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SCHAUM'S OUTLINE OF

THEORY AND PROBLEMS

OF

STATISTICS

Second Edition

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To My Mother

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Preface

Statistics, or statistical methods as it is sometimes called, is playing an increasingly important role in nearly all phases of human endeavor. Formerly dealing only with affairs of the state, thus accounting for its name, the influence of statistics has now spread to agriculture, biology, business, chemistry, communications, economics, education, electronics, medicine, physics, political science, psychology, sociology and numerous other fields of science and engineering.

The purpose of this book is to present an introduction to the general statistical principles which will be found useful to all individuals regardless of their fields of specialization. It has been designed for use either as a supplement to all current standard texts or as a textbook for a formal course in statistics. It should also be of considerable value as a book of reference for those presently engaged in applications of statistics to their own special problems of research.

Each chapter begins with clear statements of pertinent definitions, theorems and principles together with illustrative and other descriptive material. This is followed by graded sets of solved and supplementary problems which in many instances use data drawn from actual statistical situations. The solved problems serve to illustrate and amplify the theory, bring into sharp focus those fine points without which the student continually feels himself on unsafe ground, and provide the repetition of basic principles so vital to effective teaching. Numerous derivations of formulas are included among the solved problems. The large number of supplementary problems with answers serve as a complete review of the material of each chapter.

The only mathematical background needed for an understanding of the entire book is arithmetic and the elements of algebra. A review of important mathematical concepts used in the book is presented in the first chapter which may either be read at the beginning of the course or referred to later as the need arises.

The early part of the book deals with the analysis of frequency distributions and associated measures of central tendency, dispersion, skewness and kurtosis. This leads quite naturally to a discussion of elementary probability theory and applications, which paves the way for a study of sampling theory. Techniques of large sampling theory, which involve the normal distribution, and applications to statistical estimation and tests of hypotheses and significance are treated first. Small sampling theory, involving Student's t distribution, the chi-square distribution and the F distribution together with the applications appear in a later chapter. Another chapter on curve fitting and the method of least squares leads logically to the topics of correlation and regression involving two variables. Multiple and partial correlation involving more than two variables are treated in a separate chapter. These are followed by chapters on the analysis of variance and nonparametric methods, new in this second edition. Two final chapters deal with the analysis of time series and index numbers respectively.

Considerably more material has been included here than can be covered in most first courses. This has been done to make the book more flexible, to provide a more useful book of reference and to stimulate further interest in the topics. In using the book it is possible to change the order of many later chapters or even to omit certain chapters without difficulty. For example, Chapters 13–15 and 18–19 can, for the most part, be introduced immediately after Chapter 5, if it is desired to treat correlation, regression, times series, and index numbers before sampling theory. Similarly, most of Chapter 6 may be omitted if one does not wish to devote too much time to probability. In a first course all of Chapter 15 may be omitted. The present order has been used because there is an increasing tendency in modern courses to introduce sampling theory and statistical influence as early as possible.

I wish to thank the various agencies, both governmental and private, for their cooperation in supplying data for tables. Appropriate references to such sources are given throughout the book. In particular, I am indebted to Professor Sir Ronald A. Fisher, F.R.S., Cambridge; Dr. Frank Yates, F.R.S., Rothamsted; and Messrs. Oliver and Boyd Ltd., Edinburgh, for permission to use data from Table III of their book *Statistical Tables for Biological, Agricultural, and Medical Research.* I also wish to thank Esther and Meyer Scher for their encouragement and the staff of McGraw-Hill for their cooperation.

MURRAY R. SPIEGEL

CONTENTS

Chapter 1

Variables and Graphs

STATISTICS

Statistics is concerned with scientific methods for collecting, organizing, summarizing, presenting, and analyzing data as well as with drawing valid conclusions and making reasonable decisions on the basis of such analysis.

In a narrower sense, the term *statistics* is used to denote the data themselves or numbers derived from the data, such as averages. Thus we speak of employment statistics, accident statistics, etc.

POPULATION AND SAMPLE; INDUCTIVE AND DESCRIPTIVE STATISTICS

In collecting data concerning the characteristics of a group of individuals or objects, such as the heights and weights of students in a university or the numbers of defective and nondefective bolts produced in a factory on a given day, it is often impossible or impractical to observe the entire group, especially if it is large. Instead of examining the entire group, called the *population*, or *universe*, one examines a small part of the group, called a *sample*.

