VOLUME
II

The Feynman

LECTURES ON PHYSICS

FEYNMAN • LEIGHTON • SANDS

The Feynman LECTURES ON PHYSICS

MAINLY ELECTROMAGNETISM AND MATTER

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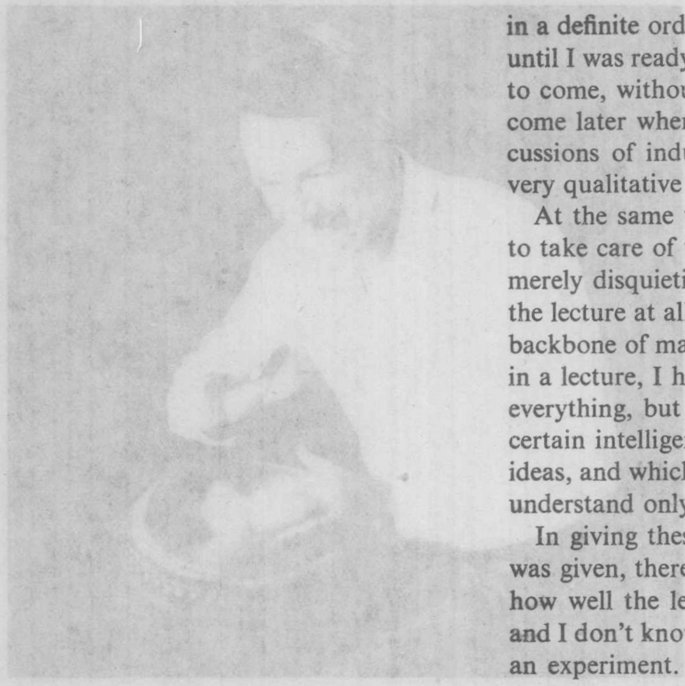
Feynman's Preface

These are the lectures in physics that I gave last year and the year before to the freshman and sophomore classes at Caltech. The lectures are, of course, not verbatim—they have been edited, sometimes extensively and sometimes less so. The lectures form only part of the complete course. The whole group of 180 students gathered in a big lecture room twice a week to hear these lectures and then they broke up into small groups of 15 to 20 students in recitation sections under the guidance of a teaching assistant. In addition, there was a laboratory session once a week.

The special problem we tried to get at with these lectures was to maintain the interest of the very enthusiastic and rather smart students coming out of the high schools and into Caltech. They have heard a lot about how interesting and exciting physics is—the theory of relativity, quantum mechanics, and other modern ideas. By the end of two years of our previous course, many would be very discouraged because there were really very few grand, new, modern ideas presented to them. They were made to study inclined planes, electrostatics, and so forth, and after two years it was quite stultifying. The problem was whether or not we could make a course which would save the more advanced and excited student by maintaining his enthusiasm.

The lectures here are not in any way meant to be a survey course, but are very serious. I thought to address them to the most intelligent in the class and to make sure, if possible, that even the most intelligent student was unable to completely encompass everything that was in the lectures—by putting in suggestions of applications of the ideas and concepts in various directions outside the main line of attack. For this reason, though, I tried very hard to make all the statements as accurate as possible, to point out in every case where the equations and ideas fitted into the body of physics, and how—when they learned more—things would be modified. I also felt that for such students it is important to indicate what it is that they should—if they are sufficiently clever—be able to understand by deduction from what has been said before, and what is being put in as something new. When new ideas came in, I would try either to deduce them if they were deducible, or to explain that it *was* a new idea which hadn't any basis in terms of things they had already learned and which was not supposed to be provable—but was just added in.

At the start of these lectures, I assumed that the students knew something when they came out of high school—such things as geometrical optics, simple chemistry ideas, and so on. I also didn't see that there was any reason to make the lectures



in a definite order, in the sense that I would not be allowed to mention something until I was ready to discuss it in detail. There was a great deal of mention of things to come, without complete discussions. These more complete discussions would come later when the preparation became more advanced. Examples are the discussions of inductance, and of energy levels, which are at first brought in in a very qualitative way and are later developed more completely.

