

ASLAM KASSIMALI



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STRUCTURAL ANALYSIS



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ASLAM KASSIMALI

Southern Illinois University — Carbondale



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In memory of *Ami*



P R E F A C E

The objective of this book is to develop an understanding of the basic principles of structural analysis. Emphasizing the intuitive classical approach, *Structural Analysis* covers the analysis of statically determinate and indeterminate beams, trusses, and rigid frames. It also presents an introduction to the matrix analysis of structures.

The book is divided into three parts. Part I presents a general introduction to the subject of structural engineering. It includes a chapter devoted entirely to the topic of loads because attention to this important topic is generally lacking in many civil engineering curricula. Part II, consisting of Chapters 3 through 10, covers the analysis of statically determinate beams, trusses, and rigid frames. The chapters on deflections (Chapters 6 and 7) are placed before those on influence lines (Chapters 8 and 9), so that influence lines for deflections can be included in the latter chapters. This part also contains a chapter on the analysis of symmetric structures (Chapter 10). Part III of the book, Chapters 11 through 17, covers the analysis of statically indeterminate structures. The format of the book is flexible to enable instructors to emphasize topics that are consistent with the goals of the course.

Each chapter of the book begins with an introductory section defining its objective and ends with a summary section outlining its salient features. An important general feature of the book is the inclusion of step-by-step procedures for analysis to enable students to make an easier transition from theory to problem solving. Numerous solved examples are provided to illustrate the application of the fundamental concepts.

A diskette containing computer software for the analysis of plane frames, continuous beams, and trusses is attached to the back cover. This interactive software can be used to simulate a variety of structural and loading configurations and to determine cause versus effect relationships between loading and various structural parameters, thereby enhancing the students' understanding of the behavior of structures. A solutions manual, containing complete solutions to text exercises, is also available for the instructor.

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Aslam Kassimali



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F_{AD} , or at joint C , which also has two unknowns, F_{BC} and F_{CD} . Let us start with joint A . The free-body diagram of this joint is shown in Fig. 4.16(d). Although we could use the sines and cosines of the angles of inclination of inclined members in writing the joint equilibrium equations, it is usually more convenient to use the slopes of the inclined members instead. The slope of an inclined member is simply the ratio of the vertical projection of the length of the member to the horizontal projection of its length. For example, from Fig. 4.16(a), we can see that member CD of the truss under consideration rises 20 ft in the vertical direction over a horizontal distance of 15 ft. Therefore, the slope of this member is 20:15, or 4:3. Similarly, we can see that the slope of member AD is 1:1. The slopes of inclined members thus determined from the dimensions of the truss are usually depicted on the diagram of the truss by means of small right-angled triangles drawn on the inclined members, as shown in Fig. 4.16(a).

Refocusing our attention on the free-body diagram of joint A in Fig. 4.16(d), we determine the unknowns F_{AB} and F_{AD} by applying the two equilibrium equations:

$$\begin{aligned}
 + \uparrow \sum F_y = 0 & \quad 34 + \frac{1}{\sqrt{2}} F_{AD} = 0 & \quad F_{AD} = -48.08 \text{ k} \\
 & & \quad = 48.08 \text{ k (C)} \\
 + \rightarrow \sum F_x = 0 & \quad 28 - \frac{1}{\sqrt{2}} (48.08) + F_{AB} = 0 & \quad F_{AB} = +6 \text{ k} \\
 & & \quad = 6 \text{ k (T)}
 \end{aligned}$$

Note that the equilibrium equations were applied in such an order so that each equation contains only one unknown. The negative answer for F_{AD} indicates that the member AD is in compression instead of in tension, as initially assumed, whereas the positive answer for F_{AB} indicates that the assumed sense of axial force (tension) in member AB was correct.

Next, we draw the free-body diagram of joint B , as shown in Fig. 4.16(e), and determine F_{BC} and F_{BD} as follows:

$$\begin{aligned}
 + \rightarrow \sum F_x = 0 & \quad -6 + F_{BC} = 0 & \quad F_{BC} = +6 \text{ k, or } F_{BC} = 6 \text{ k (T)} \\
 + \uparrow \sum F_y = 0 & \quad -F_{BD} = 0 & \quad F_{BD} = 0
 \end{aligned}$$

Applying the equilibrium equation $\sum F_x = 0$ to the free-body diagram of joint C (Fig. 4.16(f)), we obtain

$$+ \rightarrow \sum F_x = 0 \quad -6 + \frac{3}{5} F_{CD} = 0 \quad F_{CD} = +10 \text{ k, or } F_{CD} = 10 \text{ k (C)}$$

We have determined all the member forces, so the three remaining equilibrium equations, $\sum F_y = 0$ at joint C and $\sum F_x = 0$ and $\sum F_y = 0$ at joint D , can be used to check our calculations. Thus, at joint C ,

$$+ \uparrow \sum F_y = 8 - \frac{4}{5} (10) = 0 \quad \text{Checks}$$

and at joint D (Fig. 4.16(g)),

$$+ \rightarrow \sum F_x = -28 + \frac{1}{\sqrt{2}}(48.08) - \frac{3}{5}(10) = 0 \quad \text{Checks}$$

$$+ \uparrow \sum F_y = \frac{1}{\sqrt{2}}(48.08) - 42 + \frac{4}{5}(10) = 0 \quad \text{Checks}$$