A population can be *finite* or *infinite*. For example, the population consisting of all bolts produced in a factory on a given day is finite, whereas the population consisting of all possible outcomes (heads, tails) in successive tosses of a coin is infinite.

If a sample is representative of a population, important conclusions about the population can often be inferred from analysis of the sample. The phase of statistics dealing with conditions under which such inference is valid is called *inductive statistics*, or *statistical inference*. Because such inference cannot be absolutely certain, the language of *probability* is often used in stating conclusions.

The phase of statistics that seeks only to describe and analyze a given group without drawing any conclusions or inferences about a larger group is called *descriptive*, or *deductive*, *statistics*.

Before proceeding with the study of statistics, let us review some important mathematical concepts.

VARIABLES: DISCRETE AND CONTINUOUS

A *variable* is a symbol, such as X, Y, H, x, or B, that can assume any of a prescribed set of values, called the *domain* of the variable. If the variable can assume only one value, it is called a *constant*.

A variable that can theoretically assume any value between two given values is called a *continuous variable*; otherwise, it is called a *discrete variable*.

EXAMPLE 1. The number N of children in a family, which can assume any of the values 0, 1, 2, 3, ... but cannot be 2.5 or 3.842, is a discrete variable.

EXAMPLE 2. The height H of an individual, which can be 62 inches (in), 63.8 in, or 65.8341 in, depending on the accuracy of measurement, is a continuous variable.

Data that can be described by a discrete or continuous variable are called *discrete data* or *continuous data*, respectively. The number of children in each of 1000 families is an example of discrete data, while the heights of 100 university students is an example of continuous data. In general, *measurements* give rise to continuous data, while *enumerations*, or *countings*, give rise to discrete data.

It is sometimes convenient to extend the concept of variable to nonnumerical entities; for example, color C in a rainbow is a variable that can take on the "values" red, orange, yellow, green, blue, indigo, and violet. It is generally possible to replace such variables by numerical quantities; for example, denote red by 1, orange by 2, etc.

ROUNDING OF DATA

The result of rounding a number such as 72.8 to the nearest unit is 73, since 72.8 is closer to 73 than to 72. Similarly, 72.8146 rounded to the nearest hundredth (or to two decimal places) is 72.81, since 72.8146 is closer to 72.81 than to 72.82.

In rounding 72.465 to the nearest hundredth, however, we are faced with a dilemma since 72.465 is *just as far* from 72.46 as from 72.47. It has become the practice in such cases to round to the *even integer* preceding the 5. Thus 72.465 is rounded to 72.46, 183.575 is rounded to 183.58, and 116,500,000 rounded to the nearest million is 116,000,000. This practice is especially useful in minimizing *cumulative rounding errors* when a large number of operations is involved (see Problem 1.4).

SCIENTIFIC NOTATION

When writing numbers, especially those involving many zeros before or after the decimal point, it is convenient to employ the scientific notation using powers of 10.

EXAMPLE 3. $10^1 = 10$, $10^2 = 10 \times 10 = 100$, $10^5 = 10 \times 10 \times 10 \times 10 \times 10 = 100{,}000$, and $10^8 = 100{,}000{,}000$.

EXAMPLE 4. $10^0 = 1$; $10^{-1} = .1$, or 0.1; $10^{-2} = .01$, or 0.01; and $10^{-5} = .00001$, or 0.00001.

EXAMPLE 5. $864{,}000{,}000 = 8.64 \times 10^8$, and $0.00003416 = 3.416 \times 10^{-5}$.

Note that multiplying a number by 10^8, for example, has the effect of moving the decimal point of the number eight places *to the right*. Multiplying a number by 10^{-6} has the effect of moving the decimal point of the number six places *to the left*.

We often write 0.1253 rather than .1253 to emphasize the fact that a number other than zero before the decimal point has not accidentally been omitted. However, the zero before the decimal point can be omitted in cases where no confusion can result, such as in tables.

Often we use parentheses or dots to show the multiplication of two or more numbers. Thus $(5)(3) = 5 \cdot 3 = 5 \times 3 = 15$, and $(10)(10)(10) = 10 \cdot 10 \cdot 10 = 10 \times 10 \times 10 = 1000$. When letters are used to represent numbers, the parentheses or dots are often omitted; for example, $ab = (a)(b) = a \cdot b = a \times b$.