At the same time that I was aiming at the more active student, I also wanted to take care of the fellow for whom the extra fireworks and side applications are merely disquieting and who cannot be expected to learn most of the material in the lecture at all. For such students I wanted there to be at least a central core or backbone of material which he *could* get. Even if he didn't understand everything in a lecture, I hoped he wouldn't get nervous. I didn't expect him to understand everything, but only the central and most direct features. It takes, of course, a certain intelligence on his part to see which are the central theorems and central ideas, and which are the more advanced side issues and applications which he may understand only in later years.

In giving these lectures there was one serious difficulty: in the way the course was given, there wasn't any feedback from the students to the lecturer to indicate how well the lectures were going over. This is indeed a very serious difficulty, and I don't know how good the lectures really are. The whole thing was essentially an experiment. And if I did it again I wouldn't do it the same way—I hope I *don't* have to do it again! I think, though, that things worked out—so far as the physics is concerned—quite satisfactorily in the first year.

In the second year I was not so satisfied. In the first part of the course, dealing with electricity and magnetism, I couldn't think of any really unique or different way of doing it—of any way that would be particularly more exciting than the usual way of presenting it. So I don't think I did very much in the lectures on electricity and magnetism. At the end of the second year I had originally intended to go on, after the electricity and magnetism, by giving some more lectures on the properties of materials, but mainly to take up things like fundamental modes, solutions of the diffusion equation, vibrating systems, orthogonal functions, . . . developing the first stages of what are usually called "the mathematical methods of physics." In retrospect, I think that if I were doing it again I would go back to that original idea. But since it was not planned that I would be giving these lectures again, it was suggested that it might be a good idea to try to give an introduction to the quantum mechanics—what you will find in Volume III.

It is perfectly clear that students who will major in physics can wait until their third year for quantum mechanics. On the other hand, the argument was made that many of the students in our course study physics as a background for their primary interest in other fields. And the usual way of dealing with quantum mechanics makes that subject almost unavailable for the great majority of students because they have to take so long to learn it. Yet, in its real applications—especially in its more complex applications, such as in electrical engineering and chemistry—the full machinery of the differential equation approach is not actually used. So I tried to describe the principles of quantum mechanics in a way which wouldn't require that one first know the mathematics of partial differential equations. Even for a physicist I think that is an interesting thing to try to do—to present quantum mechanics in this reverse fashion—for several reasons which may be apparent in the lectures themselves. However, I think that the experiment in the quantum mechanics part was not completely successful—in large part because I really did not have enough time at the end (I should, for instance, have had three or four more lectures in order to deal more completely with such matters as energy bands and the spatial dependence of amplitudes). Also, I had never presented the subject this way before, so the lack of feedback was particularly serious. I now believe the quantum mechanics should be given at a later time. Maybe I'll have a chance to do it again someday. Then I'll do it right.

The reason there are no lectures on how to solve problems is because there were recitation sections. Although I did put in three lectures in the first year on how to solve problems, they are not included here. Also there was a lecture on inertial guidance which certainly belongs after the lecture on rotating systems, but which was, unfortunately, omitted. The fifth and sixth lectures are actually due to Matthew Sands, as I was out of town.

The question, of course, is how well this experiment has succeeded. My own point of view—which, however, does not seem to be shared by most of the people who worked with the students—is pessimistic. I don't think I did very well by the students. When I look at the way the majority of the students handled the problems on the examinations, I think that the system is a failure. Of course, my friends point out to me that there were one or two dozen students who—very surprisingly—understood almost everything in all of the lectures, and who were quite active in working with the material and worrying about the many points in an excited and interested way. These people have now, I believe, a first-rate background in physics—and they are, after all, the ones I was trying to get at. But then, "The power of instruction is seldom of much efficacy except in those happy dispositions where it is almost superfluous." (Gibbons)

Still, I didn't want to leave any student completely behind, as perhaps I did. I think one way we could help the students more would be by putting more hard work into developing a set of problems which would elucidate some of the ideas in the lectures. Problems give a good opportunity to fill out the material of the lectures and make more realistic, more complete, and more settled in the mind the ideas that have been exposed.

