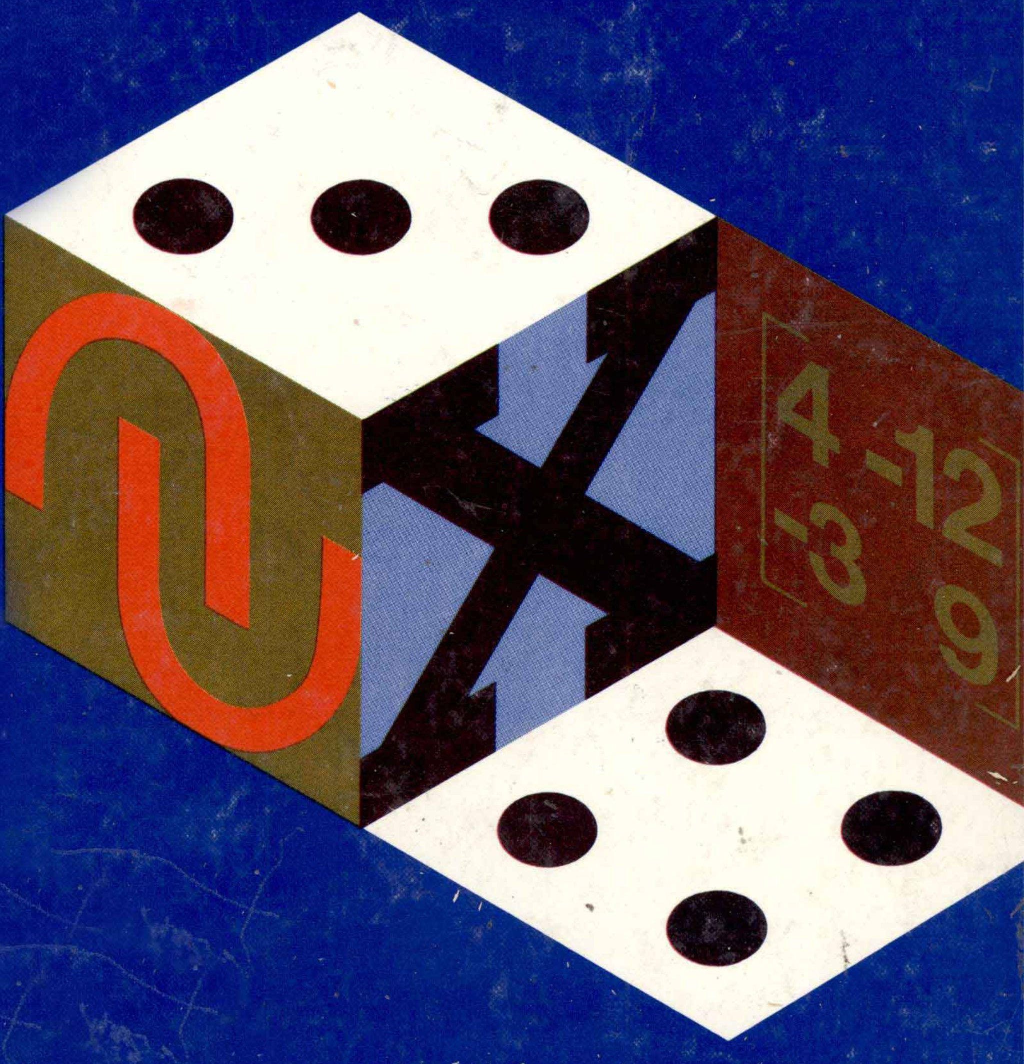
Finite Mathematics with Applications

A.W. GOODMAN and J.S. RATTI



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MACMILLAN PUBLISHING CO., INC.
New York

COLLIER MACMILLAN PUBLISHERS
London

Printed in the United States of America

Macmillan Publishing Co., Inc.
866 Third Avenue, New York, New York 10022

Collier-Macmillan Canada, Ltd., Toronto, Ontario

Library of Congress catalog card number: 72-119146

PRINTING 8 9 0 1 2 3 4

YEAR 4 5 6 7 8 9 0

preface

The concept of a course in finite mathematics is now well established. The guiding principle is to present to the reader a slice of mathematics that is interesting, meaningful, and useful and that at the same time does not involve the calculus.

With these limitations the content of a text for the course is almost uniquely determined, and we arrive at the usual topics:

Group I—Theory
- A. Logic
- B. Sets
- C. Combinatorial analysis
- D. Probability
- E. Vectors and matrices

Group II—Applications
- A. Linear Programming
- B. Game Theory
- C. Applications to social sciences
- D. Graph theory

In this text these nine topics are arranged into twelve chapters. Further, we include three appendices and an introductory chapter which cover supporting material. The table of contents gives the full details of the organization.