The scientific notation is often useful in computation, especially in locating decimal points. Use is then made of the rules

$$(10^p)(10^q) = 10^{p+q} \qquad \frac{10^p}{10^q} = 10^{p-q}$$

where p and q are any numbers.

In 10^p, p is called the *exponent* and 10 is called the *base*.

EXAMPLE 6.

$$(10^3)(10^2) = 1000 \times 100 = 100{,}000 = 10^5 \qquad \text{i.e., } 10^{3+2}$$

$$\frac{10^6}{10^4} = \frac{1{,}000{,}000}{10{,}000} = 100 = 10^2 \qquad \text{i.e., } 10^{6-4}$$

EXAMPLE 7.

$$(4{,}000{,}000)(0.0000000002) = (4 \times 10^6)(2 \times 10^{-10}) = (4)(2)(10^6)(10^{-10}) = 8 \times 10^{6-10}$$
$$= 8 \times 10^{-4} = 0.0008$$

EXAMPLE 8.

$$\frac{(0.006)(80{,}000)}{0.04} = \frac{(6\times10^{-3})(8\times10^{4})}{4\times10^{-2}} = \frac{48\times10^{1}}{4\times10^{-2}} = \left(\frac{48}{4}\right)\times10^{1-(-2)}$$
$$= 12\times10^{3} = 12{,}000$$

SIGNIFICANT FIGURES

If a height is accurately recorded as 65.4 in, it means that the true height lies between 65.35 and 65.45 in. The accurate digits, apart from zeros needed to locate the decimal point, are called the *significant digits*, or *significant figures*, of the number.

EXAMPLE 9. 65.4 has three significant figures.

EXAMPLE 10. 4.5300 has five significant figures.

EXAMPLE 11. $.0018 = 0.0018 = 1.8\times10^{-3}$ has two significant figures.

EXAMPLE 12. $.001800 = 0.001800 = 1.800\times10^{-3}$ has four significant figures.

Numbers associated with enumerations (or countings), as opposed to measurements, are of course exact and so have an unlimited number of significant figures. In some of these cases, however, it may be difficult to decide which figures are significant without further information. For example, the number 186,000,000 may have 3, 4, . . . , 9 significant figures. If it is known to have five significant figures, it would be better to record the number as either 186.00 million or 1.8600×10^{8}.

COMPUTATIONS

In performing calculations involving multiplication, division, and the extraction of roots of numbers, the final result can have no more significant figures than the numbers with the fewest significant figures (see Problem 1.9).

EXAMPLE 13. $73.24\times4.52 = (73.24)(4.52) = 331$

EXAMPLE 14. $1.648/0.023 = 72$

EXAMPLE 15. $\sqrt{38.7} = 6.22$

EXAMPLE 16. $(8.416)(50) = 420.8$ (if 50 is exact)

In performing additions and subtractions of numbers, the final result can have no more significant figures after the decimal point than the numbers with the fewest significant figures after the decimal point (see Problem 1.10).

EXAMPLE 17. $3.16 + 2.7 = 5.9$

EXAMPLE 18. $83.42 - 72 = 11$

EXAMPLE 19. $47.816 - 25 = 22.816$ (if 25 is exact)

The above rule for addition and subtraction can be extended (see Problem 1.11).

FUNCTIONS

If to each value that a variable X can assume there corresponds one or more values of a variable Y, we say that Y is a *function* of X and write $Y = F(X)$ (read "Y equals F of X") to indicate this functional dependence. Other letter (G, ϕ, etc.) can be used instead of F.

The variable X is called the *independent variable*, and Y is called the *dependent variable*.

If only one value of Y corresponds to each value of X, we call Y a *single-valued function* of X; otherwise, it is called a *multiple-valued function* of X.

EXAMPLE 20. The total population P of the United States is a function of the time t, and we write $P = F(t)$.

EXAMPLE 21. The stretch S of a vertical spring is a function of the weight W placed on the end of the spring. In symbols, $S = G(W)$.

The functional dependence (or correspondence) between variables is often depicted in a table. However, it can also be indicated by an equation connecting the variables, such as $Y = 2X - 3$, from which Y can be determined corresponding to various values of X.

If $Y = F(X)$, it is customary to let $F(3)$ denote "the value of Y when $X = 3$," to let $F(10)$ denote "the value of Y when $X = 10$," etc. Thus if $Y = F(X) = X^2$, then $F(3) = 3^2 = 9$ is the value of Y when $X = 3$.