I think, however, that there isn't any solution to this problem of education other than to realize that the best teaching can be done only when there is a direct individual relationship between a student and a good teacher—a situation in which the student discusses the ideas, thinks about the things, and talks about the things. It's impossible to learn very much by simply sitting in a lecture, or even by simply doing problems that are assigned. But in our modern times we have so many students to teach that we have to try to find some substitute for the ideal. Perhaps my lectures can make some contribution. Perhaps in some small place where there are individual teachers and students, they may get some inspiration or some ideas from the lectures. Perhaps they will have fun thinking them through—or going on to develop some of the ideas further.

RICHARD P. FEYNMAN

June, 1963

Foreword

For some forty years Richard P. Feynman focussed his curiosity on the mysterious workings of the physical world, and bent his intellect to searching out the order in its chaos. Now, he has given two years of his ability and his energy to his Lectures on Physics for beginning students. For them he has distilled the essence of his knowledge, and has created in terms they can hope to grasp a picture of the physicist's universe. To his lectures he has brought the brilliance and clarity of his thought, the originality and vitality of his approach, and the contagious enthusiasm of his delivery. It was a joy to behold.

The first year's lectures formed the basis for the first volume of this set of books. We have tried in this the second volume to make some kind of a record of a part of the second year's lectures—which were given to the sophomore class during the 1962–1963 academic year. The rest of the second year's lectures will make up Volume III.

Of the second year of lectures, the first two-thirds were devoted to a fairly complete treatment of the physics of electricity and magnetism. Its presentation was intended to serve a dual purpose. We hoped, first, to give the students a complete view of one of the great chapters of physics—from the early gropings of Franklin, through the great synthesis of Maxwell, on to the Lorentz electron theory of material properties, and ending with the still unsolved dilemmas of the electromagnetic self-energy. And we hoped, second, by introducing at the outset the calculus of vector fields, to give a solid introduction to the mathematics of field theories. To emphasize the general utility of the mathematical methods, related subjects from other parts of physics were sometimes analyzed together with their electric counterparts. We continually tried to drive home the generality of the mathematics. ("The same equations have the same solutions.") And we emphasized this point by the kinds of exercises and examinations we gave with the course.

Following the electromagnetism there are two chapters each on elasticity and fluid flow. In the first chapter of each pair, the elementary and practical aspects are treated. The second chapter on each subject attempts to give an overview of the whole complex range of phenomena which the subject can lead to. These four chapters can well be omitted without serious loss, since they are not at all a necessary preparation for Volume III.

The last quarter, approximately, of the second year was dedicated to an introduction to quantum mechanics. This material has been put into the third volume.

In this record of the Feynman Lectures we wished to do more than provide a transcription of what was said. We hoped to make the written version as clear an exposition as possible of the ideas on which the original lectures were based. For some of the lectures this could be done by making only minor adjustments of the wording in the original transcript. For others of the lectures a major reworking and rearrangement of the material was required. Sometimes we felt we should add some new material to improve the clarity or balance of the presentation. Throughout the process we benefitted from the continual help and advice of Professor Feynman.

The translation of over 1,000,000 spoken words into a coherent text on a tight schedule is a formidable task, particularly when it is accompanied by the other onerous burdens which come with the introduction of a new course—preparing for recitation sections, and meeting students, designing exercises and examinations, and grading them, and so on. Many hands—and heads—were involved. In some instances we have, I believe, been able to render a faithful image—or a tenderly retouched portrait—of the original Feynman. In other instances we have fallen far short of this ideal. Our successes are owed to all those who helped. The failures, we regret.

As explained in detail in the Foreword to Volume I, these lectures were but one aspect of a program initiated and supervised by the Physics Course Revision Committee (R. B. Leighton, Chairman, H. V. Neher, and M. Sands) at the California Institute of Technology, and supported financially by the Ford Foundation. In addition, the following people helped with one aspect or another of the preparation of textual material for this second volume: T. K. Caughey, M. L. Clayton, J. B. Curcio, J. B. Hartle, T. W. H. Harvey, M. H. Israel, W. J. Karzas, R. W. Kavanagh, R. B. Leighton, J. Mathews, M. S. Plesset, F. L. Warren, W. Whaling, C. H. Wilts, and B. Zimmerman. Others contributed indirectly through their work on the course: J. Blue, G. F. Chapline, M. J. Clauser, R. Dolen, H. H. Hill, and A. M. Title. Professor Gerry Neugebauer contributed in all aspects of our task with a diligence and devotion far beyond the dictates of duty.

