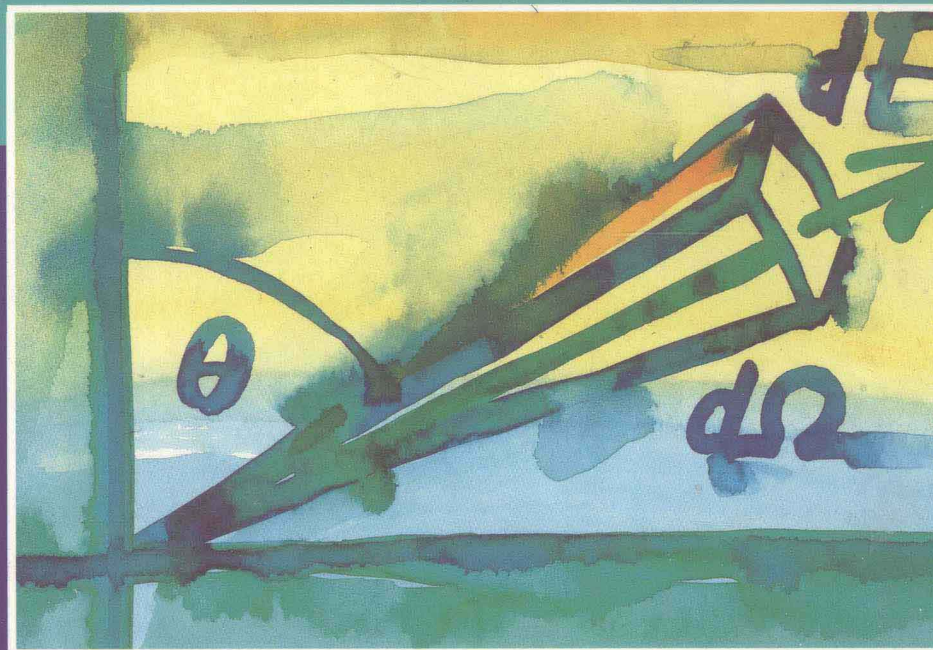

Greiner

QUANTUM MECHANICS AN INTRODUCTION

Third Edition

Springer

Walter Greiner

QUANTUM MECHANICS

An Introduction

With a Foreword by
D. A. Bromley

Third Corrected Edition
With 56 Figures,
and 87 Worked Examples and Problems

Springer

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Foreword to Earlier Series Editions

More than a generation of German-speaking students around the world have worked their way to an understanding and appreciation of the power and beauty of modern theoretical physics – with mathematics, the most fundamental of sciences – using Walter Greiner's textbooks as their guide.

The idea of developing a coherent, complete presentation of an entire field of science in a series of closely related textbooks is not a new one. Many older physicists remember with real pleasure their sense of adventure and discovery as they worked their ways through the classic series by Sommerfeld, by Planck and by Landau and Lifshitz. From the students' viewpoint, there are a great many obvious advantages to be gained through use of consistent notation, logical ordering of topics and coherence of presentation; beyond this, the complete coverage of the science provides a unique opportunity for the author to convey his personal enthusiasm and love for his subject.

The present five volume set, *Theoretical Physics*, is in fact only that part of the complete set of textbooks developed by Greiner and his students that presents the quantum theory. I have long urged him to make the remaining volumes on classical mechanics and dynamics, on electromagnetism, on nuclear and particle physics, and on special topics available to an English-speaking audience as well, and we can hope for these companion volumes covering all of theoretical physics some time in the future.

What makes Greiner's volumes of particular value to the student and professor alike is their completeness. Greiner avoids the all too common "it follows that..." which conceals several pages of mathematical manipulation and confounds the student. He does not hesitate to include experimental data to illuminate or illustrate a theoretical point and these data, like the theoretical content, have been kept up to date and topical through frequent revision and expansion of the lecture notes upon which these volumes are based.

Moreover, Greiner greatly increases the value of his presentation by including something like one hundred completely worked examples in each volume. Nothing is of greater importance to the student than seeing, in detail, how the theoretical concepts and tools under study are applied to actual problems of interest to a working physicist. And, finally, Greiner adds brief biographical sketches to each chapter covering the people responsible for the development of the theoretical ideas and/or the experimental data presented. It was Auguste Comte (1798–1857) in his *Positive Philosophy* who noted, "To understand a science it is necessary to know its history". This is all too often forgotten in modern physics teaching and the bridges that Greiner builds to the pioneering figures of our science upon whose work we build are welcome ones.