The concept of function can be extended to two or more variables (see Problem 1.17).

RECTANGULAR COORDINATES

Consider two mutually perpendicular lines $X'OX$ and $Y'OY$, called the *X and Y axes*, respectively (see Fig. 1-1), on which appropriate scales are indicated. These lines divide the plane determined by them, called the *XY plane*, into four regions denoted by I, II, III, and IV and called the first, second, third, and fourth *quadrants*, respectively.

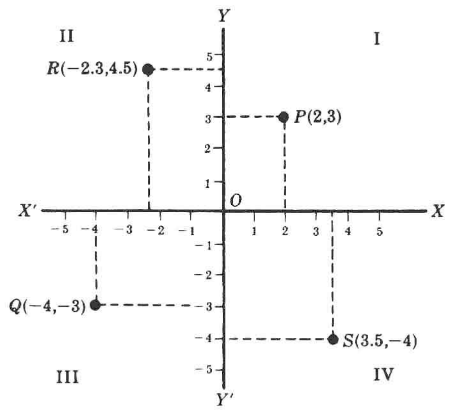

Fig. 1-1

Point O is called the *origin*, or *zero point*. Given any point P, drop perpendiculars to the X and Y axes from P. The values of X and Y at the points where the perpendiculars meet these axes are called the *rectangular coordinates*, or simply the *coordinates*, of P and are denoted by (X, Y). The coordinate X is sometimes called the *abscissa*, and Y is the *ordinate* of the point. In Fig. 1-1 the abscissa of point P is 2, the ordinate is 3, and the coordinates of P are $(2, 3)$.

Conversely, given the coordinates of a point, we can locate—or *plot*—the point. Thus the points with coordinates $(-4, -3)$, $(-2.3, 4.5)$, and $(3.5, -4)$ are represented in Fig. 1-1 by Q, R, and S, respectively.

By constructing a *Z axis* through O and perpendicular to the XY plane, we can easily extend the above ideas. In such case the coordinates of a point P would be denoted by (X, Y, Z).

GRAPHS

A *graph* is a pictorial presentation of the relationship between variables. Many types of graphs are employed in statistics, depending on the nature of the data involved and the purpose for which the graph is intended. Among these are *bar graphs*, *pie graphs*, *pictographs*, etc. These graphs are sometimes referred to as *charts* or *diagrams*. Thus we speak of bar charts, pie diagrams, etc. (see Problems 1.23, 1.24, 1.26 and 1.27).

EQUATIONS

Equations are statements of the form $A = B$, where A is called the *left-hand member* (or *side*) of the equation, and B the *right-hand member* (or *side*). So long as we apply the *same* operations to both members of an equation, we obtain *equivalent equations*. Thus we can add, subtract, multiply, or divide both members of an equation by the same value and obtain an equivalent equation, the only exception being that *division by zero is not allowed.*

EXAMPLE 22. Given the equation $2X + 3 = 9$, subtract 3 from both members: $2X + 3 - 3 = 9 - 3$, or $2X = 6$. Divide both members by 2: $2X/2 = 6/2$, or $X = 3$. This value of X is a *solution* of the given equation, as seen by replacing X by 3, obtaining $2(3) + 3 = 9$, or $9 = 9$, which is an *identity*. The process of obtaining solutions of an equation is called *solving* the equation.

The above ideas can be extended to finding solutions of two equations in two unknowns, three equations in three unknowns, etc. Such equations are called *simultaneous equations* (see Problem 1.30).

INEQUALITIES

The symbols $<$ and $>$ mean "less than" and "greater than," respectively. The symbols $\leq$ and $\geq$ mean "less than or equal to" and "greater than or equal to," respectively. They are known as *inequality symbols.*

EXAMPLE 23. $3 < 5$ is read "3 is less than 5."

EXAMPLE 24. $5 > 3$ is read "5 is greater than 3."

EXAMPLE 25. $X < 8$ is read "X is less than 8."

EXAMPLE 26. $X \geq 10$ is read "X is greater than or equal to 10."

EXAMPLE 27. $4 < Y \leq 6$ is read "4 is less than Y, which is less than or equal to 6," or "Y is between 4 and 6, excluding 4 but including 6," or "Y is greater than 4 and less than or equal to 6."

Relations involving inequality symbols are called *inequalities*. Just as we speak of members of an equation, so we can speak of *members of an inequality*. Thus in the inequality $4 < Y \leq 6$, the members are 4, Y, and 6.