The story of physics you find here would, however, not have been, except for the extraordinary ability and industry of Richard P. Feynman.

MATTHEW SANDS

March, 1964

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1

Electromagnetism

Review: Chapter 12, Vol. I, *Characteristics of Force*

1–1 Electrical forces

Consider a force like gravitation which varies predominantly inversely as the square of the distance, but which is about a *billion-billion-billion-billion* times stronger. And with another difference. There are two kinds of "matter," which we can call positive and negative. Like kinds repel and unlike kinds attract—unlike gravity where there is only attraction. What would happen?

A bunch of positives would repel with an enormous force and spread out in all directions. A bunch of negatives would do the same. But an evenly mixed bunch of positives and negatives would do something completely different. The opposite pieces would be pulled together by the enormous attractions. The net result would be that the terrific forces would balance themselves out almost perfectly, by forming tight, fine mixtures of the positive and the negative, and between two separate bunches of such mixtures there would be practically no attraction or repulsion at all.

There is such a force: the electrical force. And all matter is a mixture of positive protons and negative electrons which are attracting and repelling with this great force. So perfect is the balance, however, that when you stand near someone else you don't feel any force at all. If there were even a little bit of unbalance you would know it. If you were standing at arm's length from someone and each of you had *one percent* more electrons than protons, the repelling force would be incredible. How great? Enough to lift the Empire State Building? No! To lift Mount Everest? No! The repulsion would be enough to lift a "weight" equal to that of the entire earth!

With such enormous forces so perfectly balanced in this intimate mixture, it is not hard to understand that matter, trying to keep its positive and negative charges in the finest balance, can have a great stiffness and strength. The Empire State Building, for example, swings only eight feet in the wind because the electrical forces hold every electron and proton more or less in its proper place. On the other hand, if we look at matter on a scale small enough that we see only a few atoms, any small piece will not, usually, have an equal number of positive and negative charges, and so there will be strong residual electrical forces. Even when there are equal numbers of both charges in two neighboring small pieces, there may still be large net electrical forces because the forces between individual charges vary inversely as the square of the distance. A net force can arise if a negative charge of one piece is closer to the positive than to the negative charges of the other piece. The attractive forces can then be larger than the repulsive ones and there can be a net attraction between two small pieces with no excess charges. The force that holds the atoms together, and the chemical forces that hold molecules together, are really electrical forces acting in regions where the balance of charge is not perfect, or where the distances are very small.

You know, of course, that atoms are made with positive protons in the nucleus and with electrons outside. You may ask: "If this electrical force is so terrific, why don't the protons and electrons just get on top of each other? If they want to be in an intimate mixture, why isn't it still more intimate?" The answer has to do with the quantum effects. If we try to confine our electrons in a region that is very close to the protons, then according to the uncertainty principle they must have some mean square momentum which is larger the more we try to confine them. It is this motion, required by the laws of quantum mechanics, that keeps the electrical attraction from bringing the charges any closer together.

There is another question: "What holds the nucleus together"? In a nucleus there are several protons, all of which are positive. Why don't they push themselves apart? It turns out that in nuclei there are, in addition to electrical forces, nonelectrical forces, called nuclear forces, which are greater than the electrical forces and which are able to hold the protons together in spite of the electrical repulsion. The nuclear forces, however, have a short range—their force falls off much more rapidly than $1/r^2$. And this has an important consequence. If a nucleus has too many protons in it, it gets too big, and it will not stay together. An example is uranium, with 92 protons. The nuclear forces act mainly between each proton (or neutron) and its nearest neighbor, while the electrical forces act over larger distances, giving a repulsion between each proton and all of the others in the nucleus. The more protons in a nucleus, the stronger is the electrical repulsion, until, as in the case of uranium, the balance is so delicate that the nucleus is almost ready to fly apart from the repulsive electrical force. If such a nucleus is just "tapped" lightly (as can be done by sending in a slow neutron), it breaks into two pieces, each with positive charge, and these pieces fly apart by electrical repulsion. The energy which is liberated is the energy of the atomic bomb. This energy is usually called "nuclear" energy, but it is really "electrical" energy released when electrical forces have overcome the attractive nuclear forces.

