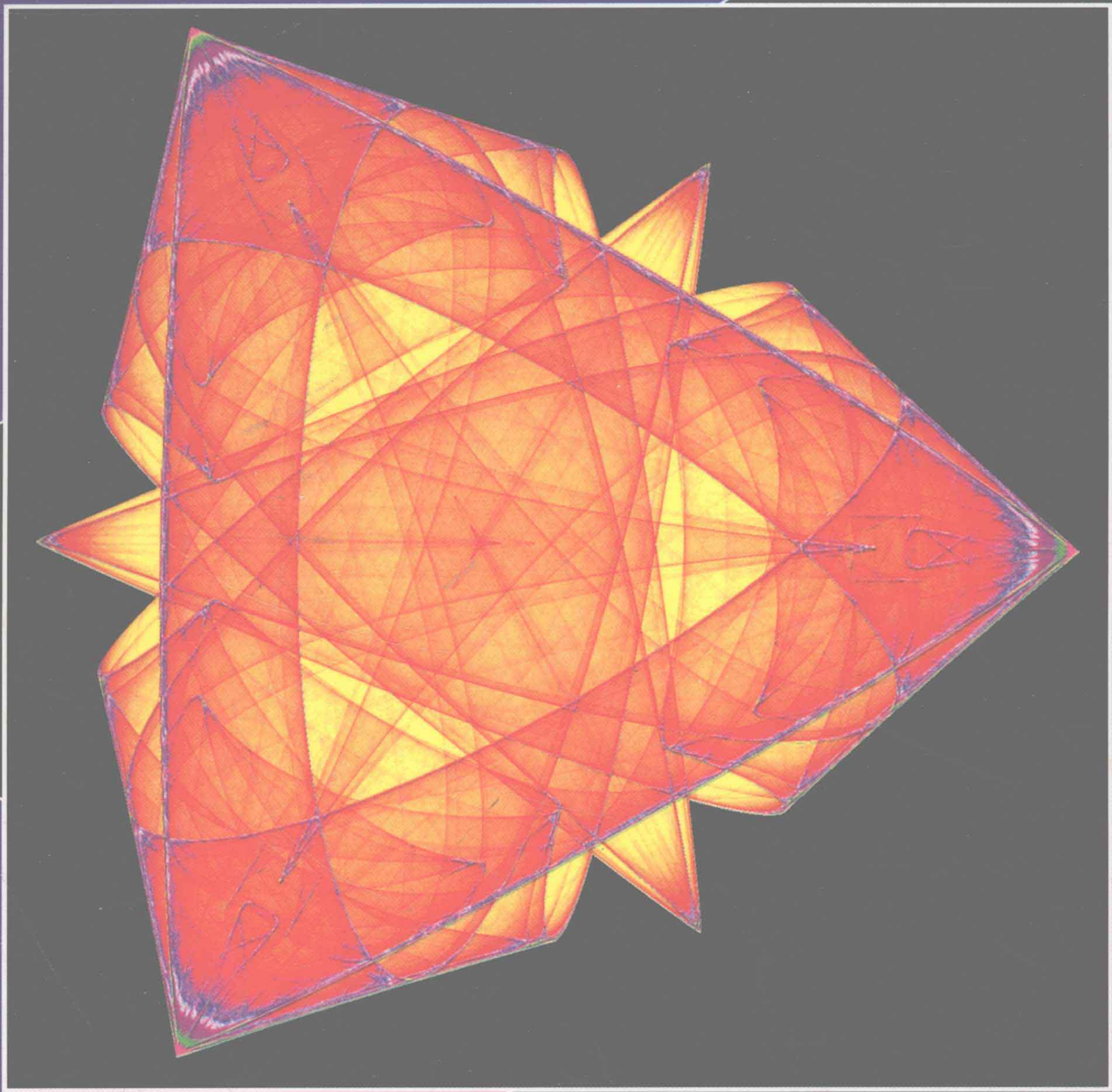
PRECALCULUS



Fourth Edition

DAVID COHEN

PRECALCULUS
A PROBLEMS-ORIENTED APPROACH

Fourth Edition

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WEST PUBLISHING COMPANY
Minneapolis / St. Paul New York Los Angeles San Francisco

FOR MY SON ANDY

Copyediting: Susan Gerstein and Pamela McMurry
Composition: The Clarinda Company
Cover Image: From *Symmetry in Chaos* by Michael Field and Martin Golubitsky. Published by Oxford University Press. © M. Field and M. Golubitsky, 1992.
Technical Illustrations: TECH*arts,* Boulder, Colorado
Text Design: Janet Bollow
Production, Prepress, Printing, and Binding by West Publishing Company.

