01413	.06413	70.761	15.593	6.088	.1643	.01179	.07179	84.802	13.929	31
32	4.765	.2099	.01328	.06328	75.299	15.803	6.453	.1550	.01100	.07100	90.890	14.084	32
33	5.003	.1999	.01249	.06249	80.064	16.003	6.841	.1462	.01027	.07027	97.343	14.230	33
34	5.253	.1904	.01176	.06176	85.067	16.193	7.251	.1379	.00960	.06960	104.184	14.368	34
35	5.516	.1813	.01107	.06107	90.320	16.374	7.686	.1301	.00897	.06897	111.435	14.498	35
40	7.040	.1420	.00828	.05828	120.800	17.159	10.286	.0972	.00646	.06646	154.762	15.046	40
45	8.985	.1113	.00626	.05626	159.700	17.774	13.765	.0727	.00470	.06470	212.744	15.456	45
50	11.467	.0872	.00478	.05478	209.348	18.256	18.420	.0543	.00344	.06344	290.336	15.762	50
55	14.636	.0683	.00367	.05367	272.713	18.633	24.650	.0406	.00254	.06254	394.172	15.991	55
60	18.679	.0535	.00283	.05283	353.584	18.929	32.988	.0303	.00188	.06188	533.128	16.161	60
65	23.840	.0419	.00219	.05219	456.798	19.161	44.145	.0227	.00139	.06139	719.083	16.289	65
70	30.426	.0329	.00170	.05170	588.529	19.343	59.076	.0169	.00103	.06103	967.932	16.385	70
75	38.833	.0258	.00132	.05132	756.654	19.485	79.057	.0126	.00077	.06077	1,300.949	16.456	75
80	49.561	.0202	.00103	.05103	971.229	19.596	105.796	.0095	.00057	.06057	1,746.600	16.509	80
85	63.254	.0158	.00080	.05080	1,245.087	19.684	141.579	.0071	.00043	.06043	2,342.982	16.549	85
90	80.730	.0124	.00063	.05063	1,594.607	19.752	189.465	.0053	.00032	.06032	3,141.075	16.579	90
95	103.035	.0097	.00049		2,040.694	19.806	253.546	.0039	.00024	.06024	4,209.104	16.601	95
100	131.501	.0076	.00038	.05038	2,610.025	19.848	339.302	.0029	.00018	.06018	5,638.368	16.618	100

TABLE 9-3 Compound Interest Factors *(Concluded)*

Examples of Use of Table and Factors

Given: \$2500 is invested now at 5 percent.
Required: Accumulated value in 10 years (i.e., the amount of a given principal).

Solution:

$$F = P(1+i)^n = \$2500 \times 1.05^{10}$$

$$\text{Compound-amount factor} = (1+i)^n = 1.05^{10} = 1.629$$

$$F = \$2500 \times 1.629 = \$4062.50$$

Given: \$19,500 will be required in 5 years to replace equipment now in use.
Required: With interest available at 3 percent, what sum must be deposited in the bank at present to provide the required capital (i.e., the principal which will amount to a given sum)?

Solution:

$$P = F\frac{1}{(1+i)^n} = \$19{,}500\frac{1}{1.03^5}$$

$$\text{Present-worth factor} = 1/(1+i)^n = 1/1.03^5 = 0.8626$$

$$P = \$19{,}500 \times 0.8626 = \$16{,}821$$

Given: \$50,000 will be required in 10 years to purchase equipment.
Required: With interest available at 4 percent, what sum must be deposited each year to provide the required capital (i.e., the annuity which will amount to a given fund)?

Solution:

$$A = F\frac{i}{(1+i)^n - 1} = \$50{,}000\frac{0.04}{1.04^{10} - 1}$$

$$\text{Sinking-fund factor} = \frac{i}{(1+i)^n - 1} = \frac{0.04}{1.04^{10} - 1} = 0.08329$$

$$A = \$50{,}000 \times 0.08329 = \$4{,}164$$

Given: \$20,000 is invested at 10 percent interest.
Required: Annual sum that can be withdrawn over a 20-year period (i.e., the annuity provided by a given capital).

