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symposia on theoretical physics

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Edited by

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Introduction

The second volume of this series is devoted to the Proceedings of the Second Anniversary Symposium under the chairmanship of the Niels Bohr Visiting Professor of the year – Professor L. Rosenfeld, Deputy Director of NORDITA, Copenhagen, and the Editor of *Nuclear Physics*. With particular appropriateness, the Symposium was inaugurated by the Honorable C. Subramaniam, Union Cabinet Minister, the founding father of the Institute.

The meeting was characterized by two features: (1) the enlargement of the scope of the discussions in theoretical physics, with the inclusion of many-body problems and statistical mechanics: (2) Seminars on pure mathematics, stimulated by the presence and participation of Professor Marshall H. Stone of Chicago as the First Ramanujan Visiting Professor at the Institute.

The year 1963 marked a new stage in the development of high-energy physics – the first successes of *SU* (3) symmetry and the eightfold way had such an impact on the scientific world that the hard, unyielding domain of strong interactions was now again open to exploration. The volume opens with two significant lectures by Sudarshan and O'Raifeartaigh on fundamental problems relating to internal symmetries. The theory of Regge poles, after its initial triumph, met with rough weather, the nature and intensity of which can be realized from the series of discussions in this volume.

In statistical mechanics, we had the privilege of having with us three leading participants, Zumino of New York, Dewitt from Berkeley, and Mohling from Colorado, whose contributions to this volume summarize their academic program during their stay at our Institute.

Marshall Stone's lecture on some current trends in mathematical research was perhaps the best possible mode of initiating studies in pure mathematics at the Institute. It was followed by a systematic account of semigroup methods in mathematical physics by Bharucha-Reid. The seminar talk on the mathematical problems of cascade theory by Srinivasan is the first in a series on stochastic processes, the rest of which will follow in succeeding volumes.

As part of the surging current of scientific literature, this volume, we hope, will convey the "integrating power of mathematics"* and the "universality of physical laws."

Alladi Ramakrishnan

* I am indebted for this phrase to Professor M. J. Lighthill, who referred to the role of mathematics in his inaugural lecture entitled "Waves in Fluids" on assuming the Royal Society Research Professorship in 1965.

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Origin of Internal Symmetries

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1. INTRODUCTION

Symmetry groups in physics seem to belong to two classes: the so-called relativity (or frame) groups, which may be called the *external symmetry groups*, defined by the geometric relations between "inertial" systems for which the laws of physics are the same, and the *internal symmetry groups*. We call the symmetry "internal" because we see only its manifestations; there is no primitive geometric characterization of the symmetry group from any fundamental dynamic principle. We shall try to see to what extent a dynamic principle can be expected to generate a symmetry group.

In this connection, two sets of quantum numbers can be distinguished—the additive quantum numbers (such as the third components of $\vec{J}$ and $\vec{T}$), which are algebraically additive, and the nonadditive ("vector") quantum numbers (such as the total angular momentum $\vec{J}$, total isotopic spin $\vec{T}$, etc.), which obey vector laws of addition and multiplication. One fact worth recalling is that the irreducible representations of a compact group are finite dimensional and are equivalent to unitary representations.

We naturally ask about the properties of particles in interaction. Suppose, for example, we consider the following (virtual) reaction:

$$N \longrightarrow N + \pi$$

From the $(NN\pi)$ vertex, we can write the invariant interactions (by using the Clebsch–Gordan coefficients) and obtain the following relationships between the various $(NN\pi)$ coupling constants (g) and among the various virtual transition probabilities:

$$
\begin{aligned}
g_{pp\pi^0} &= -g_{nn\pi^0} \\
&= \frac{1}{\sqrt{2}} g_{pn\pi^+} \\
&= \frac{1}{\sqrt{2}} g_{np\pi^-} \\
\Gamma(n \to n + \pi^0) &= \Gamma(p \to p + \pi^0) \qquad (1) \\
&= \frac{1}{2} \Gamma(p \to n + \pi^+) \\
&= \frac{1}{2} \Gamma(n \to p + \pi^-)
\end{aligned}
$$

where p and n refer to the proton and neutron, respectively. From these we conclude that for the total widths

$$\Gamma(n \to \text{any particle}) = \Gamma(p \to \text{any particle})$$

We know that the multiplet structures displayed by the known particles are consequences of the (postulated) existence of an internal symmetry. We therefore ask whether the existence of the multiplet structure conversely implies an internal symmetry.