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610 Opperman Drive
P.O. Box 64526
St. Paul, MN 55164-0526

Printed in the United States of America
00 99 98 97 96 95 94 93 8 7 6 5 4 3 2 1 0

LIBRARY OF CONGRESS CATALOGING-IN-PUBLICATION DATA

Cohen, David, 1942 Sept. 28-
Precalculus / David Cohen. — 4th ed.
p. cm.
Updated ed. of: Precalculus, a problems-oriented approach. c1990.
Includes index
ISBN 0-314-01255-9 (hard : alk. paper)
1. Mathematics. I. Cohen, David, 1942 Sept. 28- Precalculus, a problem-oriented approach. II. Title.
QA39.2.C64 1993
512.1—dc20 92-41652
CIP

PREFACE

This text is for students who are preparing to take calculus or other courses requiring a background in precalculus mathematics. As in the earlier editions, my goal has been to create a book that is *accessible* to the student. The presentation is student-oriented in three specific ways. First, I've tried to talk to, rather than lecture at, the student. Second, examples are consistently used to introduce, to explain, and to motivate concepts. And third, all of the initial exercises for each section are carefully correlated with the worked examples in that section.

AUDIENCE In writing *Precalculus,* I have assumed that the students have been exposed to intermediate algebra, but that they have not necessarily mastered that subject. Also, for many students, there may be a gap of several years between their last mathematics course and the present one. For these reasons, the review material in parts of the first two chapters and the appendix is unusually thorough.

FEATURES

1. *Word problems and applications.* Word problems and strategies for solving them are explained and developed throughout the book. Maximum–minimum problems relating to quadratic functions are discussed in detail in Section 4.4. The preceding section introduces some strategies for approaching these problems. To ensure that precalculus students gain appropriate practice and experience with these important strategies, the initial exercises in Section 4.3 make specific references to the corresponding worked examples in the text. In general, applications are integrated throughout the text. Two complete sections (5.5 and 5.6) are devoted to applications of the exponential function.
2. *Emphasis on graphing.* Graphs and techniques for graphing are developed throughout the text, and graphs are used to explain and reinforce algebraic concepts. (See, for example, Sections 3.3, 5.3, and 7.3.)
3. *Calculator Exercises.* There are two broad categories of calculator exercises in this text:
   (i) OPTIONAL GRAPHING CALCULATOR EXERCISES
   Over the past several years, all of us in the mathematics teaching community have become increasingly aware of the graphing calculator and its potential for making a positive impact in our teaching. While it is clear that vast reforms lie ahead, many of the specific details are still evolving in workshops and in classrooms across the country. And indeed, at present, even within a given school, some instructors are teaching the course with the graphing calculator, while others are not. Thus, for this 1993 edition of *Precalculus,* I have labeled the graphing calculator

exercises "optional," and I have placed them at the ends of the appropriate sections. Although these class-tested exercises can be adapted for use with any graphing calculator, they were written specifically for use with the Texas Instruments TI-81 Graphics Calculator. Since many of these exercises contain carefully detailed instructions for using this calculator, a minimum of class time is required for discussing its operation. Additionally, Section A.2 of the appendix (written by Professor Mickey Settle) contains a complete introduction to the basic features of the calculator.

(ii) (ORDINARY) CALCULATOR EXERCISES
As in the previous edition, there are many calculator exercises integrated throughout the regular exercise sets in the text. Some of these exercises reinforce or supplement the core material; some of them contain surprising results that motivate subsequent theoretical questions; and a few demonstrate that the use of a calculator cannot replace thinking or the need for mathematical proofs.