Solution:

$$A = P\frac{i(1+i)^n}{(1+i)^n - 1} = \$20{,}000\frac{0.10 \times 1.10^{20}}{1.10^{20} - 1}$$

$$\text{Capital-recovery factor} = \frac{i(1+i)^n}{(1+i)^n - 1} = \frac{0.10 \times 1.10^{20}}{1.10^{20} - 1} = 0.11746$$

$$A = \$20{,}000 \times 0.11746 = \$2349.20$$

Given: \$500 is invested each year at 8 percent interest.
Required: Accumulated value in 15 years (i.e., amount of an annuity).

Solution:

$$F = A\frac{(1+i)^n - 1}{i} = \$500\frac{1.08^{15} - 1}{0.08}$$

$$\text{Compound-amount factor} = \frac{(1-i)^n - 1}{i} = \frac{1.08^{15} - 1}{0.08} = 27.152$$

$$F = \$500 \times 27.152 = \$13{,}576$$

Given: \$8000 is required annually for 25 years.
Required: Sum that must be deposited now at 6 percent interest.

Solution:

$$P = A\frac{(1+i)^n - 1}{i(1+i)^n} = \$8000\frac{1.06^{25} - 1}{0.06 \times 1.06^{25}}$$

$$\text{Present-worth factor} = \frac{(1+i)^n - 1}{i(1+i)^n} = \frac{1.06^{25} - 1}{0.06 \times 1.06^{25}} = 12.783$$

$$P = \$8000 \times 12.783 = \$102{,}264$$

*Factors presented for two interest rates only. By using the appropriate formulas, values for other interest rates may be calculated.

Annual Cost or Payment A series of equal annual payments A invested at a fractional interest rate i at the end of each year over a period of n years may be used to build up a future sum of money F. These relations are given by

$$F = A\left[\frac{(1+i)^n - 1}{i}\right] \tag{9-41}$$

or

$$F = A/f_{AF} \tag{9-42}$$

where the annuity future-worth factor is

$$f_{AF} = i/[(1+i)^n - 1]$$

Values for f_{AF} are readily available in tables.

Equation (9-41) can be combined with Eq. (9-34) to yield

$$P = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] \tag{9-43}$$

$$P = A/f_{AP} \tag{9-44}$$

where P is the present worth of the series of future equal annual payments A and the annuity present-worth factor is

$$f_{AP} = [i(1+i)^n]/[(1+i)^n - 1]$$

Values for f_{AP} are also available in tables.

Alternatively, the annual payment A required to build up a future sum of money F with a present value of P is given by

$$A = Ff_{AF} \tag{9-45}$$

$$A = Pf_{AP} \tag{9-46}$$

Equation (9-41) represents the future sum of a series of uniform annual payments that are invested at a stated interest rate over a period of years. This procedure defines an ordinary annuity. Other forms of annuities include the annuity due, in which payments are made at the beginning of the year instead of at the end; and the deferred annuity, in which the first payment is deferred for a definite number of years.

Capitalized Cost A piece of equipment of fixed-capital cost C_{FC} will have a finite life of n years. The capitalized cost of the equipment C_K is defined by

$$(C_K - C_{FC})(1+i)^n = C_K - S \tag{9-47}$$

C_K is in excess of C_{FC} by an amount which, when compounded at an annual interest rate i for n years, will have a future worth of C_K less the salvage or scrap value S. If the renewal cost of the equipment remains constant at $(C_{FC} - S)$ and the interest rate remains constant at i, then C_K is the amount of capital required to replace the equipment in perpetuity.

Equation (9-47) may be rewritten as

$$C_K = \left[C_{FC} - \frac{S}{(1+i)^n}\right]\left[\frac{(1+i)^n}{(1+i)^n - 1}\right] \tag{9-48}$$

or

$$C_K = (C_{FC} - Sf_d)f_k \tag{9-49}$$

where f_d is the discount factor and f_k, the capitalized-cost factor, is

$$f_k = [(1+i)^n]/[(1+i)^n - 1]$$

Values for each factor are available in tables.

Example 1: Capitalized Cost of Equipment A piece of equipment has been installed at a cost of \$100,000 and is expected to have a working life of 10 years with a scrap value of \$20,000. Let us calculate the capitalized cost of the equipment based on an annual compound-interest rate of 5 percent.