Greiner's lectures, which underlie these volumes, are internationally noted for their clarity, their completeness and for the effort that he has devoted to making physics an integral whole; his enthusiasm for his science is contagious and shines through almost every page.

These volumes represent only a part of a unique and Herculean effort to make all of theoretical physics accessible to the interested student. Beyond that, they are of enormous value to the professional physicist and to all others working with quantum phenomena. Again and again the reader will find that, after dipping into a particular volume to review a specific topic, he will end up browsing, caught up by often fascinating new insights and developments with which he had not previously been familiar.

Having used a number of Greiner's volumes in their original German in my teaching and research at Yale, I welcome these new and revised English translations and would recommend them enthusiastically to anyone searching for a coherent overview of physics.

Yale University
New Haven, CT, USA
1989

D. Allan Bromley
Henry Ford II Professor of Physics

Preface to the Third Edition

The text *Quantum Mechanics – An Introduction* has found many friends among physics students and researchers so that the need for a third edition has arisen. There was no need for a major revision of the text but I have taken the opportunity to make several amendments and improvements. A number of misprints and minor errors have been corrected and a few clarifying remarks have been added at various places. A few figures have been added or revised, in particular the three-dimensional density plots in Chap. 9.

I am grateful to several colleagues for helpful comments, in particular to Prof. R.A. King (Calgary) who supplied a comprehensive list of corrections. I also thank Dr. A. Scherdin for help with the figures and Dr. R. Mattiello who has supervised the preparation of the third edition of the book. Furthermore I acknowledge the agreeable collaboration with Dr. H. J. Kölsch and his team at Springer-Verlag, Heidelberg.

Frankfurt am Main
July 1994

Walter Greiner

Preface to the Second Edition

Like its German companion, the English edition of our textbook series has also found many friends, so that it has become necessary to prepare a second edition of this volume. There was no need for a major revision of the text. However, I have taken the opportunity to make several minor changes and to correct a number of misprints. Thanks are due to those colleagues and students who made suggestions to improve the text. I am confident that this textbook will continue to serve as a useful introduction to the fascinating topic of quantum mechanics.

Frankfurt am Main,
November 1992

Walter Greiner

Preface to the First Edition

Quantum Mechanics – An Introduction contains the lectures that form part of the course of study in theoretical physics at the Johann Wolfgang Goethe University in Frankfurt. There they are given for students in physics and mathematics in their fourth semester. They are preceded by Theoretical Mechanics I (in the first semester), Theoretical Mechanics II (in the second semester), and Classical Electrodynamics (in the third semester). Quantum Mechanics I – An Introduction then concludes the foundations laid for our students of the mathematical and physical methods of theoretical physics. Graduate work begins with the courses Thermodynamics and Statistical Mechanics, Quantum Mechanics II – Symmetries, Relativistic Quantum Mechanics, Quantum Electrodynamics, the Gauge Theory of Weak Interactions, Quantum Chromodynamics, and other, more specialized lectures.

As in all the other fields mentioned, we present quantum mechanics according to the inductive method, which comes closest to the methodology of the research physicist: starting with some key experiments, which are idealized, the basic ideas of the new science are introduced step by step. In this book, for example, we present the concepts of "state of a system" and "eigenstate", which then straightforwardly lead to the basic equation of motion, i.e. to the Schrödinger equation; and, by way of a number of classic, historically important observations concerning the quantization of physical systems and the various radiation laws, we infer the duality of waves and particles, which we understand with Max Born's conception of a "guiding field".

Quantum mechanics is then further developed with respect to fundamental problems (uncertainty relations; many-body systems; quantization of classical systems; spin within the phenomenological Pauli theory and through linearization of wave equations; etc.), applications (harmonic oscillator; hydrogen atom; Stern-Gerlach, Einstein-de Haas, Frank-Hertz, and Rabi experiments), and its mathematical structure (elements of representation theory; introduction of the S matrix, of Heisenberg, Schrödinger, and interaction pictures; eigendifferentials and the normalization of continuum wave functions; perturbation theory; etc.). Also, the elements of angular-momentum algebra are explained, which are so essential in many applications of atomic and nuclear physics. These will be presented in a much broader theoretical context in *Quantum Mechanics – Symmetries*.

Obviously an introductory course on quantum theory cannot (and should not) cover the whole field. Our selection of problems was carried out according to their physical importance, their pedagogical value, and their historical impact on the development of the field.