4. *Analytic Geometry.* The basic equations for lines and circles are introduced in Chapter 2 and used throughout the book. In Chapter 10, conic sections are discussed in greater detail than in most other precalculus books. The unifying focus–directrix property of the conics is developed in Section 10.6. In the past, analytic geometry traditionally served as the capstone in the student's preparation for calculus. It was here that the student had a real opportunity to sharpen his or her algebraic and analytic skills. Chapter 10 is intended to reaffirm, rather than dismiss, that tradition.

5. *Trigonometry.* The development of most topics in this text proceeds from the specific to the general, and the trigonometry follows this pattern, too. Chapter 6 introduces the right-triangle definitions (in Section 6.1) before the unit circle definitions (in Section 6.4).

The unit-circle definitions of the trigonometric functions involve simple geometric ideas that should be presented as such, and not cluttered with excessive notation or terminology (which will never appear in subsequent science or calculus courses). In presenting these definitions in Sections 6.4 and 7.2, the text sets a pace that is deliberately measured. Rather than rushing on, we take time to help the student understand the definitions. Experience shows that this pays dividends later on when the student studies the more analytical portions of trigonometry in Chapter 7.

6. *Graded Exercise Sets.* As a convenience for the instructor, most of the exercise sets (except for the chapter review sets) are divided into three categories: A, B, and C. The group-A exercises are based directly on the examples and definitions in that section of the text. Moreover, these problems treat topics in roughly the same order in which they appear in the text. The group-B exercises serve several key functions. Some of them require students to use several different techniques or topics in a single solution. Some group-B problems, while not conceptually more difficult than their group-A counterparts, require lengthier calculations or algebraic manipulations. Finally, some group-B exercises are included simply because they are interesting or because they allow the students to see the material from a different perspective. The group-C exercises contain the more challenging problems. (In many cases, though, detailed hints are provided.)

7. *End-of-Chapter Material.* Each chapter concludes with a detailed chapter summary, an extensive chapter-review exercise set, and a chapter test. Additionally, in Chapters 2 through 12, there are *Writing Mathematics* sections. In the *Writing Mathematics* questions, the student is asked to organize his or her thoughts and respond in complete sentences. Some of the questions are simply *true*-or-*false* questions that can be explained in just a sentence or two. In other cases, a more elaborate response is required. For example, in Chapter 2 (page 92), the *Writing Mathematics* section describes a geometric technique for solving quadratic equations and the student is asked to explain why the method is valid. In Chapter 8, the *Writing Mathematics* section describes a new proof for the law of cosines from the *College Mathematics Journal* and the student is asked to fill in the details. (See page 496.)

CHANGES IN THIS EDITION

Comments and suggestions from students, instructors, and reviewers have helped me to revise this text in a number of ways that I believe will make the book more useful to the instructor and more accessible to the student. As already mentioned, two new features of the book are the graphing calculator exercises and the *Writing Mathematics* sections. Other major changes occur in the following areas.

Chapter 1 *Algebra Background for Precalculus* There are three structural changes that have shortened this chapter. The geometric material on rectangular coordinates has been moved into Chapter 2; the review of exponents and radicals is given in two appendices at the back of the book; and the introduction to the complex-number system now appears at the beginning of Chapter 11, *Roots of Polynomial Equations*. The section on factoring (Section 1.3) has been revised. There are more examples and exercises, and many of these emphasize patterns.

Chapter 2 *Coordinates and Graphs* This is a self-contained review or reintroduction to elementary analytic geometry. Section 2.2 (*Graphs and Equations, a Second Look*) emphasizes the connection between graphing equations and solving equations. This section also contains a discussion of the six basic graphs:

$$y = |x| \qquad y = x^3 \qquad y = \sqrt{x}$$
$$y = x^2 \qquad y = 1/x \qquad y = \sqrt{1-x^2}$$

(In the previous edition, these graphs were discussed later, in the context of functions.) The material on symmetry (in Section 2.4) has been expanded to include symmetry about the line $y = x$. This simplifies the presentation of inverse functions in the next chapter.