We may ask, finally, what holds a negatively charged electron together (since it has no nuclear forces). If an electron is all made of one kind of substance, each part should repel the other parts. Why, then, doesn't it fly apart? But does the electron have "parts"? Perhaps we should say that the electron is just a point and that electrical forces only act between *different* point charges, so that the electron does not act upon itself. Perhaps. All we can say is that the question of what holds the electron together has produced many difficulties in the attempts to form a complete theory of electromagnetism. The question has never been answered. We will entertain overselves by discussing this subject some more in later chapters.

As we have seen, we should expect that it is a combination of electrical forces and quantum-mechanical effects that will determine the detailed structure of materials in bulk, and, therefore, their properties. Some materials are hard, some are soft. Some are electrical "conductors"—because their electrons are free to move about; others are "insulators"—because their electrons are held tightly to individual atoms. We shall consider later how some of these properties come about, but that is a very complicated subject, so we will begin by looking at the electrical forces only in simple situations. We begin by treating only the laws of electricity—including magnetism, which is really a part of the same subject.

We have said that the electrical force, like a gravitational force, decreases inversely as the square of the distance between charges. This relationship is called Coulomb's law. But it is not precisely true when charges are moving—the electrical forces depend also on the motions of the charges in a complicated way. One part of the force between moving charges we call the *magnetic* force. It is really one aspect of an electrical effect. That is why we call the subject "electromagnetism."

There is an important general principle that makes it possible to treat electromagnetic forces in a relatively simple way. We find, from experiment, that the force that acts on a particular charge—no matter how many other charges there are or how they are moving—depends only on the position of that particular charge, on the velocity of the charge, and on the amount of charge. We can write the force $\boldsymbol{F}$ on a charge q moving with a velocity $\boldsymbol{v}$ as

$$\boldsymbol{F} = q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}). \tag{1.1}$$

We call $\boldsymbol{E}$ the *electric field* and $\boldsymbol{B}$ the *magnetic field* at the location of the charge. The important thing is that the electrical forces from all the other charges in the universe can be summarized by giving just these two vectors. Their values will depend on *where* the charge is, and may change with *time*. Furthermore, if we replace that charge with another charge, the force on the new charge will be just in proportion to the amount of charge so long as all the rest of the charges in the

Lower case Greek letters and commonly used capitals

α		alpha
β		beta
γ	Γ	gamma
δ	Δ	delta
ϵ		epsilon
ζ		zeta
η		eta
θ	Θ	theta
ι		iota
κ		kappa
λ	Λ	lambda
μ		mu
ν		nu
ξ	Ξ	xi (ksi)
o		omicron
π	Π	pi
ρ		rho
σ	Σ	sigma
τ		tau
υ	Υ	upsilon
ϕ	Φ	phi
χ		chi (khi)
ψ	Ψ	psi
ω	Ω	omega

world do not change their positions or motions. (In real situations, of course, each charge produces forces on all other charges in the neighborhood and may cause these other charges to move, and so in some cases the fields *can* change if we replace our particular charge by another.)

We know from Vol. I how to find the motion of a particle if we know the force on it. Equation (1.1) can be combined with the equation of motion to give

$$\frac{d}{dt}\left[\frac{mv}{(1 - v^2/c^2)^{1/2}}\right] = F = q(E + v \times B). \tag{1.2}$$

So if E and B are given, we can find the motions. Now we need to know how the E's and B's are produced.

One of the most important simplifying principles about the way the fields are produced is this: Suppose a number of charges moving in some manner would produce a field E_1, and another set of charges would produce E_2. If both sets of charges are in place at the same time (keeping the same locations and motions they had when considered separately), then the field produced is just the sum

$$E = E_1 + E_2. \tag{1.3}$$

This fact is called *the principle of superposition* of fields. It holds also for magnetic fields.

This principle means that if we know the law for the electric and magnetic fields produced by a *single* charge moving in an arbitrary way, then all the laws of electrodynamics are complete. If we want to know the force on charge A we need only calculate the E and B produced by each of the charges B, C, D, etc., and then add the E's and B's from all the charges to find the fields, and from them the forces acting on charge A. If it had only turned out that the field produced by a single charge was simple, this would be the neatest way to describe the laws of electrodynamics. We have already given a description of this law (Chapter 28, Vol. I) and it is, unfortunately, rather complicated.