Therefore, we substitute values into Eq. (9-48) to give

$$C_K = \left[\$100{,}000 - \frac{\$20{,}000}{(1+0.05)^{10}}\right]\left[\frac{(1+0.05)^{10}}{(1+0.05)^{10} - 1}\right]$$

$$C_K = [\$100{,}000 - (\$20{,}000/1.62889)](2.59009)$$

$$C_K = \$227{,}207$$

Modern Measures of Profitability An investment in a manufacturing process must earn more than the cost of capital for it to be worthwhile. The larger the additional earnings, the more profitable the venture and the greater the justification for putting the capital at risk. A profitability estimate is an attempt to quantify the desirability of taking this risk.

The ways of assessing profitability to be considered in this section are (1) discounted-cash-flow rate of return (DCFRR), (2) net present value (NPV) based on a particular discount rate, (3) equivalent maximum investment period (EMIP), (4) interest-recovery period (IRP), and (5) discounted breakeven point (DBEP).

Cash Flow Let us consider a project in which C_{FC} = \$1,000,000, C_{WC} = \$90,000, and C_L = \$10,000. Hence, C_{TC} = \$1,100,000 from Eq. (9-14). If all this capital expenditure occurs in Year 0 of the project, then A_{TC} = \$1,100,000 in Year 0 and $-A_{TC}$ = −\$1,100,000. From Eq. (9-4), it is seen that any capital expenditure makes a negative contribution to the net annual cash flow A_{CF}.

Let us consider another project in which the fixed-capital expenditure is spread over 2 years, according to the following pattern:

$$C_{FC} = C_{FC0} + C_{FC1}$$

Year 0	Year 1
C_{FC0} = \$400,000	C_{FC1} = \$600,000
C_L = 10,000	C_{WC} = 90,000
A_{TC} = 410,000	A_{TC} = 690,000

In the final year of the project, the working capital and the land are recovered, which in this case cost a total of \$100,000. Thus, in the final year of the project, A_{TC} = −\$100,000 and $-A_{TC}$ = +\$100,000. From Eq. (9-4), it is seen that any capital recovery makes a positive contribution to the net annual cash flow.

During the development and construction stages of a project, A_{CI} and A_{IT} are both zero in Eqs. (9-2) and (9-4). For this period, the cash flow for the project is negative and is given by

$$A_{CF} = -A_{TC} \tag{9-50}$$

Figure 9-9 shows the cash-flow stages in a project. The expenditure during the research and development stage is normally relatively small. It will usually include some preliminary process design and a market survey. Once the decision to go ahead with the project has been taken, detailed process-engineering design will commence, and the rate of expenditure starts to increase. The rate is increased still further when equipment is purchased and construction gets under way. There is no return on the investment until the plant is started up. Even during startup, there is some additional expenditure. Once the plant is operating smoothly, an inflow of cash is established. During the early stages of a project, there may be a tax credit because of the existence of expenses without corresponding income.

Discounted Cash Flow The present value P of a future sum of money F is given by

$$P = Ff_d \tag{9-51}$$

where $f_d = 1/(1+i)^n$, the discount factor. Values for this factor are readily available in tables. For example, \$90,909 invested at an annual interest rate of 10 percent becomes \$100,000 after 1 year. Similarly, \$38,554 invested at 10 percent becomes \$100,000 after 10 years.

Thus, cash flow in the early years of a project has a greater value than the same amount in the later years of a project. Therefore, it pays to receive money as soon as possible and to delay paying out money for as long as possible.

Time is taken into account by using the annual discounted cash flow A_{DCF}, which is related to the annual cash flow A_{CF} and the discount factor f_d by

$$A_{DCF} = A_{CF}f_d \tag{9-52}$$

Thus, at the end of any year n,

$$(A_{DCF})_n = (A_{CF})_n/(1+i)^n$$

The sum of the annual discounted cash flows over n years, $\sum A_{DCF}$, is known as the net present value (NPV) of the project:

$$(\text{NPV}) = \sum_0^n (A_{DCF})_n \tag{9-53}$$

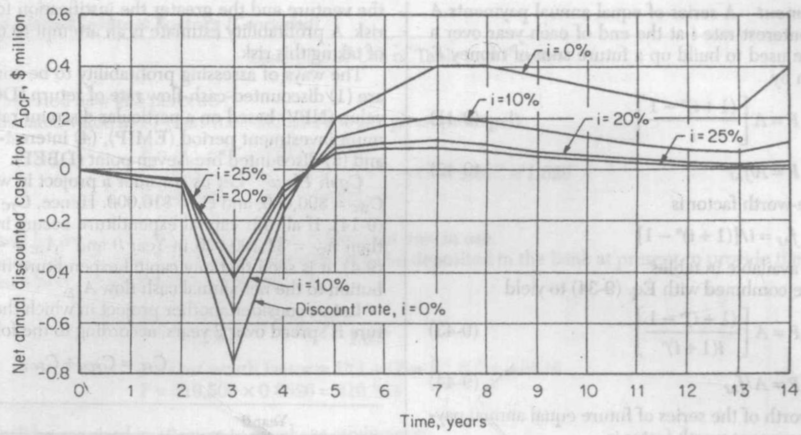

FIG. 9-9 Effect of discount rate on cash flows.

The value of (NPV) is directly dependent on the choice of the fractional interest rate i. An interest rate can be selected to make (NPV) = 0 after a chosen number of years. This value of i is found from

$$\sum_{0}^{n} (A_{DCF})_n = \frac{(A_{CF})_0}{(1+i)^0} + \frac{(A_{CF})_1}{(1+i)^1} + \cdots + \frac{(A_{CF})_n}{(1+i)^n} = 0 \qquad (9\text{-}54)$$

Equation (9-54) may be solved for i either graphically or by an iterative trial-and-error procedure. The value of i given by Eq. (9-54) is known as the discounted-cash-flow rate of return (DCFRR). It is also known as the profitability index, true rate of return, investor's rate of return, and interest rate of return.

Cash-Flow Curves Figure 9-9 shows the cash-flow stages in a project together with their discounted-cash-flow values for the data given in Table 9-4. In addition to cash-flow and discounted-cash-flow curves, it is also instructive to plot cumulative-cash-flow and cumulative-discounted-cash-flow curves. These are shown in Fig. 9-10 for the data in Table 9-4.

The cost of capital may also be considered as the interest rate at which money can be invested instead of putting it at risk in a manufacturing process. Let us consider the process data listed in Table 9-4 and plotted in Fig. 9-10. If the cost of capital is 10 percent, then the appropriate discounted-cash-flow curve in Fig. 9-10 is *abcdef*. Up to point e, or 8.49 years, the capital is at risk. Point e is the discounted breakeven point (DBEP). At this point, the manufacturing process has paid back its capital and produced the same return as an equivalent amount of capital invested at a compound-interest rate of 10 percent. Beyond the breakeven point, the capital is no longer at risk and any cash flow above the horizontal baseline, $\sum A_{DCF} = 0$, is in excess of the return on an equivalent amount of capital invested at a compound-interest rate of 10 percent. Thus, the greater the area above the baseline, the more profitable the process.

When (NPV) and (DCFRR) are computed, depreciation is not considered as a separate expense. It is simply used as a permitted writing-down allowance to reduce the annual amount of tax in accordance with the rules applying in the country of earning. The tax payable is deducted in accordance with Eq. (9-2) in the year in which it is paid, which may differ from the year in which the corresponding income was earned.

A (DCFRR) of, say, 15 percent implies that 15 percent per year will be earned on the investment, in addition to which the project generates sufficient money to repay the original investment plus any interest payable on borrowed capital plus all taxes and expenses.

It is not normally possible to make a comprehensive assessment of profitability with a single number. The shape of the cumulative-cash-flow and cumulative-discounted-cash-flow curves both before and after the breakeven point is an important factor.