Recently, there have been a good many attempts to explain the internal symmetry by some direct dynamic calculations. If we start with a multiplet of N vector mesons of equal masses and assume that the interactions among these vector mesons are essentially trilinear in character, we can make a dynamic scheme in terms of a straightforward and self-consistent bootstrap mechanism between these (equally massive) vector mesons. One such attempt was made by Capps,[1] who found that the interactions among these N equally massive vector mesons obey unitary symmetry (i.e., invariance under the group SU_3). Capps was surprised to find this relationship between unitary symmetry and a self-consistent bootstrap calculation. It looked as though unitary symmetry could be derived from first principles. However, it is possible that the symmetry would have emerged from the assumption of the existence of a multiplet degenerate in mass before the interaction and the postulate that this multiplet structure is preserved even in the presence of interactions, so that the particles exhibit the same mass degeneracy even in the presence of the interaction if they have equal masses. Such arguments have been used by Sakurai,[2] who tried to prove that the emergence of the symmetry is not a consequence of a sophisticated dynamic calculation, but rather the immediate consequence of the assumptions:

1. Equality of masses of the particles.
2. Existence of a degenerate multiplet before the interaction.
3. The presence of interaction not altering the equality of masses (or the multiplicity of the particles).

He directly shows, as an example, that if we equate the contributions of certain self-energy diagrams we arrive at the required symmetry.

Thus, if we equate the second-order self-energy of the nucleons (assuming equality of masses before the interaction) as shown in Fig. 1a and that of the pions as shown in Fig. 1b, we obtain

$$2g_{pp\pi^0}^2 = 2g_{nn\pi^0}^2 = g_{pn\pi^+}^2 = g_{np\pi^-}^2$$

In this calculation the multiplets are treated "on the same footing," and the "total width" for each component comes out to be equal. By taking fourth-order diagrams also, we can further deduce

$$g_{pp\pi^0} = -g_{nn\pi^0}$$

Thus, the symmetries may well be explained if we assume the equality of masses and multiplicity of the particles and postulate that these properties remain unchanged even in the presence of interaction.

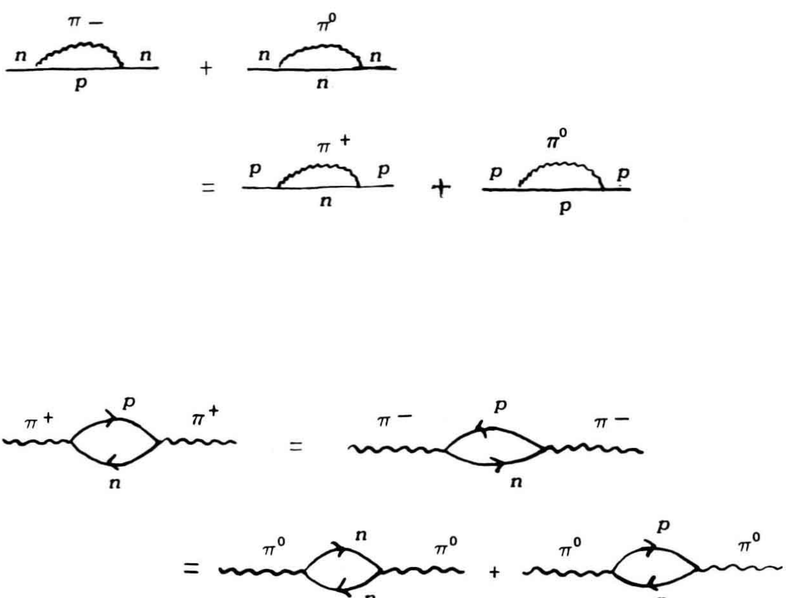

Fig. 1

2. GAUGE FIELDS

The consequence of the existence of symmetries and the postulated invariance of interactions under the gauge transformations of the second kind is the existence of vector gauge fields coupled linearly to conserved quantities (such as electric charge, etc.).