Chapter 3 *Functions* New material has been added on the average rate of change of a function. (See pages 113–114.) Section 3.3, *Techniques in Graphing*, has been streamlined and the presentation is now, I believe, easier to teach and easier to learn. (Some of the material on translation and coordinates is presented again in Chapter 10, but from a different viewpoint.) Following NCTM guidelines, exercises on iteration of functions are now included. (See, for example, Exercises 49–52 on page 138.)

Chapter 4 *Polynomial and Rational Functions. Applications to Optimization* The material on graphing polynomials in factored form has been revised, now making use of the earlier work (in Chapter 1) on solving inequalities. Also, more details are given about how to analyze a graph in the vicinity of an x-intercept.

Chapter 5 *Exponential and Logarithmic Functions* The chapter introduces exponential functions from a more practical or contemporary viewpoint. We indicate how these functions are used in modeling real-life situations. See pages 227 and 228 and Example 3 on page 277. In Section 5.2, the constant e is now introduced both in terms of the limit of $[1 + (1/n)]^n$ and in terms of slopes of tangent lines. In the previous edition, the applications were scattered throughout the sections. Now they are presented in the last two sections of the chapter: Section 5.5 discusses compound interest; Section 5.6 uses the function $\mathcal{N} = \mathcal{N}_0 e^{kt}$ to model population growth and radioactive decay.

Chapter 6 *Trigonometric Functions of Angles* The section "Algebra and the Trigonometric Functions" from the third edition has been expanded and reorganized. Some of this material now appears much earlier (in Section 6.2). The treatment of the law of sines and the law of cosines has been moved into Chapter 8, along with several other applications.

Chapter 7 *Trigonometric Functions of Real Numbers* The graphs of the trigonometric functions are explained in Sections 7.3 through 7.5. In Section 7.3, the initial discussion of periodic functions is now illustrated with examples from astronomy, medicine, and music. The discussion in Section 7.3 is limited to the graphs of $y = \sin x$ and $y = \cos x$. The approximation $\sin x \approx x$ is introduced both for its utility and for the insight it yields about the shape of $y = \sin x$.

Chapter 8 *Additional Topics in Trigonometry* This chapter collects the applications that were spread over three different chapters in the previous edition: the laws of sines and cosines, vectors, and polar coordinates. The treatment of polar coordinates is expanded from one to two sections. In drawing polar curves, we make use of the Cartesian graphs for sine and cosine that were studied in the previous chapter. Also in this chapter, there is a new section on parametric equations.

Chapter 10 *Analytic Geometry* There are two new sections here, as well as some reorganization. The new sections are *The Focus–Directrix Property of Conics* (10.6) and *The Conics in Polar Coordinates* (10.7). The material on the parabola has been split into two sections. The second section, labeled "optional," shows how to find the tangent to a parabola using the methods of algebra rather than calculus. The section on rotation of axes has been moved to the end of the chapter.

Chapter 11 *Roots of Polynomial Equations* The chapter now begins with an introduction to the complex-number system. (In the previous edition, this introduction appeared at the end of Chapter 1.) The chapter ends with DeMoivre's theorem and the calculation of nth roots. (In the previous edition, this material was in the last chapter, *Additional Topics in Algebra*.)

SUPPLEMENTARY MATERIALS

1. The *Student's Solutions Manual*, by Ross Rueger, contains complete solutions for the odd-numbered exercises and for all test questions at the end of each chapter.
2. The *Instructor's Solutions Manual*, by Ross Rueger, contains answers or solutions for every even-numbered exercise in *Precalculus*.
3. The *Computer-Generated Testing Programs*, WestTest™ 3.0, are available to schools adopting *Precalculus*. (There are versions for both the Macintosh and the IBM PCs or compatibles running DOS or Windows 3.0.)
4. The *Test Bank*, by Charles Heuer, is a package that contains five versions of a

chapter test, as well as two multiple-choice versions, for each chapter in *Precalculus*.

5. *GraphToolz*, by Tom Saxton, is a software program for graphing and evaluating functions. This ingenious, easy-to-use software, along with *Drill and Enrichment Exercises*, by David Cohen, can make many of the topics in *Precalculus* really seem to come alive. *GraphToolz* is free to qualified adopters. (Available for the MacIntosh family of computers.)
6. *Transparency masters* for many of the key figures or tables appearing in the text are available to schools adopting *Precalculus*.
7. *College Algebra: In Simplest Terms* (a series of video tapes produced by Annenberg/CPB Collection) is available to qualified adopters. This series covers the major topics in college algebra, and it includes some excellent applications of precalculus mathematics.