D. H. Allen [*Chem. Eng.*, **74**, 75–78 (July 3, 1967)] accounted for the shape of the cumulative-undiscounted-cash-flow curve up to the

TABLE 9-4 Annual Cash Flows and Discounted Cash Flows for a Project

			Discounted at 10%			Discounted at 20%			Discounted at 25%		
Year	A_{CF}, $	$\sum A_{CF}$, $	f_d	A_{DCF}, $	$\sum A_{DCF}$, $	f_d	A_{DCF}, $	$\sum A_{DCF}$, $	f_d	A_{DCF}, $	$\sum A_{DCF}$, $
0	−10,000	−10,000	1.00000	−10,000	−10,000	1.00000	−10,000	−10,000	1.00000	−10,000	−10,000
1	−30,000	−40,000	0.90909	−27,273	−37,273	0.83333	−25,000	−35,000	0.80000	−24,000	−34,000
2	−60,000	−100,000	0.82645	−49,587	−86,860	0.69444	−41,666	−76,666	0.64000	−38,400	−72,400
3	−750,000	−850,000	0.75131	−563,483	−650,343	0.57870	−434,025	−510,691	0.51200	−384,000	−456,400
4	−150,000	−1,000,000	0.68301	−102,452	−752,795	0.48225	−72,338	−583,029	0.40960	−61,440	−517,840
5	+200,000	−800,000	0.62092	+124,184	−628,611	0.40188	+80,376	−502,653	0.32768	+65,536	−452,304
6	+300,000	−500,000	0.56447	+169,341	−459,270	0.33490	+100,470	−402,183	0.26214	+78,642	−373,662
7	+400,000	−100,000	0.51316	+205,264	−254,006	0.27908	+111,632	−290,551	0.20972	+83,888	−289,774
8	+400,000	+300,000	0.46651	+186,604	−67,402	0.23257	+93,028	−197,523	0.16777	+67,108	−222,666
9	+360,000	+660,000	0.42410	+152,676	+85,274	0.19381	+69,772	−127,751	0.13422	+48,319	−174,347
10	+320,000	+980,000	0.38554	+123,373	+208,647	0.16151	+51,683	−76,068	0.10737	+34,358	−139,989
11	+280,000	+1,260,000	0.35049	+98,137	+306,784	0.13459	+37,685	−38,383	0.08590	+24,052	−115,937
12	+240,000	+1,500,000	0.31863	+76,471	+383,255	0.11216	+26,918	−11,465	0.06872	+16,493	−99,444
13	+240,000	+1,740,000	0.28966	+69,518	+452,773	0.09346	+22,430	+10,965	0.05498	+13,195	−86,249
14	+400,000	+2,140,000	0.26333	+105,332	+558,105	0.07789	+31,156	+42,121	0.04398	+17,592	−68,657

NOTE: A_{CF} is net annual cash flow, A_{DCF} is net annual discounted cash flow, f_d is discount factor at stated interest. $\sum A_{CF}$ is cumulative cash flow, and $\sum A_{DCF}$ is cumulative discounted cash flow.

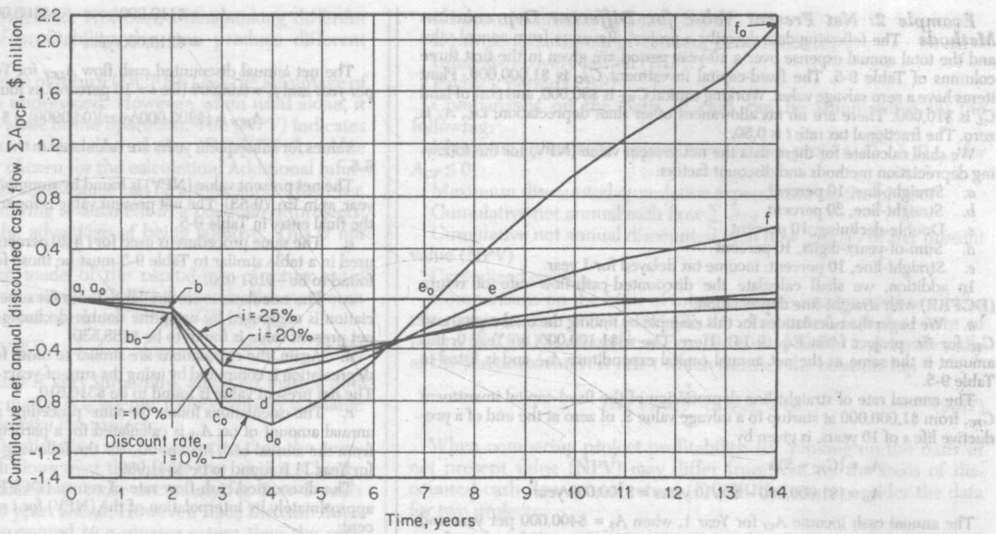

FIG. 9-10 Effect of discount rate on cumulative cash flows.