It turns out that the form in which the laws of electrodynamics are simplest are not what you might expect. It is *not* simplest to give a formula for the force that one charge produces on another. It is true that when charges are standing still the Coulomb force law is simple, but when charges are moving about the relations are complicated by delays in time and by the effects of acceleration, among others. As a result, we do not wish to present electrodynamics only through the force laws between charges; we find it more convenient to consider another point of view—a point of view in which the laws of electrodynamics appear to be the most easily manageable.

1–2 Electric and magnetic fields

First, we must extend, somewhat, our ideas of the electric and magnetic vectors, E and B. We have defined them in terms of the forces that are felt by a charge. We wish now to speak of electric and magnetic fields *at a point* even when there is no charge present. We are saying, in effect, that since there are forces "acting on" the charge, there is still "something" there when the charge is removed. If a charge located at the point (x, y, z) at the time t feels the force F given by Eq. (1.1) we associate the vectors E and B with *the point* in space (x, y, z). We may think of $E(x, y, z, t)$ and $B(x, y, z, t)$ as giving the forces that *would be* experienced at the time t by a charge located at (x, y, z), *with the condition* that placing the charge there *did not disturb* the positions or motions of all the other charges responsible for the fields.

Following this idea, we associate with *every* point (x, y, z) in space two vectors E and B, which may be changing with time. The electric and magnetic fields are, then, viewed as *vector functions* of x, y, z, and t. Since a vector is specified by its components, each of the fields $E(x, y, z, t)$ and $B(x, y, z, t)$ represent three mathematical functions of x, y, z, and t.

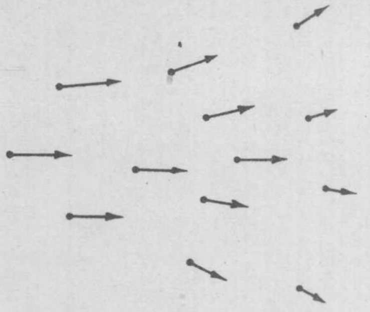

Fig. 1–1. A vector field may be represented by drawing a set of arrows whose magnitudes and directions indicate the values of the vector field at the points from which the arrows are drawn.

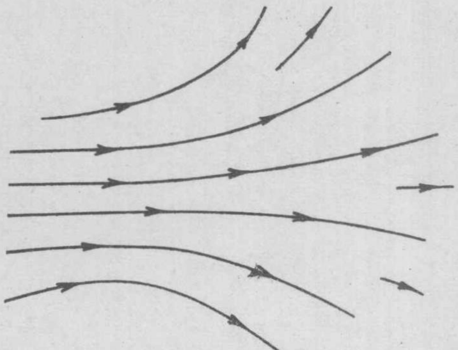

Fig. 1–2. A vector field can be represented by drawing lines which are tangent to the direction of the field vector at each point, and by drawing the density of lines proportional to the magnitude of the field vector.

It is precisely because $\boldsymbol{E}$ (or $\boldsymbol{B}$) can be specified at every point in space that it is called a "field." A "field" is any physical quantity which takes on different values at different points in space. Temperature, for example, is a field—in this case a scalar field, which we write as $T(x, y, z)$. The temperature could also vary in time, and we would say the temperature field is time-dependent, and write $T(x, y, z, t)$. Another example is the "velocity field" of a flowing liquid. We write $\boldsymbol{v}(x, y, z, t)$ for the velocity of the liquid at each point in space at the time t. It is a vector field.

Returning to the electromagnetic fields—although they are produced by charges according to complicated formulas, they have the following important characteristic: the relationships between the values of the fields at *one point* and the values at a *nearby point* are very simple. With only a few such relationships in the form of differential equations we can describe the fields completely. It is in terms of such equations that the laws of electrodynamics are most simply written.

There have been various inventions to help the mind visualize the behavior of fields. The most correct is also the most abstract: we simply consider the fields as mathematical functions of position and time. We can also attempt to get a mental picture of the field by drawing vectors at many points in space, each of which gives the field strength and direction at that point. Such a representation is shown in Fig. 1–1. We can go further, however, and draw lines which are everywhere tangent to the vectors—which, so to speak, follow the arrows and keep track of the direction of the field. When we do this we lose track of the *lengths* of the vectors, but we can keep track of the strength of the field by drawing the lines far apart when the field is weak and close together when it is strong. We adopt the convention that the *number of lines per unit area* at right angles to the lines is proportional to the *field strength*. This is, of course, only an approximation, and it will require, in general, that new lines sometimes start up in order to keep the number up to the strength of the field. The field of Fig. 1–1 is represented by field lines in Fig. 1–2.