Unlike the electromagnetic field, which by itself is neutral and interacts only with charged fields (and is thus coupled to the electric current), gauge vector fields may themselves carry the properties. The isospin gauge field, for instance, itself carries the properties of isospin, and it is hence nonlinearly self-coupled. We may even consider a situation in which the gauge vector field alone carries isotopic spin and is consequently self-coupled. Thus, if we can write $L = j^{\mu} A_{\mu}$ for the Lagrangian of the electromagnetic interaction, where A_{μ} is the electromagnetic field and j^{μ} is the current to which it is coupled, what can we write for the Lagrangian of the interaction of the gauge vector field? Since the gauge vector field is coupled to itself, we naturally expect that the interaction can be written as a product of these fields *B*. Then how many *B* can enter the product? The simplest possibility (which we may take to be basic) is the trilinear interaction between vector particles. This is because the current is bilinear in *B* field and coupled to another *B* field, making a trilinear vertex.

Cutkosky[3] has given a simple model in which he assumes that there are a number (N) of vector mesons which have the same mass, i.e., he assumes a multiplet structure. Then, with a number of additional plausible assumptions, he shows that a Lie group could be associated with these particles. The assumptions made are:

1. The vector mesons arise as self-consistent bound states of pairs of vector mesons.

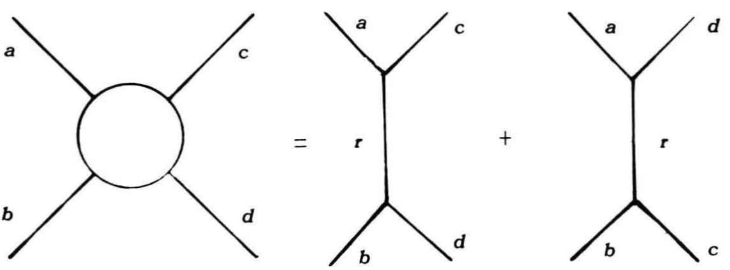

Fig. 2

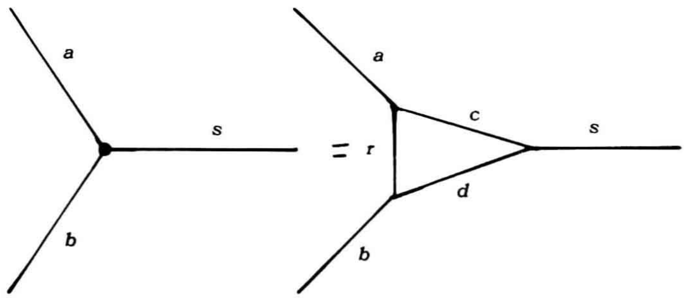

Fig. 3

2. The binding force is mediated by the exchange of single vector mesons; for example, the long-range part of the force is given by the one-particle exchange diagrams shown in Fig. 2.
3. The renormalized coupling constants are well approximated by the simplest irreducible vertex part, with the "bare-coupling constants" set equal to zero, as shown in Fig. 3.
4. Parity is conserved in strong interactions, and strong interactions are invariant under charge conjugation.
5. Electric charge is conserved.
6. The dependence of the vertex on the internal labels F_{abc} is antisymmetric in all pairs of indices.

If we represent the particles by real vector fields $B_\mu(\mu = 1 \ldots N)$, the invariant interaction has the form

$$F_{abc}\, B_a\, B_b\, B_c$$

and F_{abc} is antisymmetric. We then look for the eigenfunctions of F_{abc}. The Born-approximation scattering amplitude is proportional to

$$V_{ab,cd} = (F_{adr}\, F_{bcr} - F_{acr}\, F_{bdr}) \tag{2}$$

corresponding to the two diagrams in Fig. 2 and taking into account the antisymmetric nature of F.

Since all the particles which are together, and also all the exchanged particles, have the same mass (which we have normalized to unity), it is clear that we can obtain N degenerate bound states only if V has N degenerate eigenvalues. Also, the F themselves must be eigenvectors of V, in view of postulate (3):

$$V_{ab,cd}\, F_{cds} = \lambda F_{abs} \tag{3}$$