It is impossible for the authors to present a universal course outline because of the natural variations in the local calendars, previous preparation of the students, and objectives of the course. However, the following comments may prove helpful.

The text has 74 exercises (including the 6 in the appendices). Consequently the entire book can be covered by a class meeting three times a week, either for two semesters or for three quarters. If the time allotted is significantly less (one semester or two quarters), then some subset of the material must be selected.

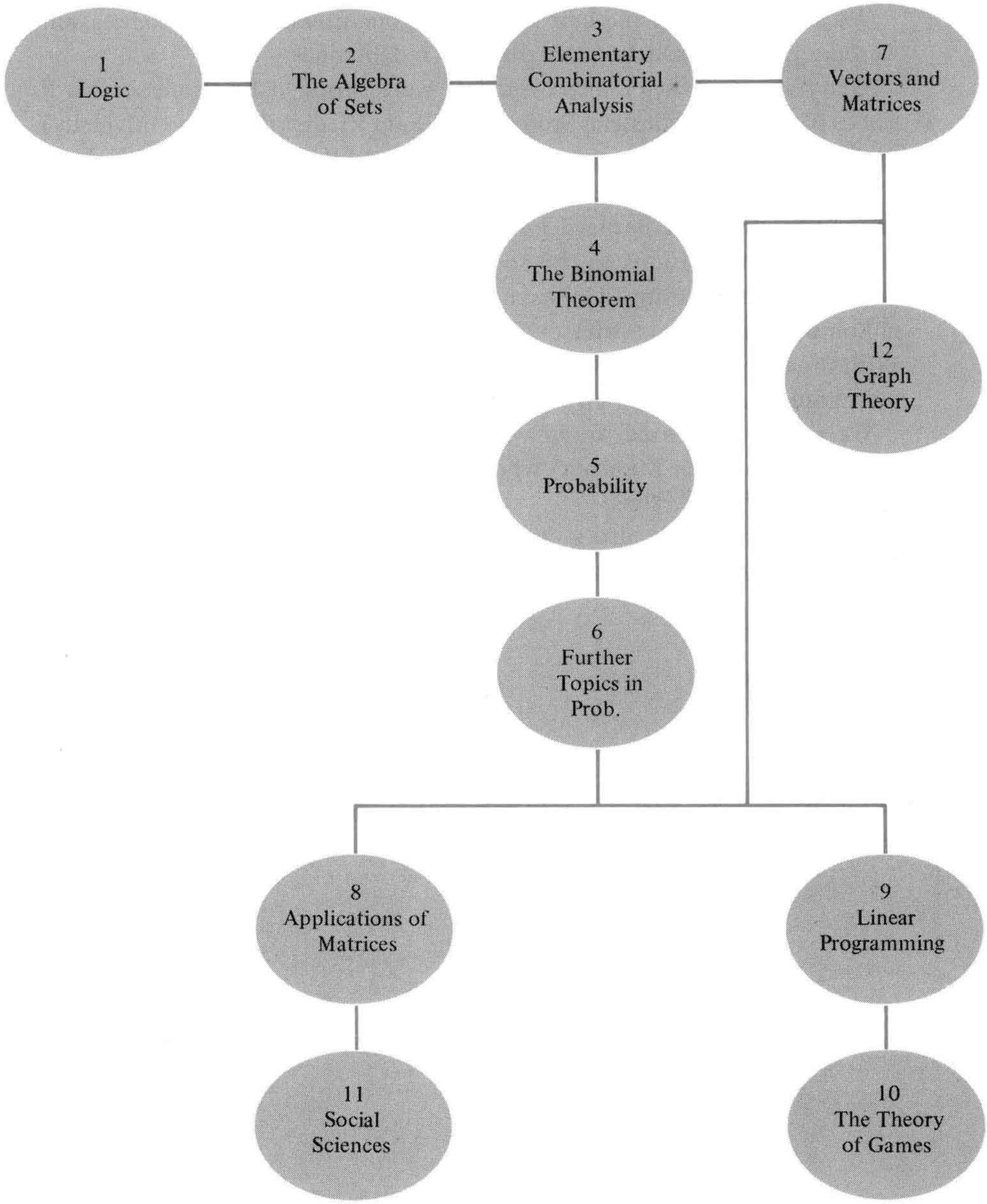
The two chapters on logic and sets are not absolutely essential for the remainder of the book, and the authors feel that these two chapters can safely (but not wisely) be omitted. These two topics (logic and sets) are interesting in their own right and serve well to train the student to think clearly. If logic and sets are to be omitted, the teacher should seriously consider covering Sections 6 and 7 of Chapter 2 ("The number of elements in a set" and "Tree diagrams") since these items form a natural introduction to the combinatorial analysis in Chapters 3 and 4. If there is time to cover only one of the first two chapters, then the authors recommend that the chapter on logic be omitted.

