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NUMERICAL MATHEMATICS AND COMPUTING

Fourth Edition

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The University of Texas at Austin



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PREFACE

In preparing the fourth edition of this book, we have adhered to the basic objective of the previous editions—namely, to acquaint students of science and engineering with the potentialities of the modern computer for solving numerical problems that may arise in their professions. A secondary objective is to give students an opportunity to hone their skills in programming and problem solving. A third objective is to help students arrive at an understanding of the important subject of *errors* that inevitably accompany scientific computing and to arm them with methods for detecting, predicting, and controlling these errors. A final, fourth objective is to familiarize students with the intelligent use of powerful and versatile systems such as Matlab®, Maple®, Mathematica®, and others in attacking numerical problems and obtaining not only numeric but graphical and sometimes symbolic results.*

With regard to these objectives, let us quote from *Numerical Computation in the Information Age* by John Guckenheimer, President of the Society for Industrial and Applied Mathematics (*SIAM News*, June 1998).

We are conducting ever more complex computations built upon the assumption that the underlying numerical methods are mature and reliable. Often these assumptions are not valid. . . . We regard the existing base as static and invest in the development of problem-solving environments and high-level languages. This is needed, but we also need to maintain our investment in continuing research on algorithms themselves.

What this implies for numerical methods and analysis courses is that students should study algorithms and learn not only how they work but also how they can fail. Critical thinking and constant skepticism are attitudes we want students to acquire. Any extensive numerical calculation, even when carried out by state-of-the-art software, should be subjected to independent verification, if possible.

*Matlab is a registered trademark of The MathWorks, Inc., Maple is a registered trademark of Waterloo Maple Software, and Mathematica is a registered trademark of Wolfram Research, Inc.

Since this book is to be accessible to students who are not necessarily advanced in their formal study of mathematics, we have tried to achieve an elementary style of presentation. Toward this end, we have provided numerous examples and fragments of pseudocode for illustrative purposes. Believing that most students at this level need a *survey* of the subject of numerical mathematics, we have presented a wide diversity of topics, including some rather advanced ones that play an important role in current scientific computing.

Features in the Fourth Edition

All sections of the book have been revised to some degree. Major new features are as follows:

- A broadened and intensified discussion of mathematical computer systems such as Maple, Matlab, Mathematica, and others and of their use in numerical methods.
- An expanded discussion of the IEEE standard for floating-point numbers on 32-bit machines.
- An overview of Neville's algorithm for polynomial interpolation.
- An elementary discussion of conditioning in linear systems of equations.
- A new section on iterative methods for solving systems of linear equations.
- A discussion of the singular value decomposition and some of its applications.
- A discussion of Bezier curves and how they are generated using B splines or Bernstein polynomials.
- Additional discussion of stiff ordinary differential equations.
- The Nelder-Meade algorithm and the method of simulated annealing for minimization problems.
- An expansion of Appendix A on linear algebra.
- A new Appendix B providing a survey of mathematical software on the World Wide Web from archives of programs to sources for commercial software.

Suggestions for Use

Numerical Mathematics and Computing, Fourth Edition, can be used in a variety of ways, depending on the emphasis the instructor prefers. Problems have been supplied in abundance to enhance the book's versatility. They are divided into two categories: *Problems* and *Computer Problems*. In the first category, there are more than 800 exercises in analysis that require pencil, paper, and possibly a calculator. In the second category, there are approximately 450 problems that involve writing a program and testing it on a computer. Readers can often follow a model or example in the text to assist them in working out exercises, but in other cases they must proceed on their own from a mathematical description given in the text or in the problems. In most of the computer problems, there is something to be learned beyond simply writing code—a *moral*, if you like. Some computing problems are

designed to give experience in using either mathematical software systems, pre-coded programs, or *black-box* library codes.

When exploring the problems in this book, the reader may wish to utilize one or more of the useful scientific and engineering software tools available for computation, visualization, and data analysis. Noteworthy examples are the computer algebra system Maple, the Matlab system with a variety of application tool boxes, and the Mathematica system for doing mathematics by the computer. In particular, the Symbolic Math Toolbox in Matlab provides access to the entire Maple kernel using an extension of the Matlab language. See Appendix B for information on these and on some of the other mathematical software systems.

The pseudocode displayed in this text has been coded in several programming languages and is available over the Internet as explained in Appendix B. Also, teachers who adopt the book can obtain from the publisher a copy of the Instructor's Solution Manual.

The arrangement of chapters reflects our own view of how the material might best unfold for a student new to the subject. However, there is very little mutual dependence among the chapters, and the instructor can order the sequence of their presentation in various ways. Most courses will certainly have to omit some sections and chapters.

Our own recommendations for courses based on this text are as follows:

- A one-term course carefully covering Chapters 1 through 9 (possibly omitting Sections 4.2, 6.4, 7.3–7.4, 8.3, and 9.3, for example), followed by a selection of material from the remaining chapters as time permits.
- A one-term survey rapidly covering all chapters in the text, omitting some of the more difficult sections.
- A two-term course carefully covering all chapters.

Acknowledgments

In preparing the fourth edition, we have been able to profit from advice and suggestions kindly offered by a large number of colleagues, students, and users of the third edition. It is our pleasure to thank them and others who helped with the task of preparing the new edition or previous editions.

Valuable comments and suggestions were made by our colleagues and friends. In particular, David Young has been very generous with suggestions for improving the accuracy and clarity of the exposition. Assistance in preparing this fourth edition or previous editions was provided in various capacities by Adarsh Beohar, Jason Brazile, Alan Donoho, Katherine Hua Guo, Kwang-il In, Hidajaty Thajeb, and Bi Roubolo Vona. Some parts of previous editions were typed with great care and attention to detail by Katy Burrell, Kata Carbone, and Belinda Trevino. In particular, we wish to thank Baharen Momken for checking the Matlab and Maple programs and Vincent Tsao for help with the index.

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We would appreciate any comments, questions, criticisms, or corrections that readers may communicate to us. E-mail is especially efficient. Our addresses are:

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*Ward Cheney
David Kincaid*

Dedicated to David M. Young, Jr.,
on the occasion of his 75th birthday

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