breakeven point e_0 in Fig. 9-10 by using a parameter known as the equivalent maximum investment period (EMIP), which is defined as

$$(\text{EMIP}) = \frac{\text{area } (a_0 \text{ to } e_0)}{(\sum A_{CF})_{max}} \quad \text{for} \quad A_{CF} \leq 0 \qquad (9\text{-}55)$$

where the area (a_0 to e_0) refers to the area below the horizontal baseline ($\sum A_{CF} = 0$) on the cumulative-cash-flow curve in Fig. 9-10. The sum $(\sum A_{CF})_{max}$ is the maximum cumulative expenditure on the project, which is given by point d_0 in Fig. 9-10. (EMIP) is a time in years. It is the equivalent period during which the total project debt would be outstanding if it were all incurred at one instant and all repaid at one instant. Clearly, the shorter the (EMIP), the more attractive the project.

Allen accounted for the shape of the cumulative-cash-flow curve beyond the breakeven point by using a parameter known as the interest-recovery period (IRP). This is the time period (illustrated in Fig. 9-11) that makes the area (e_0 to f_0) above the horizontal baseline equal to the area (a_0 to e_0) below the horizontal baseline on the cumulative-cash-flow curve.

C. G. Sinclair [*Chem. Process. Eng.*, **47**, 147 (1966)] has considered similar parameters to the (EMIP) and (IRP) based on a cumulative-discounted-cash-flow curve.

Consideration of the cash-flow stages in Fig. 9-10 shows the factors that can affect the (EMIP) and (IRP). If the required capital investment is increased, it is necessary to increase the rate of income after startup for the (EMIP) to remain the same. In order to have the (EMIP) small, it is necessary to keep the research and development, design, and construction stages short.

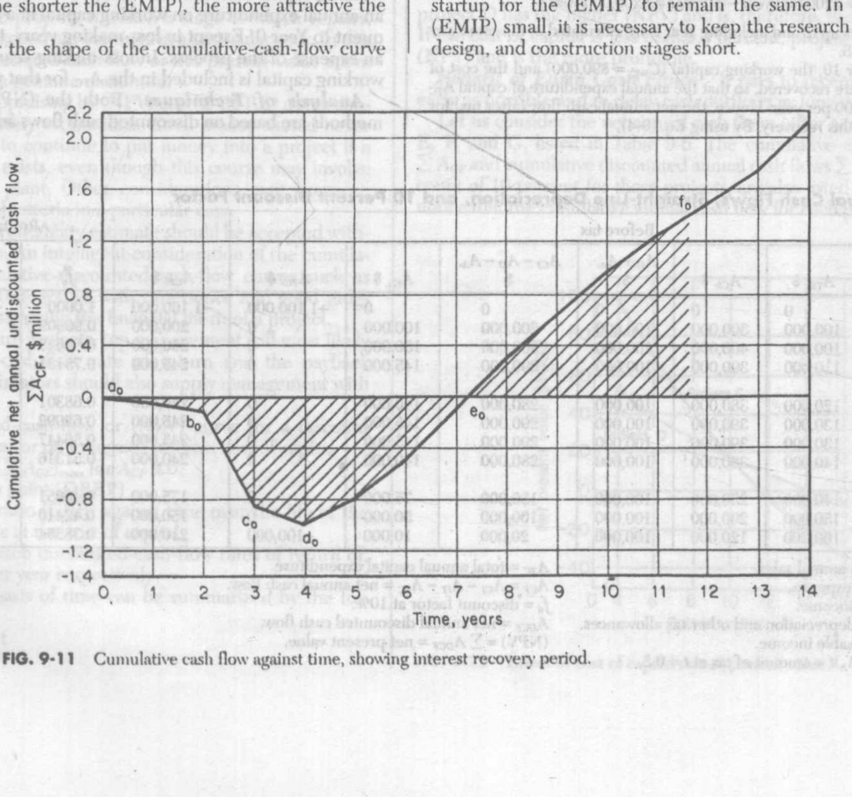

FIG. 9-11 Cumulative cash flow against time, showing interest recovery period.