It is not intended that the teacher cover the three appendices. This material is included merely for the comfort and convenience of the reader. However, the teacher should feel free to incorporate these appendices into his lectures if he feels there is a need.

The following table may be useful as a guide to those who are planning a course based on this text.

Time allotted	*Chapters to be omitted*	*Number of exercises to be covered*
Two quarters Three credits	1 and 2	50
	or 9, 10, 11, and 12	53
One semester Four credits	1 and 2	50
	or 9, 10, and 11	58
One semester Three credits	1, 2, 11, and 12	43
	or 7 through 12	41
One quarter Four credits	6 through 12	35
	or 1, 2, and 9 through 12	35

We follow the usual custom of placing a star (★) on those sections which may be omitted without loss of continuity. However, a competent teacher will certainly make his own selection. We also use a system of stars to mark those problems which may be omitted without loss of continuity or which are relatively difficult. The following diagram gives the relative dependence of the various chapters.



Custom also dictates that answers be supplied only to the odd-numbered problems. Here we respect this well-established practice, but we do not follow it slavishly. Answers to all the odd-numbered problems are indeed supplied, but wherever we feel that there are good reasons, we also give the answer to an even-numbered problem. As a result, answers are supplied in the text for approximately two-thirds of the problems, and answers for the complementary set are available (as usual) in a supplementary pamphlet.

We are grateful for the help of many of our friends and colleagues, and we take pleasure in acknowledging our indebtedness.

Many suggestions for improvements were supplied by Professor M. P. Fobes (College of Wooster), Professor William Ziemer (Indiana University), Professor Klaus Witz (University of Illinois), and Professor Bernard Kolman (Drexel University). The two chapters on probability were carefully examined by Professor Bernard Lindgren (University of Minnesota) and the chapter on applications to the social sciences was reviewed by Professor Gordon Bower (Stanford University), Professor Douglas Nelson (University of South Florida), and Professor G. H. Mellish (University of South Florida). With the able assistance of these kind critics, we managed to remove many inaccuracies and to clean up some (we even hope all) of the obscure writing.

We are also indebted to our very capable student Miss Cynthia Strong (now Mrs. Norman Mansour) who worked every problem in the text and corrected our many slips.

Finally, our thanks also go to Mrs. Janelle Fortson for her cheerfulness and accuracy in typing the entire manuscript, together with numerous revisions.

Tampa, Florida

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CHAPTER

0

preliminaries

The student who is about to start a solid course in finite mathematics should be familiar with a large portion of elementary mathematics.

We resist the temptation to make a brief list of these elementary items for review. First, such a list is usually dull and boring, second it should be unnecessary, and third the list (together with review problems) is never brief.

Our objective here is to touch on just a few selected points that may have been slighted, omitted, or misunderstood.

1. mathematical symbols

The reader is already familiar with the more common mathematical symbols such as $+$, $-$, $=$, etc. To appreciate the advantages offered by good symbolism, one might compare the statement "For every pair of real numbers x and y

$$x + y = y + x\text{''} \tag{1}$$

with the nonsymbolic form: "For every pair of real numbers, the sum is independent of the order in which the addition of the two numbers is performed."

The energetic reader (who still doubts the advantages of symbols) should try to restate, without using symbols, the problem "Solve the equation $x^2 - 7x + 10 = 0$."

Certainly the use of symbols shortens the labor of writing and speeds the thinking process. One might deduce from this that the more symbols we have available, the better off we are. To some extent this is true, but we cannot introduce 50 new symbols all at once. The new symbols must be introduced gradually, and with each new symbol the student must be given ample time to practice using it, and to master it, before being confronted with the next one. For the reader's convenience, a list of the symbols used in this book is given just before the index.

It would be nice if each symbol represented a unique element or concept. Thus A should always represent area, C should always represent a constant, x should always represent an unknown, etc. But such an arrangement is impossible, because there are many more concepts in mathematics than there are symbols. Some symbols must bear the burden of being employed quite often with a variety of meanings. Here the best rule we can make is this: In any particular problem or proof, do not use one symbol with two different meanings.

2. subscripts and superscripts

Suppose that we have a problem involving the areas of four triangles (the exact nature of the problem is unimportant). It is natural to use the letter A to represent the area of the first triangle and to use a for the area of the second triangle. For the third area we might use α (Greek letter alpha) because it corresponds to the English a. But now we are stuck for a suitable choice of a symbol for the area of the fourth triangle. The way out is quite simple. We return to the letter A and put little numbers called *subscripts* just below the letter, thus: A_1, A_2, A_3, A_4, and use these to represent the areas of the four triangles. These symbols are read A sub-one, A sub-two, etc. If we are in a hurry we may say A-one, A-two, etc. Clearly the device of adding subscripts greatly enlarges the number of symbols available for our use.