ACKNOWLEDGMENTS Many students and teachers from both colleges and high schools have made useful constructive suggestions about the text and exercises, and I thank them for that. Special thanks go to Professor Charles Heuer for his careful work in checking the text and the exercise solutions for accuracy. I am grateful to Professor Mickey Settle for allowing us to reproduce his copyrighted work, *Using the TI-81 Graphics Calculator*, in Section A.2 of the appendix. In preparing and revising the manuscript, I received valuable suggestions and comments from the following reviewers.

Wayne Andrepont
University of Southwestern Louisiana

Edward Bradley
California Polytechnic State University, San Luis Obispo

Curtis Chipman
Oakland University

Philip Farmer
Diablo Valley College

Ray Glenn
Tallahassee Community College

Harvey Greenwald
California Polytechnic State University, San Luis Obispo

John Gregory
Southern Illinois University

Giles Maloof
Boise State University

I would like to thank Professor Ross Rueger, who wrote the supplementary manuals and created the answer section for the text; and Susan Gerstein and Pam McMurry, who copyedited the manuscript. Finally, to Stacy Lenzen, Deanna Quinn, Peter Marshall, and Jane Bass at West Publishing Company, thank you for all your work and help in bringing this manuscript into print.

David Cohen
Lunada Bay, California, 1993

PRECALCULUS

Fourth Edition

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CHAPTER ONE

ALGEBRA BACKGROUND FOR PRECALCULUS

There are topics [in algebra] *whose consideration prepares a student for a deeper understanding.*

Leonhard Euler (1707–1783) in *Introduction to Analysis of the Infinite,* translated by John D. Blanton (New York: Springer–Verlag, 1988)

Perhaps Pythagoras was a kind of magician to his followers because he taught them that nature is commanded by numbers. There is a harmony in nature, he said, a unity in her variety, and it has a language: numbers are the language of nature.

Jacob Bronowski in *The Ascent of Man* (Boston: Little, Brown, and Company, 1973)

INTRODUCTION

In Chapters 1 and 2 we review several key topics from algebra and coordinate geometry that form the foundation for our work in precalculus. Although you are probably already familiar with some of this material from previous courses, do not be lulled into a false sense of security. Now, in your second exposure to these topics, you really have the opportunity to master them. Take advantage of this opportunity; it will pay great dividends both in this course and in calculus.

1.1 SETS OF REAL NUMBERS

Natural numbers have been used since time immemorial; fractions were employed by the ancient Egyptians as early as 1700 B.C.*; and the Pythagoreans, in ancient Greece, about 400* B.C.*, discovered numbers, like* $\sqrt{2}$*, which cannot be fractions.*

Stefan Drobot in *Real Numbers* (Englewood Cliffs, NJ: Prentice-Hall, Inc., 1964)

What secrets lie hidden in decimals?

Stephen P. Richards in *A Number for Your Thoughts* (New Providence, NJ: S. P. Richards, 1982)

Here, as in your previous mathematics courses, most of the numbers we deal with are *real numbers.* These are the numbers used in everyday life, in the sciences, in industry, and in business. Perhaps the simplest way to define a real number is this: A **real number** is any number that can be expressed in decimal form. Some examples of real numbers are

$$7\ (= 7.000\ldots) \qquad -\tfrac{2}{3}\ (= -0.\overline{6}) \qquad \sqrt{2}\ (= 1.4142\ldots)$$

(The bar above the 6 in the decimal $-0.\overline{6}$ indicates that the 6 repeats indefinitely.)

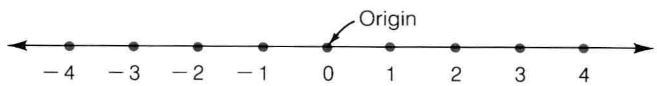
Certain sets of real numbers are referred to often enough to be given special names. These are summarized in the box that follows.