Example 2: Net Present Value for Different Depreciation Methods The following data describe a project. Revenue from annual sales and the total annual expense over a 10-year period are given in the first three columns of Table 9-5. The fixed-capital investment C_{FC} is \$1,000,000. Plant items have a zero salvage value. Working capital C_{WC} is \$90,000, and cost of land C_L is \$10,000. There are no tax allowances other than depreciation; i.e., A_A is zero. The fractional tax rate t is 0.50.

We shall calculate for these data the net present value (NPV) for the following depreciation methods and discount factors:

a. Straight-line, 10 percent
b. Straight-line, 20 percent
c. Double-declining, 10 percent
d. Sum-of-years-digits, 10 percent
e. Straight-line, 10 percent; income tax delayed for 1 year

In addition, we shall calculate the discounted-cash-flow rate of return (DCFRR) with straight-line depreciation.

a. We begin the calculations for this example by finding the total capital cost C_{TC} for the project from Eq. (9-14). Here, C_{TC} = \$1,100,000. In Year 0, this amount is the same as the net annual capital expenditure A_{TC} and is listed in Table 9-5.

The annual rate of straight-line depreciation of the fixed-capital investment C_{FC}, from \$1,000,000 at startup to a salvage value S, of zero at the end of a productive life s of 10 years, is given by

$$A_D = (C_{FC} - S)/s$$

$$A_D = (\$1{,}000{,}000 - \$0)/10 \text{ years} = \$100{,}000/\text{year}$$

The annual cash income A_{CI} for Year 1, when A_S = \$400,000 per year and A_{TE} = \$100,000 per year, is, from Eq. (9-1), \$300,000 per year. Values for subsequent years are calculated in the same way and listed in Table 9-4.

Annual amount of tax A_{IT} for Year 1, when A_{CI} = \$300,000 per year, A_D = \$100,000 per year, A_A = \$0 per year, and $t = 0.5$, is found from Eq. (9-3) to be

$$A_{IT} = [(\$300{,}000 - \$100{,}000 - \$0)/\text{year}](0.5)$$

$$= \$100{,}000/\text{year}$$

Values for subsequent years are calculated in the same way and listed in Table 9-4.

Net annual cash flow (after tax) A_{CF} for Year 0, when A_{CI} = \$0 per year, A_{IT} = \$0 per year, and A_{TC} = \$1,100,000 per year, is found from Eq. (9-4) to be

$$A_{CF} = \$0/\text{year} - \$1{,}100{,}000/\text{year} = -\$1{,}100{,}000/\text{year}$$

Net annual cash flow (after tax) A_{CF} for Year 1, when A_{CI} = \$300,000 per year, A_{IT} = \$100,000 per year, and A_{TC} = \$0 per year, is found from Eqs. (9-2) and (9-4) to be

$$A_{CF} = \$200{,}000/\text{year} - \$0/\text{year} = \$200{,}000/\text{year}$$

Values for the years up to and including Year 9 are calculated in the same way and listed in Table 9-5.

At the end of Year 10, the working capital (C_{WC} = \$90,000) and the cost of land (C_L = \$10,000) are recovered, so that the net annual expenditure of capital A_{TC} in Year 10 is −\$100,000 per year. Hence, the net annual cash flow (after tax) for Year 10 must reflect this recovery. By using Eq. (9-4),

$$A_{CF} = \$110{,}000/\text{year} - (-\$100{,}000/\text{year})$$

$$= \$210{,}000/\text{year}$$

The net annual discounted cash flow A_{DCF} for Year 1, when A_{CF} = \$200,000 per year and $f_d = 0.90909$ (for $i = 10$ percent), is found from Eq. (9-52) to be

$$A_{DCF} = (\$200{,}000/\text{year})(0.90909) = \$181{,}820/\text{year}$$

Values for subsequent years are calculated in the same way and listed in Table 9-5.

The net present value (NPV) is found by summing the values of A_{DCF} for each year, as in Eq. (9-53). The net present value is found to be \$276,210, as given by the final entry in Table 9-5.

b. The same procedure is used for $i = 20$ percent. The discount factors to be used in a table similar to Table 9-5 must be those for 20 percent. The (NPV) is found to be −\$151,020.

c. The calculations are similar to those for subexample *a* except that depreciation is computed by using the double-declining method of Eq. (9-19). The net present value is found to be \$288,530.

d. Again, the calculations are similar to those for subexample *a* except that depreciation is computed by using the sum-of-years-digits method of Eq. (9-20). The net present value is found to be \$316,610.

e. The calculations follow the same procedure as for subexample *a*, but the annual amount of tax A_{IT} is calculated for a particular year and then deducted from the annual cash income A_{CI} for the following year. The net present value for Year 11 is found to be \$341,980.