In the preceding paragraphs, the analysis of a truss has been carried out by drawing a free-body diagram and writing the two equilibrium equations for each of its joints. However, the analysis of trusses can be considerably expedited if we can determine some (preferably all) of the member forces by inspection—that is, without drawing the joint free-body diagrams and writing the equations of equilibrium. This approach can be conveniently used for the joints at which at least one of the two unknown forces is acting in the horizontal or vertical direction. When both of the unknown forces at a joint have inclined directions, it usually becomes necessary to draw the free-body diagram of the joint and determine the unknowns by solving the equilibrium equations simultaneously. To illustrate this procedure, consider again the truss of Fig. 4.16(a). The free-body diagram of the entire truss is shown in Fig. 4.16(c), which also shows the support reactions computed previously. Focusing our attention on joint A in this figure, we observe that in order to satisfy the equilibrium equation $\sum F_y = 0$ at joint A , the vertical component of F_{AD} must push downward into the joint with a magnitude of 34 k to balance the vertically upward reaction of 34 k. The fact that member AD is in compression is indicated on the diagram of the truss by drawing arrows near joints A and D pushing into the joints, as shown in Fig. 4.16(c). Because the magnitude of the vertical component of F_{AD} has been found to be 34 k and since the slope of member AD is 1:1, the magnitude of the horizontal component of F_{AD} must also be 34 k; therefore, the magnitude of the resultant force F_{AD} is $F_{AD} = \sqrt{(34)^2 + (34)^2} = 48.08$ k. The components of F_{AD} , as well as F_{AD} itself are shown on the corresponding sides of a right-angled triangle drawn on member AD , as shown in Fig. 4.16(c). With the horizontal component of F_{AD} now known, we observe (from Fig. 4.16(c)) that in order to satisfy the equilibrium equation $\sum F_x = 0$ at joint A , the force in member AB (F_{AB}) must pull to the right on the joint with a magnitude of 6 k to balance the horizontal component of F_{AD} of 34 k acting to the left and the horizontal reaction of 28 k acting to the right. The magnitude of F_{AB} is now written on member AB , and the arrows, pulling away on the joints, are drawn near joints A and B to indicate that member AB is in tension.

Next, we focus our attention on joint B of the truss. It should be obvious from Fig. 4.16(c) that in order to satisfy $\sum F_y = 0$ at B , the force in member BD must be zero. To satisfy $\sum F_x = 0$, the force in member BC must have a magnitude of 6 k, and it must pull to the right on joint B , indicating tension in member BC . This latest information is recorded in

the diagram of the truss in Fig. 4.16(c). Considering now the equilibrium of joint C , we can see from the figure that in order to satisfy $\sum F_y = 0$, the vertical component of F_{CD} must push downward into the joint with a magnitude of 8 k to balance the vertically upward reaction of 8 k. Thus, member CD is in compression. Since the magnitude of the vertical component F_{CD} is 8 k and since the slope of member CD is 4:3, the magnitude of the horizontal component of F_{CD} is equal to $(3/4)(8) = 6$ k; therefore, the magnitude of F_{CD} itself is $F_{CD} = \sqrt{(6)^2 + (8)^2} = 10$ k. Having determined all the member forces, we check our computations by applying the equilibrium equations $\sum F_x = 0$ at joint C and $\sum F_x = 0$ and $\sum F_y = 0$ at joint D . The horizontal and vertical components of the member forces are already available in Fig. 4.16(c), so we can easily check by inspection to find that these equations of equilibrium are indeed satisfied. We must recognize that all the arrows shown on the diagram of the truss in Fig. 4.16(c) indicate forces acting at the joints (not at the ends of the members).

Identification of Zero-Force Members

Because trusses are usually designed to support several different loading conditions, it is not uncommon to find members with zero forces in them when a truss is being analyzed for a particular loading condition. Zero-force members are also added to trusses to brace compression members against buckling and slender tension members against vibrating. The analysis of trusses can be expedited if we can identify the zero-force members by inspection. Two common types of member arrangements that result in zero-force members are the following:

1. If only two noncollinear members are connected to a joint that has no external loads or reactions applied to it, then the force in both members is zero.
2. If three members, two of which are collinear, are connected to a joint that has no external loads or reactions applied to it, then the force in the member that is not collinear is zero.

The first type of arrangement is shown in Fig. 4.17(a). It consists of two noncollinear members AB and AC connected to a joint A . Note that no external loads or reactions are applied to the joint. From this figure we can see that in order to satisfy the equilibrium equation $\sum F_y = 0$, the y component of F_{AB} must be zero; therefore, $F_{AB} = 0$. Because the x component of F_{AB} is zero, the second equilibrium equation, $\sum F_x = 0$, can be satisfied only if F_{AC} is also zero.

The second type of arrangement is shown in Fig. 4.17(b), and it consists of three members, AB , AC , and AD , connected together at a joint A . Note that two of the three members, AB and AD , are collinear. We can see from the figure that since there is no external load or reaction applied to the joint to balance the y component of F_{AC} , the equilibrium equation $\sum F_y = 0$ can be satisfied only if F_{AC} is zero.