PROPERTY SUMMARY SETS OF REAL NUMBERS

NAME	DEFINITION AND COMMENTS	EXAMPLES
Natural numbers	These are the ordinary counting numbers, 1, 2, 3, and so on.	1, 4, 29, 1066
Integers	These are the natural numbers along with their negatives and zero.	−26, 0, 1, 1993
Rational numbers	As the name suggests, these are the real numbers that are *ratios* of two integers (with nonzero denominators, of course). It can be proved that a real number is rational if and only if its decimal expansion terminates (e.g., 3.15) or repeats (e.g., $2.\overline{43}$).	$4\ (=\frac{4}{1})$, $-\frac{2}{3}$, $1.7\ (=\frac{17}{10})$, $4.\overline{3}$, $4.\overline{173}$
Irrational numbers	These are the real numbers that are not rational. Section A.4 of the Appendix contains a proof of the fact that the number $\sqrt{2}$ is irrational. The proof that π is irrational is more difficult. The first person to prove that π is irrational was the Swiss mathematician J. H. Lambert (1728–1777).	$\sqrt{2}$, $3+\sqrt{2}$, $3\sqrt{2}$, π, $4+\pi$, 4π

As you've seen in previous courses, the real numbers can be represented as points on a *number line*, as shown in Figure 1. As indicated in Figure 1, the point associated with the number zero is referred to as the **origin.**

FIGURE 1



The fundamental fact here is that there is a **one-to-one correspondence** between the set of real numbers and the set of points on the line. This means that each real number is identified with exactly one point on the line; conversely, with each point on the line we identify exactly one real number. The real number associated with a given point is called the **coordinate** of the point. As a practical matter, we're usually more interested in relative locations than precise locations on a number line. For instance, since π is approximately 3.1, we show π slightly to the right of 3 in Figure 2. Similarly, since $\sqrt{2}$ is approximately 1.4, we show $\sqrt{2}$ slightly less than halfway from 1 to 2 in Figure 2.

It is often convenient to use number lines that show reference points other than the integers used in Figure 2. For instance, Figure 3(a) displays a number line with reference points that are multiples of π. In this case, of course, it is the integers that we then locate approximately. For example, in Figure 3(b) we show the approximate location of the number 1 on such a line.

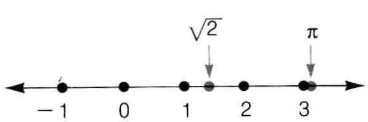


FIGURE 2

FIGURE 3

(a) (b)

Two of the most basic relations for real numbers are **less than** and **greater than**, symbolized by $<$ and $>$, respectively. For ease of reference, we review these and two related symbols in the box that follows.

PROPERTY SUMMARY NOTATION FOR LESS THAN AND GREATER THAN

NOTATION	DEFINITION	EXAMPLES
$a<b$	a is less than b. On a number line, oriented as in Figure 1, 2, or 3, the point a lies to the left of b.	$2<3$; $-4<1$
$a\leq b$	a is less than or equal to b.	$2\leq 3$; $3\leq 3$
$b>a$	b is greater than a. On a number line oriented as in Figure 1, 2, or 3, the point b lies to the right of a. [$b>a$ is equivalent to $a<b$.]	$3>2$; $0>-1$
$b\geq a$	b is greater than or equal to a.	$3\geq 2$; $3\geq 3$

In general, relationships involving real numbers and any of the four symbols $<, \leq, >$, and $\geq$ are called **inequalities.** One of the simplest uses of inequalities is in defining certain sets of real numbers called *intervals*. Roughly speaking, any uninterrupted portion of the number line is referred to as an **interval.** In the definitions that follow, you'll see notation such as $a<x<b$. This means that *both* of the inequalities $a<x$ and $x<b$ hold; in other words, the number x is between a and b.

(a) The open interval (a, b) contains all real numbers from a to b, excluding a and b.

(b) The closed interval [a, b] contains all real numbers from a to b, including a and b.

FIGURE 4

DEFINITION Open Intervals and Closed Intervals

The **open interval** (a, b) consists of all real numbers x such that $a<x<b$. See Figure 4(a).