A valid inequality remains valid when:

1. The same number is added to or subtracted from each member.

EXAMPLE 28. Since $15 > 12$, $15+3 > 12+3$ (i.e., $18 > 15$) and $15-3 > 12-3$ (i.e., $12 > 9$).

2. Each member is multiplied or divided by the same *positive* number.

EXAMPLE 29. Since $15 > 12$, $(15)(3) > (12)(3)$ (i.e., $45 > 36$) and $15/3 > 12/3$ (i.e., $5 > 4$).

3. Each member is multiplied or divided by the same *negative* number, provided that the inequality symbols are reversed.

EXAMPLE 30. Since $15 > 12$, $(15)(-3) < (12)(-3)$ (i.e., $-45 < -36$) and $15/(-3) < 12/(-3)$ (i.e., $-5 < -4$).

LOGARITHMS

Every positive number N can be expressed as a power of 10; that is, we can always find p such that $N = 10^p$. We call p the *logarithm of N to the base 10*, or the *common logarithm of N*, and we write briefly $p = \log N$, or $p = \log_{10} N$. For example, since $1000 = 10^3$, $\log 1000 = 3$. Similarly, since $0.01 = 10^{-2}$, $\log 0.01 = -2$.

When N is a number between 1 and 10 (i.e., 10^0 and 10^1), $p = \log N$ is a number between 0 and 1 and can be found from the table of logarithms in Appendix VII.

EXAMPLE 31. To find log 2.36 in Appendix VII, we glance down the *left* column headed N until we come to the first two digits, 23. Then we proceed *right* to the column headed 6. We find the entry 3729. Thus $\log 2.36 = 0.3729$ (i.e., $2.36 = 10^{0.3729}$).

The logarithms of *all* positive numbers can be found from the logarithms of numbers between 1 and 10.

EXAMPLE 32. From Example 31, $2.36 = 10^{0.3729}$. Multiplying successively by 10, we have $23.6 = 10^{1.3729}$, $236 = 10^{2.3729}$, $2360 = 10^{3.3729}$, and so forth. Thus $\log 2.36 = 0.3729$, $\log 23.6 = 1.3729$, $\log 236 = 2.3729$, and $\log 2360 = 3.3729$.

EXAMPLE 33. Since $2.36 = 10^{0.03729}$, we find on successive divisions by 10 that $0.236 = 10^{0.3729-1} = 10^{-0.6271}$, $0.0236 = 10^{0.3729-2} = 10^{-1.6271}$, and so forth.

Often we write $0.3729 - 1$ as $9.3729 - 10$, or $\bar{1}.3729$; and $0.3729 - 2$ as $8.3729 - 10$, or $\bar{2}.3729$; and so forth. With this notation, we have

$$\log 0.236 = 9.3729 - 10 = \bar{1}.3729 = -0.6271$$

$$\log 0.0236 = 8.3729 - 10 = \bar{2}.3729 = -1.6271$$

and so forth.

The decimal part .3729 in all these logarithms is called the *mantissa*. The remaining part, before the decimal of the mantissa [i.e., 1, 2, 3, and $\bar{1}$ and $\bar{2}$ (or $9 - 10$ and $8 - 10$, respectively)] is called the *characteristic*.

The following rules are easily demonstrated:

1. For a number greater than 1, the characteristic is positive and is one *less* than the number of digits before the decimal point.

EXAMPLE 34. The characteristics of the logarithms of 2360, 236, 23.6, and 2.36 are 3, 2, 1, and 0, and the required logarithms are 3.3729, 2.3729, 1.3729, and 0.3729.

2. For a number less than 1, the characteristic is negative and is one *more* than the number of zeros immediately following the decimal point.

EXAMPLE 35. The characteristics of the logarithms of 0.236, 0.0236, and 0.00236 are −1, −2, and −3, and the required logarithms are $\bar{1}.3729$, $\bar{2}.3729$, and $\bar{3}.3729$, or 9.3729 − 10, 8.3729 − 10, and 7.3729 − 10, respectively.

If logarithms of four-digit numbers (such as 2.364 and 758.2) are required, the method of *interpolation* can be used (see Problem 1.36).