The discounted-cash-flow rate of return (DCFRR) can readily be obtained approximately by interpolation of the (NPV) for $i = 10$ percent and $i = 20$ percent:

$$(\text{DCFRR}) = 0.100 + [(\$276{,}210)(0.20 - 0.10)]/[\$276{,}210 - (-\$151{,}020)]$$

$$(\text{DCFRR}) = 0.164, \text{ or } 16.4 \text{ percent}$$

The calculation of (DCFRR) usually requires a trial-and-error solution of Eq. (9-57), but rapidly convergent methods are available [N. H. Wild, *Chem. Eng.*, **83**, 153–154 (Apr. 12, 1976)]. For simplicity linear interpolation is often used.

A comparison of the (NPV) values for a 10 percent discount factor shows clearly that double-declining depreciation is more advantageous than straight-line depreciation and that sum-of-years-digits depreciation is more advantageous than the double-declining method. However, a significant advantage is obtained by delaying the payment of tax for 1 year even with straight-line depreciation.

This example is a simplified one. The cost of the working capital is assumed to be paid for in Year 0 and returned in Year 10. In practice, working capital increases with the production rate. Thus there may be an annual expenditure on working capital in a number of years subsequent to Year 0. Except in loss-making years, this is usually treated as an expense of the process. In loss-making years the cash injection for working capital is included in the A_{TC} for that year.

Analysis of Techniques Both the (NPV) and the (DCFRR) methods are based on discounted cash flows and in that sense are vari-

TABLE 9-5 Annual Cash Flows, Straight-Line Depreciation, and 10 Percent Discount Factor

	Before tax						After tax				
Year	A_S, \$	A_{TE}, \$	A_{CI}, \$	$A_D + A_A$, \$	$A_{CI} - A_D - A_A$, \$	A_{IT}, \$	A_{TC}, \$	A_{CF}, \$	f_d	A_{DCF}, \$	(NPV), \$
0	0	0	0	0	0	0	+1,100,000	−1,100,000	1.0000	−1,100,000	−1,100,000
1	400,000	100,000	300,000	100,000	200,000	100,000	0	200,000	0.90909	181,820	−918,180
2	500,000	100,000	400,000	100,000	300,000	150,000	0	250,000	0.82645	206,610	−711,570
3	500,000	110,000	390,000	100,000	290,000	145,000	0	245,000	0.75131	184,070	−527,500
4	500,000	120,000	380,000	100,000	280,000	140,000	0	240,000	0.68301	163,920	−363,580
5	520,000	130,000	390,000	100,000	290,000	145,000	0	245,000	0.62092	152,120	−211,460
6	520,000	130,000	390,000	100,000	290,000	145,000	0	245,000	0.56447	138,300	−73,160
7	520,000	140,000	380,000	100,000	280,000	140,000	0	240,000	0.51316	123,160	+50,000
8	390,000	140,000	250,000	100,000	150,000	75,000	0	175,000	0.46651	81,640	+131,640
9	350,000	150,000	200,000	100,000	100,000	50,000	0	150,000	0.42410	63,610	+195,250
10	280,000	160,000	120,000	100,000	20,000	10,000	−100,000	210,000	0.38554	80,960	+276,210

A_S = revenue from annual sales.
A_{TE} = total annual expense.
A_{CI} = annual cash income.
$A_D + A_A$ = annual depreciation and other tax allowances.
$A_{CI} - A_D - A_A$ = taxable income.
$A_{IT} = (A_{CI} - A_D - A_A)t$ = amount of tax at $t = 0.5$.
A_{TC} = total annual capital expenditure.
$A_{CF} = A_{CI} - A_{IT} - A_{TC}$ = net annual cash flow.
f_d = discount factor at 10%.
A_{DCF} = net annual discounted cash flow.
(NPV) = $\sum A_{DCF}$ = net present value.