The **closed interval** $[a, b]$ consists of all real numbers x such that $a\leq x\leq b$. See Figure 4(b).

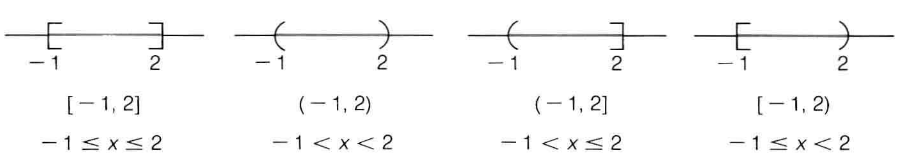
Notice that the brackets in Figure 4(b) are used to indicate that the numbers a and b are included in the interval $[a, b]$, whereas the parentheses in Figure 4(a) indicate that a and b are excluded from the interval (a, b). At times you'll see notation such as $[a, b)$. This stands for the set of all real numbers x such that $a\leq x<b$. Similarly, $(a, b]$ denotes the set of all numbers x such that $a<x\leq b$.

EXAMPLE 1 Show each interval on a number line, and specify inequalities describing the numbers x in each interval.

$[-1, 2] \quad (-1, 2) \quad (-1, 2] \quad [-1, 2)$

Solution See Figure 5.

FIGURE 5



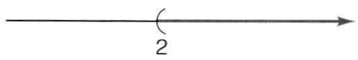
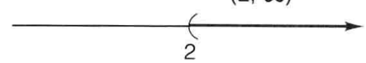
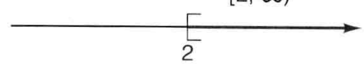
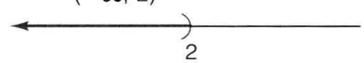
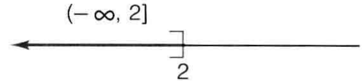
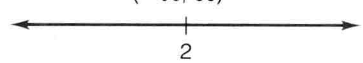


FIGURE 6
The set of all real numbers x such that $x > 2$.

In addition to the four types of intervals shown in Figure 5, we can also consider **unbounded intervals.** These are intervals that extend indefinitely in one direction or the other, as shown, for example, in Figure 6. We also have a convenient notation for unbounded intervals; for example, we indicate the unbounded interval in Figure 6 with the notation $(2, \infty)$.

COMMENT AND CAUTION The symbol ∞ is read *infinity*. It is not a real number, and its use in the context $(2, \infty)$ is only to indicate that the interval has no right-hand boundary. In the box that follows we define the five types of unbounded intervals. Note that the last interval, $(-\infty, \infty)$, is actually the entire real-number line.

PROPERTY SUMMARY **UNBOUNDED INTERVALS**

NOTATION	DEFINING INEQUALITY	EXAMPLE
(a, ∞)	$x > a$	$(2, \infty)$  2
$[a, \infty)$	$x \geq a$	$[2, \infty)$  2
$(-\infty, a)$	$x < a$	$(-\infty, 2)$  2
$(-\infty, a]$	$x \leq a$	$(-\infty, 2]$  2
$(-\infty, \infty)$		$(-\infty, \infty)$  2

EXAMPLE 2 Indicate each set of real numbers on a number line: **(a)** $(-\infty, 4]$; **(b)** $(-3, \infty)$.

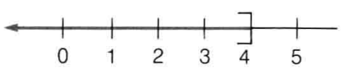


FIGURE 7
$(-\infty, 4]$

Solution

(a) The interval $(-\infty, 4]$ consists of all real numbers that are less than or equal to 4. See Figure 7.

(b) The interval $(-3, \infty)$ consists of all real numbers that are greater than -3. See Figure 8.

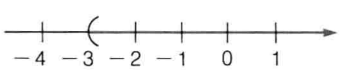


FIGURE 8
$(-3, \infty)$

We conclude this section by mentioning that our treatment of the real-number system has been rather informal, and we have not derived any of the rules of arithmetic and algebra using the most basic properties of the real numbers. However, we do list those basic properties and derive some of their consequences in Secion A.3 of the Appendix. (For example, in the appendix we prove that the product of two negative numbers is positive.