1–3 Characteristics of vector fields

There are two mathematically important properties of a vector field which we will use in our description of the laws of electricity from the field point of view. Suppose we imagine a closed surface of some kind and ask whether we are losing "something" from the inside; that is, does the field have a quality of "outflow"? For instance, for a velocity field we might ask whether the velocity is always outward on the surface or, more generally, whether more fluid flows out (per unit time) than comes in. We call the net amount of fluid going out through the surface per unit time the "flux of velocity" through the surface. The flow through an element of a surface is just equal to the component of the velocity perpendicular to the surface times the area of the surface. For an arbitrary closed surface, the *net outward flow*—or *flux*—is the average outward normal component of the velocity, times the area of the surface:

$$\text{Flux} = (\text{average normal component})\cdot(\text{surface area}). \tag{1.4}$$

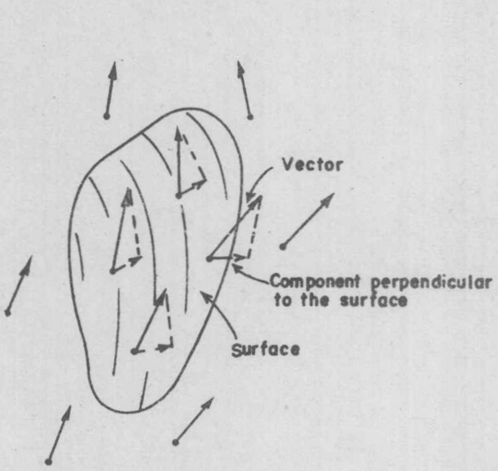

Fig. 1–3. The flux of a vector field through a surface is defined as the average value of the normal component of the vector times the area of the surface.

In the case of an electric field, we can mathematically define something analogous to an outflow, and we again call it the flux, but of course it is not the flow of any substance, because the electric field is not the velocity of anything. It turns out, however, that the mathematical quantity which is the average normal component of the field still has a useful significance. We speak, then, of the *electric flux*—also defined by Eq. (1.4). Finally, it is also useful to speak of the flux not only through a completely closed surface, but through any bounded surface. As before, the flux through such a surface is defined as the average normal component of a vector times the area of the surface. These ideas are illustrated in Fig. 1–3.

There is a second property of a vector field that has to do with a line, rather than a surface. Suppose again that we think of a velocity field that describes the flow of a liquid. We might ask this interesting question: Is the liquid circulating?

By that we mean: Is there a net rotational motion around some loop? Suppose that we instantaneously freeze the liquid everywhere except inside of a tube which is of uniform bore, and which goes in a loop that closes back on itself as in Fig. 1–4. Outside of the tube the liquid stops moving, but inside the tube it may keep on moving because of the momentum in the trapped liquid—that is, if there is more momentum heading one way around the tube than the other. We define a quantity called the *circulation* as the resulting speed of the liquid in the tube times its circumference. We can again extend our ideas and define the "circulation" for any vector field (even when there isn't anything moving). For any vector field the *circulation around any imagined closed curve* is defined as the average tangential component of the vector (in a consistent sense) multiplied by the circumference of the loop (Fig. 1–5).