ANTILOGARITHMS

In the exponential form $2.36 = 10^{0.3729}$, the number 2.36 is called the *antilogarithm* of 0.3729, or antilog 0.3729. It is the number whose logarithm is 0.3729. It follows at once that antilog 1.3729 = 23.6, antilog 2.3729 = 236, antilog 3.3729 = 2360, antilog 9.3729 − 10 = antilog $\bar{1}.3729$ = 0.236, and antilog 8.3729 − 10 = antilog $\bar{2}.3729$ = 0.0236. The antilog of any number can be found by reference to Appendix VII.

EXAMPLE 36. To find antilog 8.6284 − 10, look up the mantissa .6284 in the body of the table. Since it appears in the row marked 42 and the column headed 5, the required digits of the number are 425. Since the characteristic is 8 − 10, the required number is 0.0425.

Similarly, antilog 3.6284 = 4250, and antilog 5.6284 = 425,000.

If the mantissas are not found in Appendix VII, interpolation can be used (see Problem 1.37).

COMPUTATIONS USING LOGARITHMS

These computations employ the following properties:

$$\log MN = \log M + \log N$$

$$\log \frac{M}{N} = \log M - \log N$$

$$\log M^p = p \log M$$

By combining these results, we find, for example,

$$\log \frac{A^p B^q C^r}{D^s E^t} = p \log A + q \log B + r \log C - s \log D - t \log E$$

See Problems 1.38 to 1.45.

Solved Problems

VARIABLES

1.1 State which of the following represent discrete data and which represent continuous data:

(*a*) Numbers of shares sold each day in the stock market

(*b*) Temperatures recorded every half hour at a weather bureau

(*c*) Lifetimes of television tubes produced by a company

(*d*) Yearly incomes of college professors

(*e*) Lengths of 1000 bolts produced in a factory

SOLUTION

(*a*) Discrete; (*b*) continuous; (*c*) continuous; (*d*) discrete; (*e*) continuous.

1.2 Give the domain of each of the following variables, and state whether the variables are continuous or discrete:

(*a*) Number G of gallons (gal) of water in a washing machine

(*b*) Number B of books on a library shelf

(*c*) Sum S of points obtained in tossing a pair of dice

(*d*) Diameter D of a sphere

(*e*) Country C in Europe

SOLUTION

(*a*) *Domain:* Any value from 0 gal to the capacity of the machine. *Variable:* Continuous.

(*b*) *Domain:* 0, 1, 2, 3, ... up to the largest number of books that can fit on a shelf. *Variable:* Discrete.

(*c*) *Domain:* Points obtained on a single die can be 1, 2, 3, 4, 5, or 6. Hence the sum of points on a pair of dice can be 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, or 12, which is the domain of S. *Variable:* Discrete.

(*d*) *Domain:* If we consider a point as a sphere of zero diameter, the domain of D is all values from zero upward. *Variable:* Continuous.

(*e*) *Domain:* England, France, Germany, etc., which can be represented numerically by 1, 2, 3, etc. *Variable:* Discrete.

ROUNDING OF DATA

1.3 Round each of the following numbers to the indicated accuracy:

(*a*)	48.6	nearest unit	(*f*)	143.95	nearest tenth
(*b*)	136.5	nearest unit	(*g*)	368	nearest hundred
(*c*)	2.484	nearest hundredth	(*h*)	24,448	nearest thousand
(*d*)	0.0435	nearest thousandth	(*i*)	5.56500	nearest hundredth
(*e*)	4.50001	nearest unit	(*j*)	5.56501	nearest hundredth

SOLUTION

(*a*) 49; (*b*) 136; (*c*) 2.48; (*d*) 0.044; (*e*) 5; (*f*) 144.0; (*g*) 400; (*h*) 24,000; (*i*) 5.56; (*j*) 5.57.

1.4 Add the numbers 4.35, 8.65, 2.95, 12.45, 6.65, 7.55, and 9.75 (*a*) directly, (*b*) by rounding to the nearest tenth according to the "even integer" convention, and (*c*) by rounding so as to increase the digit before the 5.

SOLUTION

	(*a*)		(*b*)		(*c*)
	4.35		4.4		4.4
	8.65		8.6		8.7
	2.95		3.0		3.0
	12.45		12.4		12.5
	6.65		6.6		6.7
	7.55		7.6		7.6
	9.75		9.8		9.8
Total	52.35	Total	52.4	Total	52.7

Note that procedure (*b*) is superior to procedure (*c*) because *cumulative rounding errors* are minimized in procedure (*b*).