Students profit in the fourth semester at Frankfurt from the solid mathematical education of the first two years of studies. Nevertheless, in these lectures, new mathematical tools and methods and their use have also to be discussed. Within this category belong the solution of special differential equations (especially of the hypergeometrical and confluent hypergeometrical differential equations), a reminder of the elements of matrix calculus, the formulation of eigenvalue problems, and the explanation of (simple) perturbation methods. As in all the lectures, this is done in close connection with the physical problems encountered. In this way the student gets a feeling for the practical usefulness of the mathematical methods. Very many worked examples and exercises illustrate and round off the new physics and mathematics.

Furthermore, biographical and historical footnotes anchor the scientific development to the general side of human progress and evolution. In this context I thank the publishers Harri Deutsch and F.A. Brockhaus (*Brockhaus Enzyklopädie*, F.A. Brockhaus, Wiesbaden – marked by BR) for giving permission to extract the biographical data of physicists and mathematicians from their publications.

The lectures are now in their 5th German edition. Over the years many students and collaborators have helped to work out exercises and illustrative examples. For the first English edition I enjoyed the help of Maria Berenguer, Snježana Butorac, Christian Derreth, Dr. Klaus Geiger, Dr. Matthias Grabiak, Carsten Greiner, Christoph Hartnack, Dr. Richard Hermann, Raffaele Mattiello, Dieter Neubauer, Jochen Rau, Wolfgang Renner, Dirk Rischke, Thomas Schönfeld, and Dr. Stefan Schramm. Miss Astrid Steidl drew the graphs and pictures. To all of them I express my sincere thanks.

I would especially like to thank Mr. Béla Waldhauser, Dipl.-Phys., for his overall assistance. His organizational talent and his advice in technical matters are very much appreciated.

Finally, I wish to thank Springer-Verlag; in particular, Dr. H.-U. Daniel, for his encouragement and patience, and Mr. Mark Seymour, for his expertise in copy-editing the English edition.

Frankfurt am Main
July 1989

Walter Greiner

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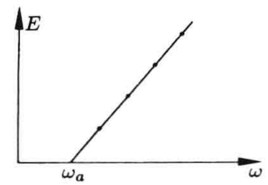

Fig. 1.2. The linear increase of photoelectron energy with frequency ω of the incident light

energy. This is in clear contradiction to classical wave theory, where the energy of a wave is given by its intensity. If we carry out the experiment with monochromatic light of different frequencies, a linear dependence between energy and frequency is obtained, as shown in Fig. 1.2:

$$E \propto (a + b\omega) \quad . \tag{1.1}$$

The proportionality factor, i.e. the slope of the straight line, is found to be Planck's constant h divided by 2π, so that

$$E = \hbar(\omega - \omega_a) = h(\nu - \nu_a) \tag{1.2}$$

with $h = 2\pi\hbar = 6.6 \times 10^{-34}\,\mathrm{Ws}^2$.

Einstein interpreted this effect by postulating discrete quanta of light (photons) with energy $\hbar\omega$. Increasing the intensity of the light beam also increases the number of photons, which can break off correspondingly more electrons from the metal.

In these experiments, a frequency limit ω_a appears, which depends on the kind of metal. Below this frequency limit, no electrons can leave the metal. This means that a definite *escape energy* $\hbar\omega$ is needed to raise electrons from the surface of the metal.

The light quantum that has to be postulated to understand the photoelectric effect moves with the velocity of light. Hence it follows from Einstein's *Theory of Relativity* that the rest mass of the photon is equal to zero.

If we set the rest mass equal to zero in the general relation for the total energy

$$E^2 = (m_0 c^2)^2 + p^2 c^2 = \hbar^2 \omega^2 \tag{1.3}$$

and express the frequency by the wave number $k = \omega/c$, the momentum of the photon follows as

$$p = \hbar k = \hbar\omega/c \quad , \tag{1.4}$$

or written as a vector identity, assuming that the direction of the momentum of the photon should correspond to the propagation direction of the light wave,

$$\boldsymbol{p} = \hbar \boldsymbol{k} \quad . \tag{1.5}$$

1.3 The Compton Effect

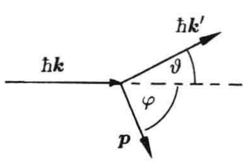

Fig. 1.3. Conservation of momentum in Compton scattering

When X-rays are scattered by electrons, a shift in frequency can be observed, the amount of this shift depending on the scattering angle. This effect was discovered by ***Compton*** in 1923 and explained on the basis of the photon picture simultaneously by Compton himself and ***Debye***.

Figure 1.3 illustrates the kinematical situation. We assume the electron is unbound and at rest before the collision. Then the conservation of energy and