We can also use *superscripts*. Thus we might write $A^{(1)}$, $A^{(2)}$, $A^{(3)}$, and $A^{(4)}$ to denote the altitudes of the triangles. Here we want to avoid A_1, because presumably in the problem at hand it has already been assigned the meaning of area. The symbols $A^{(1)}$, $A^{(2)}$, . . . are read A upper-one, A upper-two, etc. The superscripts are enclosed in parentheses to distinguish them from powers. Thus $A^{(2)} \neq A^2$, because the latter is AA or A squared, whereas $A^{(2)}$ is merely a symbol; in this example it is the symbol for the altitude of the second triangle.

A subscript or superscript is also called an *index*. The indices may be represented by a letter such as k. As a matter of shorthand, we can indicate the four altitudes $A^{(1)}$, $A^{(2)}$, $A^{(3)}$, $A^{(4)}$ by merely writing $A^{(k)}$ $(k = 1,2,3,4)$.

3. the three dots notation

In Appendix 3 on mathematical induction we shall prove the beautiful formula

$$1 + 3 + 5 + \cdots + (2n - 1) = n^2. \tag{2}$$

But to follow the proof, one must *first understand* what the formula means. Many students do not understand the three dots $(\cdots)$ which appear in equation (2). These dots mean that the terms proceed in the manner indicated by the first few already given. For example, if $n = 6$, then $2n - 1 = 11$, and equation (2) states that

$$1 + 3 + 5 + 7 + 9 + 11 = 6^2.$$

In this case the three dots represent $7 + 9$. But if $n = 10$, then equation (2) states that

$$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 = 10^2.$$

and in this case the three dots represent

$$7 + 9 + 11 + 13 + 15 + 17.$$

The three dots do not always mean that terms are *missing*. If $n = 3$, then $1 + 3 + 5 + \cdots + (2n - 1)$ *means* $1 + 3 + 5$. If $n = 2$, this same expression *means* $1 + 3$. If $n = 1$, the left-hand side of equation (2) has only *one* term, namely 1.

Why do we use the three dots? In the first place it is shorter. Thus if $2n - 1 = 99$ in equation (2), we would waste considerable time in writing all of the terms. If $2n - 1 = 9{,}999{,}999$ in equation (2), it would take about 58 days to write all of the terms of the sum, writing at the rate of one number per second and not stopping to eat or sleep.

Further, the three dots notation is a necessity, because our assertion is that equation (2) is true for any n. It would be impossible to express this idea properly if we did not have some notation such as the three dots.

The three dots notation is not restricted to sums or numbers. It may be used to indicate a sequence

$$x_1, x_2, x_3, \ldots, x_n,$$

where the notation merely means that we have n elements in some particular order and that the kth element in the sequence is x_k.

The three dots can also be used for products. For example, either

$$P = 1 \cdot 2 \cdot 3 \cdot \cdots \cdot n$$

or

$$P = 1 \times 2 \times 3 \times \cdots \times n$$

means that P is the product of all the integers from 1 to n inclusive. The symbol $n!$ (read "n factorial") is always attached to this product (for further details see Chapter 3).

4. what is finite mathematics?

Roughly speaking, finite mathematics consists of those parts of mathematics that are concerned with finite sets. Perhaps the reader is now comfortably waiting for the definition of a finite set, but here he may be disappointed. Of course the idea of a finite set is certainly easy, and we all have a strong intuitive understanding of this concept. But the precise definition is not simple, and we prefer to avoid it because it is never really needed in this book. For the reader who absolutely insists we offer

Definition 1. Finite Set. A set is said to be finite if it cannot be put into one-to-one correspondence with a proper subset of itself. A set that is not finite is said to be infinite.

We shall have much more to say about finite sets in Chapter 2, and, indeed, finite sets form the underlying theme of the book. However, we shall often use infinite sets in our examples and problems. The most important of these infinite sets are

$$N = \{1,2,3,4,5, \ldots\}, \tag{3}$$

the set of all natural numbers;

$$Z = \{0, \pm 1, \pm 2, \pm 3, \ldots\}, \tag{4}$$

the set of all integers; and

$$P = \{2,3,5,7,11, \ldots\}, \tag{5}$$

the set of all primes (see Section 5).

It may seem improper to introduce sets with *infinitely* many elements in a book called Finite Mathematics. However, it is very convenient to use