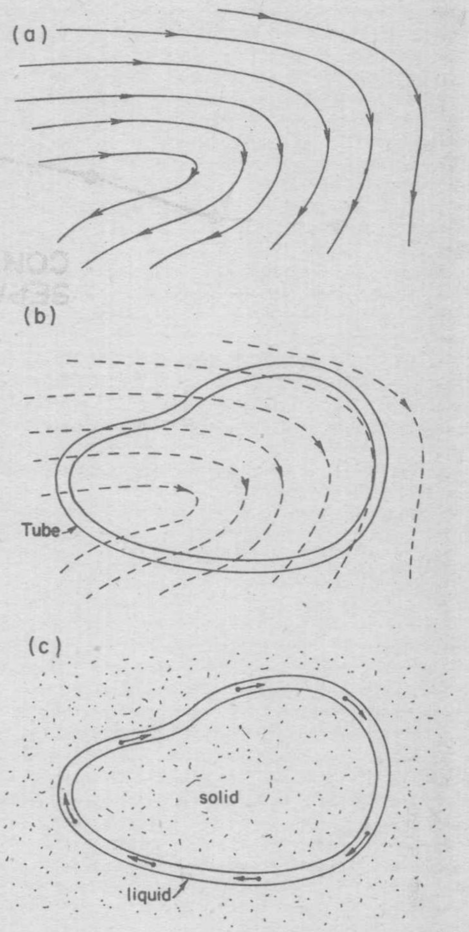

Fig. 1–4. (a) The velocity field in a liquid. Imagine a tube of uniform cross section that follows an arbitrary closed curve as in (b). If the liquid were suddenly frozen everywhere except inside the tube, the liquid in the tube would circulate as shown in (c).

$$\text{Circulation} = (\text{average tangential component})\cdot(\text{distance around}). \tag{1.5}$$

You will see that this definition does indeed give a number which is proportional to the circulation velocity in the quickly frozen tube described above.

With just these two ideas—flux and circulation—we can describe all the laws of electricity and magnetism at once. You may not understand the significance of the laws right away, but they will give you some idea of the way the physics of electromagnetism will be ultimately described.

1–4 The laws of electromagnetism

The first law of electromagnetism describes the flux of the electric field:

$$\text{The flux of } \boldsymbol{E} \text{ through any closed surface} = \frac{\text{the net charge inside}}{\epsilon_0}, \tag{1.6}$$

where ϵ_0 is a convenient constant. (The constant ϵ_0 is usually read as "epsilon-zero" or "epsilon-naught".) If there are no charges inside the surface, even though there are charges nearby outside the surface, the *average* normal component of $\boldsymbol{E}$ is zero, so there is no net flux through the surface. To show the power of this type of statement, we can show that Eq. (1.6) is the same as Coulomb's law, provided only that we also add the idea that the field from a single charge is spherically symmetric. For a point charge, we draw a sphere around the charge. Then the average normal component is just the value of the magnitude of $\boldsymbol{E}$ at any point, since the field must be directed radially and have the same strength for all points on the sphere. Our rule now says that the field at the surface of the sphere, times the area of the sphere—that is, the outgoing flux—is proportional to the charge inside. If we were to make the radius of the sphere bigger, the area would increase as the square of the radius. The average normal component of the electric field times that area must still be equal to the same charge inside, and so the field must decrease as the square of the distance—we get an "inverse square" field.

If we have an arbitrary stationary curve in space and measure the circulation of the electric field around the curve, we will find that it is not, in general, zero (although it is for the Coulomb field). Rather, for electricity there is a second law that states: for any surface S (not closed) whose edge is the curve C,

$$\text{Circulation of } \boldsymbol{E} \text{ around } C = \frac{d}{dt}(\text{flux of } \boldsymbol{B} \text{ through } S). \tag{1.7}$$

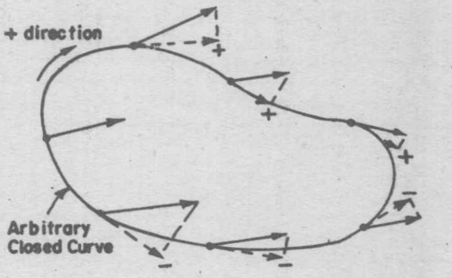

Fig. 1–5. The circulation of a vector field is the average tangential component of the vector (in a consistent sense) times the circumference of the loop.

We can complete the laws of the electromagnetic field by writing two corresponding equations for the magnetic field $\boldsymbol{B}$.

$$\text{Flux of } \boldsymbol{B} \text{ through any closed surface} = 0. \tag{1.8}$$

For a surface S bounded by the curve C,

$$c^2(\text{circulation of } \boldsymbol{B} \text{ around } C) = \frac{d}{dt}(\text{flux of } \boldsymbol{E} \text{ through } S) + \frac{\text{flux of electric current through } S}{\epsilon_0}. \tag{1.9}$$